

# Choice Theory – A Synopsis

14.123 Microeconomic Theory III  
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## Road map

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### 1. Basic Concepts:

- ▶ 1. Choice
- ▶ 2. Preference
- ▶ 3. Utility

### 2. Weak Axiom of Revealed Preferences

### 3. Preference as a representation of choice

### 4. Ordinal Utility Representation

### 5. Continuity



## Basic Concepts

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- ▶  $X$  = Set of Alternatives
  - ▶ Mutually exclusive
  - ▶ Exhaustive
- ▶  $A$  = non-empty set of available alternatives
- ▶ Choice Function:  $c : A \mapsto c(A) \subseteq A$ .
  - ▶  $c(A)$  is non-empty
- ▶ Preference: A relation  $\succsim$  on  $X$  that is
  - ▶ complete :  $\forall x, y \in X$ , either  $x \succsim y$  or  $y \succsim x$ ;
  - ▶ transitive :  $\forall x, y, z \in X$ , [ $x \succsim y$  and  $y \succsim z$ ]  $\Rightarrow x \succsim z$ .
- ▶ Utility Function:  $U : X \rightarrow \mathbb{R}$



## Choice Function

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- ▶  $c : A \mapsto c(A) \subseteq A$
- ▶ It describes what alternatives DM may choose under each set of constraints
- ▶ Feasibility:  $c(A) \subseteq A$ .
- ▶ Exhaustive:  $c(A)$  is non-empty
- ▶ Mutually exclusive: only one alternative is chosen



## Preference

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- ▶ Preference Relation: A relation  $\succsim$  on  $X$  s.t.
  - ▶ complete :  $\forall x, y \in X$ , either  $x \succsim y$  or  $y \succsim x$ ;
  - ▶ transitive :  $\forall x, y, z \in X$ ,  $[x \succsim y \text{ and } y \succsim z] \Rightarrow x \succsim z$ .
- ▶  $x \succsim y$  means: DM finds  $x$  at least as good as  $y$
- ▶ **Preferences do not depend on A!**
- ▶ Strict Preference:  $x \succ y \leftrightarrow [x \succsim y \text{ and not } y \succsim x]$
- ▶ Indifference:  $x \sim y \leftrightarrow [x \succsim y \text{ and } y \succsim x]$ .
- ▶ Choice induced by preference:

$$c_{\succsim}(A) = \{x \in A \mid x \succsim y \forall y \in A\}$$


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## Choice v. Preference

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**Definition:** A choice function  $c$  is represented by  $\succsim$  iff  $c = c_{\succsim}$ .

**Theorem:** Assume that  $X$  is finite. A choice function  $c$  is represented by some preference relation  $\succsim$  if and only if  $c$  satisfies WARP.

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## Weak Axiom of Revealed Preference

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**Axiom (WARP):** For all  $A, B \subseteq X$  and  $x, y \in A \cap B$ ,  
if  $x \in c(A)$  and  $y \in c(B)$ , then  $x \in c(B)$ .

- ▶ WARP: DM has well-defined preferences
  - ▶ That govern the choice
  - ▶ don't depend on the set  $A$  of feasible alternatives



## Ordinal Utility Representation

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**Ordinal Representation:**  $U : X \rightarrow \mathbb{R}$  is an ordinal representation of  $\succsim$  iff:

$$x \succsim y \Leftrightarrow U(x) \geq U(y) \quad \forall x, y \in X.$$

**Fact:** If  $U$  represents  $\succsim$  and  $f: \mathbb{R} \rightarrow \mathbb{R}$  is strictly increasing, then  $f \circ U$  represents  $\succsim$ .

**Theorem:** Assume  $X$  is finite (or countable). A relation has an ordinal representation if and only if it is complete and transitive.

**Example:** Lexicographic preference relation on unit square does not have an ordinal representation.



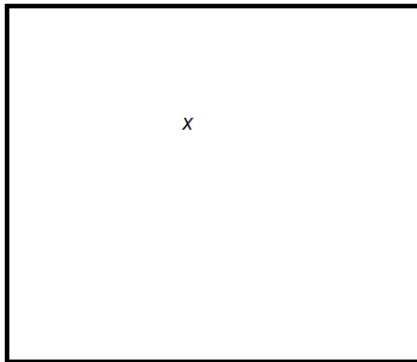
## Continuous Representation

**Definition:** A preference relation  $\succsim$  is said to be continuous iff  $\{y \mid y \succsim x\}$  and  $\{y \mid x \succsim y\}$  are closed for every  $x$  in  $X$ .

**Theorem:** Assume  $X$  is a compact, convex subset of a separable metric space. A preference relation has a continuous ordinal representation if and only if it is continuous.



## Indifference Sets of a Continuous Preference



- ▶  $I(x) = \{y \mid x \sim y\}$
- ▶  $I(x)$  is closed.
- ▶ If
  - ▶  $x' \succ x \succ x''$
  - ▶  $\phi: [0, 1] \rightarrow X$  continuous
  - ▶  $\phi(1) = x'$ ;  $\phi(0) = x''$ ,
- ▶ Then,  $\exists t \in [0, 1]$  such that  $\phi(t) \sim x$ .



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