Lecture 11: Critiques of Expected Utility

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Expected Utility and Its Discontents

Expected utility (EU) is the workhorse model of choice under uncertainty.

From very early on, EU has been subject to several important critiques.

Today:

- Survey some of the most important critiques of EU.
- Describe some extensions/alternatives that have been developed to accommodate these critiques.

Just barely scratch surface of several large and active literatures in decision theory.

To learn more, take a decision theory class.
(Like Econ 2059 at Harvard.)

A Caveat: Positive vs. Normative Economic Theory

Economic theory is both **positive** (descriptive) and **normative** (prescriptive).

Normative response to critiques:
"Good thing we have decision theory to teach people not to make dumb mistakes."

Positive response to critiques:
"How can we extend the theory to accommodate more realistic behavior?"

Both responses can have value.

But more to say about extending theory.

Critique I: The Allais Paradox

Which lottery would you choose?

A: Win 1 million dollars for sure.

B: Win 5 million dollars with probability 10%, 1 million dollars with 89%, nothing with 1%.

The Allais Paradox

Which lottery would you choose?

C: Win 1 million dollars with probability 11%, nothing with 89%.

D: Win 5 million dollars with probability 10%, nothing with 90%.

The Allais Paradox

A: 1M for sure.
B: 5M wp 10%, 1M wp 89%, 0 wp 1%.

C: 1M wp 11%, 0 wp 89%.
D: 5M wp 10%, 0 wp 90%.

Most subjects both choose A in first problem, D in second problem.

The paradox: **this is inconsistent with EU.**

Normalize $u(0)$ to 0.
Choose A iff

$$\begin{aligned} u(1) &\geq 0.1u(5) + 0.89u(1) \\ &\iff \\ 0.11u(1) &\geq 0.1u(5) \end{aligned}$$

This is condition for choosing **C**.

Choosing A from (A,B) or choosing C from (C,D) amounts to preferring 1M for sure to a $\frac{10}{11}$ chance of 5M and a $\frac{1}{11}$ chance of 0.

Rank-Dependent Expected Utility

How to extend EU to accommodate Allais-type behavior?

One way: **rank-dependent EU**.

1. Rank consequences in order of utility
2. Apply an increasing **probability weighting function** $w : [0, 1] \to [0, 1]$ to cdf F to form new cdf $w \circ F$.
3. Choose to maximize

$$U(F) = \int u(x)\, dw(F(x)).$$

Ex. w inverse S-shaped $\implies$ decision-maker overweights probabilities of unlikely events with extreme values

Rank-Dependent EU and Allais

Normalize $u(0)$ to 0. Then:

$$\begin{aligned}
U(A) &= u(1) \\
U(B) &= [w(0.9) - w(0.01)]\, u(1) + [1 - w(0.9)]\, u(5) \\
U(C) &= [1 - w(.89)]\, u(1) \\
U(D) &= [1 - w(0.9)]\, u(5)
\end{aligned}$$

Rank-Dependent EU and Allais

Suppose w approaches extreme inverse-S shape, given by

$$\begin{aligned} w(0) &= 0 \\ w(p) &= 0.5 \text{ for all } p \in (0,1) \\ w(p) &= 1 \end{aligned}$$

Then:

$$\begin{aligned} u(A) &\to u(1) \\ u(B) &\to 0.5u(5) \\ u(C) &\to 0.5u(1) \\ u(D) &\to 0.5u(5) \end{aligned}$$

Rationalizes Allais behavior if $u(5) \in (u(1), 2u(1))$.

Can also rationalize extreme risk-loving behavior like buying lottery tickets.

Critique II: The Ellsberg Paradox

An urn contains 300 balls.
Exactly 100 of the balls are red.
The other 200 are some unknown mix of blue and green.

A ball is randomly drawn from the urn.
You choose a color.

If the ball is of the color you choose, you win $100.
What color would you choose?

If the ball is **not** of the color you choose, you win $100.
What color would you choose?

The Ellsberg Paradox

100 red balls.
200 blue or green balls.

The paradox:
If asked to bet which color came up, subjects say red.
If asked to bet which color didn't come up, subjects say red.

This is inconsistent with EU:
if subject believes that ball is red, blue, and green with probabilities p_R, p_B, and p_G, then strict preference for red in first question means

$$p_R > p_B, p_G$$

but strict preference for red in second question means

$$p_R < p_B, p_G$$

Risk vs. Ambiguity

Old idea in economics: there's a difference between "quantifiable uncertainty" (**risk**) and "unquantifiable uncertainty" (**ambiguity**).

The Ellsberg paradox is often cited as support for this idea.

This interpretation is **controversial**.

Nonetheless, has spawn large literature on developing, testing, and applying models of ambiguity and ambiguity-aversion.

Most famous model: **maxmin expected utility** (Gilboa and Schmeidler, 1989).

Maxmin Expected Utility

1. Decision-maker has **set** of beliefs P.
2. Chooses action a to maximize expected utility according to **worst-case** belief

$$U(a) = \min_{p \in P} E_p [u(a)]$$

According to standard definitions, this decision-maker is **irrational**: she acts as if her behavior affects the underlying probability distribution, which is exogenously given.

Maxmin Expected Utility and Ellsberg

Suppose P is set of beliefs where $p_R = 1/3$, p_B and p_G can be anything between 0 and 2/3 (that add to 2/3).

For first question:

$$\begin{aligned} U(R) &= \min_{p\in P} p_R = \frac{1}{3} \\ U(B) &= \min_{p\in P} p_B = 0 \\ U(G) &= \min_{p\in P} p_G = 0 \end{aligned}$$

Choose red.

Maxmin Expected Utility and Ellsberg

For second question:

$$
\begin{aligned}
U(R) &= \min_{p \in P} (1 - p_R) = \frac{2}{3} \\
U(B) &= \min_{p \in P} (1 - p_B) = \frac{1}{3} \\
U(G) &= \min_{p \in P} (1 - p_G) = \frac{1}{3}
\end{aligned}
$$

Choose red again.

Decision-maker acts as if choice affects number of blue and green balls, which is impossible.

There are other explanations of the Ellsberg paradox that are arguably closer to expected utility.

Critique III: The Rabin Calibration Paradox

Under EU, decision-maker is approximately risk-neutral for small gambles
(unless kink in u, which happens almost nowhere if u concave).

Ex. always wants to invest some money in any positive expected value asset.

Implication: if decision-maker rejects a small positive expected value gamble, she must be **extremely** risk-averse.

If think through examples, **plausible risk-aversion at small stakes implies implausible risk-aversion at large stakes.**

The Rabin Calibration Paradox: CARA Example

Suppose decision-maker has CARA preferences with risk-aversion coefficient α:

$$u(x) = -\exp(-\alpha x)$$

Suppose rejects gamble that gives +\$100 wp 0.6, -\$100 wp 0.4

Seems reasonable.

But implies:

$$-p\exp(-100\alpha) - (1-p)\exp(100\alpha) \geq -1$$

$$\iff$$

$$\alpha \geq \frac{1}{100}\log\frac{p}{1-p} = \frac{1}{100}\log\frac{3}{2}$$

The Rabin Calibration Paradox: CARA Example

If $\alpha \geq \frac{1}{100} \log \frac{3}{2}$, then expected utility from gamble that gives -\$200 wp 0.5 and **any** gain wp 0.5 is at most

$$\begin{aligned} & -\frac{1}{2} \exp(-\infty) - \frac{1}{2} \exp(200\alpha) \\ \leq \quad & -\frac{1}{2} \exp\left(2 \log \frac{3}{2}\right) \\ = \quad & -\frac{9}{8} \\ < \quad & -1 = -\exp(0) \end{aligned}$$

Decision-maker must reject -\$200 wp 0.5, $+\$\infty$ wp 0.5.

This is absurd.

he Rabin Calibration Paradox: Other Examples

Is the problem CARA utility?

No: for **any** concave function, if decision-maker rejects small gambles over a fairly small range of wealth levels, imposes enough curvature on utility function to force absurd risk-aversion over large gambles.

- Suppose at any initial wealth level decision-maker rejects 50-50 bet on -\$100, $+$\$110.
  Then must reject 50-50 bet on \$-1,000, $+\$\infty$.
- Suppose at any initial wealth level below \$350,000, decision-maker rejects 50-50 bet on -\$100, $+$\$105.
  Then at initial wealth \$340,000, must reject 50-50 bet on -\$4,000, $+$\$635,000.

Rabin Paradox: Resolution?

Rabin argues EU bad explanation for risk-aversion over small stakes.

One resolution: maybe people assign extra weight to small losses.

Ex. For 50-50 bet on -\$100, +\$105, losses already look pretty bad. Pain doesn't scale up to bet on -\$4,000, +635,000.

This **loss-aversion** makes people act as if always at kink.

This asymmetry between gains and losses a feature of **prospect theory**.

Prospect Theory

Decision-maker has a **reference point** x_0, weights gains and losses relative to x_0 differently.

Has reference-dependent utility function of form

$$u(x|x_0) = v(x - x_0)$$

where v satisfies:

- Concavity on $\mathbb{R}_+$ (risk-aversion toward gains)
- Convexity on $\mathbb{R}_-$ (risk-loving toward losses)
- Kink at 0 (loss-aversion)

Can also combine with probability weighting function as in rank-dependent EU.

Prospect Theory, the Rabin Paradox, and Framing Effects

Prospect theory can accommodate Rabin behavior, as kink allows rejection of small, positive expected value gambles.

Reference point also lets prospect theory accommodate **framing effects**, where behavior depends on how choice is presented.

But in practice can be hard to determine what reference point should be.

Critiques of EU: Summary

Allais paradox suggests people may overweight extreme probabilities of small events
Can model with rank-dependent EU.

Ellsberg paradox suggests people may be averse to ambiguity.
Can model with maxmin EU.

Rabin paradox suggests people may overweight small losses.
Can model with prospect theory.

All of this just scratches the surface of decision theory/behavioral economics.