14.12 Game Theory
Muhamet Yildiz
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Homework 5

Due on 12/7/2012 (Recitation)

1. Alice and Bob have inherited a factory from their parents. The value of the factory is v_A for Alice and v_B for Bob, where v_A and v_B are independently and uniformly distributed over $[0,1]$, and each of them knows his or her own value. Simultaneously, Alice and Bob bid b_A and b_B, respectively, and the highest bidder wins the factory and pays the other sibling's bid. (If the bids are equal, the winner is determined by a coin toss.) Find a symmetric, linear Bayesian Nash equilibrium of this game.

2. An object is to be sold to one of n bidders through a first-price auction. The value of object for bidder i is $v_i \geq 0$, which is privately known by the bidder i. The values $(v_1, \ldots, v_n)$ are independently and identically distributed with probability density function f and cumulative distribution function F. Write this formally as a Bayesian game and compute the symmetric Bayesian Nash equilibria in increasing differentiable strategies.

3. There are n hunters. Simultaneously each hunter chooses between Stag and Rabbit. If a hunter chooses Rabbit, his payoff is θ regardless of what other hunters do. If he chooses Stag, he gets $v > 0$ if at least k other hunters also choose Stag and gets 0 otherwise. The possible values for θ is $[a, b]$ where $a < 0$ and $b > v$.

   (a) Assume that θ is known and compute the set of pure strategy Nash equilibria.

   (b) Assume that θ is not known. Each player i observes a signal

   $$x_i = \theta + \varepsilon\eta_i$$

   where $0 < \varepsilon \ll \min\{-a, b - v\}$ and $(\eta_1, \ldots, \eta_n)$ are independently and identically distributed with uniform distribution on $[-1, 1]$. Find the set of symmetric, monotone Bayesian Nash equilibria. (That is, find the possible values for a cutoff x^* such that each player i chooses Stag if and only if $x_i \leq x^*$ in equilibrium.)

   Hint: By symmetry, the probability that x_i is the mth highest among $(x_1, \ldots, x_n)$ is $1/n$.

   (c) Briefly discuss your answers.

4. Alice owns a car, and Bob may buy it. The car can be a Lemon or a Peach. Alice knows whether it is Lemon or Peach, while Bob assigns probability 1/2 to each case. The value of a Lemon is \$1,000 for Alice and \$3,000 for Bob, and the value of a Peach is \$5,000 for Alice and \$10,000 for Bob. Alice sets a price $p \in P$, and Bob decides whether to accept the price. If the price is accepted, they trade the car at price p; no trade occurs otherwise. For each set P below find a sequential equilibrium and verify that it is indeed a sequential equilibrium.

   (a) $P = \{2000, 7000\}$

   (b) $P = [0, \infty)$.

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14.12 Economic Applications of Game Theory

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