Lecture 5
Rationalizability

14.12 Game Theory

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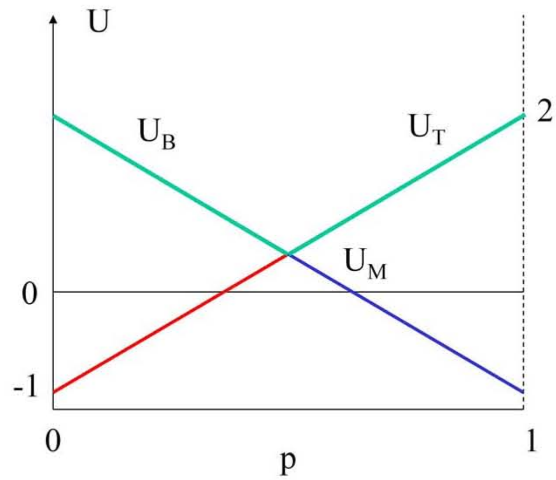
Recall: A Game

$U_M = 0$

$U_T = 2p-(1-p) = 3p-1$

$U_B = -p+2\ (1-p) = 2-3p$

	L	R
T	(2,0)	(-1,1)
M	(0,10)	(0,0)
B	(-1,-6)	(2,0)
	p	1-p



Recap: Rationality & Dominance

- Belief: A probability distribution p_{-i} on **others'** strategies;
- Mixed Strategy: A probability distribution σ_i on **own** strategies;
- Playing s_i^* is rational ⇔ s_i^* is a best response to a belief p_{-i}: $\forall s_i$

$$\sum_{s_{-i}} u_i(s_i^*, s_{-i}) p_{-i}(s_{-i}) \geq \sum_{s_{-i}} u_i(s_i, s_{-i}) p_{-i}(s_{-i})$$

- σ_i dominates s_i^{**} ⇔ $\forall s_{-i}$

$$\sum_{s_i} u_i(s_i, s_{-i}) \sigma_i(s_i) > u_i(s_i^{**}, s_{-i})$$

- Theorem: Playing s_i^* is rational ⇔ s_i^* is not dominated.

Assume

	L	R
T	(2,0)	(-1,1)
M	(0,10)	(0,0)
B	(-1,-6)	(2,0)

Player 1 is rational

Player 2 is rational

Player 2 is rational and
Knows that Player 1 is rational

Player 1 is rational,
knows that 2 is rational
knows that 2 knows that
1 is rational

Assume

1 \ 2	L	m	R
T	(3,0)	(1,1)	(0,3)
M	(1,0)	(0,10)	(1,0)
B	(0,3)	(1,1)	(3,0)

P1 is rational

P2 is rational and
knows that P1 is rational

P1 is rational and
knows all these

Rationalizability

The play is rationalizable, provided that …

Important

- Eliminate only the **strictly** dominated strategies
  - Ignore weak dominance
- Make sure to eliminate the strategies dominated by mixed strategies as well as pure

Beauty Contest

- There are n students.
- Simultaneously, each student submits a number x_i between 0 and 100.
- The payoff of student *i* is $100 - (x_i - 2\bar{x}/3)^2$ where

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}.$$

Rationalizability in Beauty Contest

If X_{-i} = Expected value of sum of x_j with $j \neq i$, best strategy is $(2/3)X_{-i}/(n-2/3)$

After Round 1:

$$\left[0, \frac{2}{3}\frac{n-1}{n-2/3}100\right]$$

After Round 2:

$$\left[0, \left(\frac{2}{3}\frac{n-1}{n-2/3}\right)^2 100\right]$$

After Round *k*:

$$\left[0, \left(\frac{2}{3}\frac{n-1}{n-2/3}\right)^k 100\right]$$

Rationalizability = {0}.

with m mischievous students

Payoff for mischievous: $(x_i - 2x/3)^2$

Round 1: only 0 and 100 survive for mischievous; same as before for normal

Rounds 2 to $k(m,n)$-1**:** no elimination for mischievous; same as before for normal

Round $k(m,n)$**:** eliminate 0 for mischievous; same as before for normal

Round $k > k(m,n)$**:**

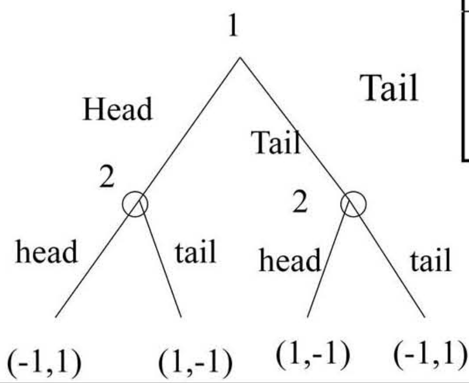
- Strategies for normal after round $k = [L_k, H_k]$

$$L_k = \frac{2}{3}\frac{100m + (n-m-1)L_{k-1}}{n-2/3} \qquad H_k = \frac{2}{3}\frac{100m + (n-m-1)H_{k-1}}{n-2/3}$$

Ratinalizability = mischievous 100, normal 200m/(n+2m)

Matching pennies with perfect information

1 \ 2	HH	HT	TH	TT
Head	(-1,1)	(-1,1)	(1,-1)	(1,-1)
Tail	(1,-1)	(-1,1)	(1,-1)	(-1,1)



A summary

- If players are rational and cautious, they play the dominant-strategy equilibrium whenever it exists
  - But, typically, it does not exist
- If rationality is common knowledge, a rationalizable strategy is played
  - Typically, there are too many rationalizable strategies
- **Nash Equilibrium**: the players correctly guess the other players' strategies (or conjectures).

14.12 Economic Applications of Game Theory
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