14.123 Microeconomics III—Problem Set 3

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Instructions. You are encouraged to work in groups, but everybody must write their own solution to the problem that is for grade. Good Luck!

1. (For Grade) There are finitely many states $s \in S$. The set of outcomes is $[0, \infty)$, the amount of consumption. Consider an expected utility maximizer with utility function $u(c) = \sqrt{c}$. Suppose that for each state $s \in S$, there is an asset A_s that pays 1 unit of consumption if the state is s and 0 otherwise (these are called Arrow-Debreu securities). Suppose also that we know the preference of the decision maker among these assets and the constant consumtion levels; e.g., we know how he compares an asset A_s to consuming c at every state.

   (a) Derive the decision maker's preference relation among all acts from the above information.

   (b) Assume that the decision maker has a fixed amount of money M, which he cannot consume unless he invests in the Arrow-Debreu secuirites above, assuming that these secrities are perfectly divisible, and the price of a unit of A_s is some $p_s > 0$. Derive the demand of the decision maker for these securities as a function of the price vector $p = (p_s)_{s\in S}$.

2. Ann is an expected utility maximizer, but she does not know her preferences, which she can learn by costly contemplation. To model this situation, take $S = [0, 1]$, and let $Z \subseteq \mathbb{R}$ be a finite set of consequences with at leat two elements. Assume that Ann's von Neumann utility function is

$$u(z) = z \qquad \forall z \in Z,$$

   and her belief on S is represented by uniform distribution. For any n and some fixed $c > 0$, by spending cn utils, Ann can obtain a partition

$$P_n = \{[0, 1/2^n], (1/2^n, 2/2^n), \ldots, (k/2^n, (k+1)/2^n), \ldots, [(2^n - 1)/2^n, 1]\}$$

   and observe the cell $I_n(s) \in P_n$ in which the true state s lies. After the observation, she assigns uniform distribution on $I_n(s)$ and can choose an act $f : S \to Z$ under the new belief. Her eventual payoff is $u(f(s)) - cn$. Now imagine that, given any two acts f and g, Ann first chooses n and, after observing the cell in which s lies, she chooses one of the acts f and g. She does so in order to maximize her expected payoff minus the cost cn, knowing all along that she will choose one of the acts f and g optimally based on her observation. Write $f \succeq_s g$ if Ann may end up choosing f when the true state happens to be s. Check which of the postulates P1-P5 of Savage is satisfied by $\succeq_s$ for any fixed s.

3. Under the assumptions P1-P5, prove or disprove the following statements.

(a) For any partition $A_1, \ldots, A_n$ of S, and for any acts $f, g \in F$,

$$[f \succeq g \text{ given } A_k \text{ for all } A_k] \Rightarrow f \succeq g.$$

(b) If $A_1 \dot{\succeq} B_1$, $A_2 \dot{\succeq} B_2$, and $A_1 \cap A_2 = \varnothing$, then $A_1 \cup A_2 \dot{\succeq} B_1 \cup B_2$.

(c) For any given event D, define "$\dot{\succeq}$ given D" by $A \dot{\succeq} B$ given D iff $A \cap D \dot{\succeq} B \cap D$. The relation $\dot{\succeq}$ given D is a qualitative probability.