14.06 Midterm

Spring 2004

You have 1.5 hours. You must answer all questions. No books or notes are allowed. Total points: 100. Answer question 1 in either continuous or discrete time. Good luck!

Question 1 (Continuous Time)

Consider the neoclassical growth model, in *continuous* time. Assume that there is no population growth and no exogenous technological change; and normalize the size of the population and labor to 1. We examine *decentralized* competitive allocations.

We introduce a government in the model. The only role of the government is to redistribute income (there is no government spending). The government imposes a proportional tax on household income and uses the collected taxes to finance a lump-sum transfer to each household. Denote the time-invariant tax rate with $\tau \in [0, 1)$ and the transfer with T_t.

The budget of the representative household is thus

$$c_t + i_t = (1 - \tau)(r_t k_t + w_t) + T_t, \tag{1}$$

where c_t denotes consumption, i_t investment, k_t capital, r_t the interest rate, and w_t the wage rate. The capital stock accumulates according to

$$\dot{k}_t = i_t - \delta k_t, \tag{2}$$

where $\delta \in (0, 1)$. The household's lifetime utility is given by

$$\mathcal{U} = \int_0^\infty e^{-\rho t} u(c_t) dt, \quad u(c) = \frac{c^{1-1/\theta}}{1 - 1/\theta}, \tag{3}$$

where $\rho > 0$ is the discount rate and $\theta > 0$ is the elasticity of intertemporal substitution. The household maximizes (3) subject to (1) and (2), taking w_t and r_t, as well as T_t and τ, as given.

Firms are perfectly competitive and maximize profits. The technology is Cobb-Douglas. Output at time t is thus

$$y_t = f(k_t) = k_t^\alpha$$

where k_t denotes the capital stock, y_t denotes output or income, and $\alpha \in (0, 1)$. Note that, in equilibrium,

$$y_t = r_t k_t + w_t.$$

Finally, the government budget requires that the tax revenue covers the cost of the transfer, so that

$$T_t = \tau \left(r_t k_t + w_t \right) = \tau y_t.$$

(a) Derive the Euler condition that characterizes the optimal consumption behavior of households? How does consumption growth depends on the tax rate τ? Interpret.   `15 points`

(b) What is r_t in equilibrium? Does a higher k_t increase or reduce r_t? Interpret.   `10 points`

(c) Consider now the general equilibrium. Write down the system of two first-order differential equations that characterizes the dynamics of consumption and capital. (*Remark:* These two conditions are the Euler condition and the resource constraints, expressed in such a way that the system depends only on $c, k, \dot{c}, \dot{k}$, and the exogenous parameters, namely $\tau, \theta, \alpha, \delta$, and ρ.)   `10 points`

(d) Solve for the steady-state value of capital. How does it depend on τ? Interpret the effect of the tax rate on capital accumulation.   `10 points`

(e) Draw the phase diagram (that is, the $\dot{c} = 0$ and $\dot{k} = 0$ loci together with the saddle path) for two cases: $\tau = \tau_{high}$ and $\tau = \tau_{low}$, where $\tau_{low} < \tau_{high}$. Suppose the economy has been forever in the steady state with $\tau = \tau_{high} > 0$. Suddenly and unexpectedly, at some time $t = t_0$, the government reduces the tax to τ_{low}. Following the tax cut, τ is expected to stay at τ_{low} forever. How does consumption respond at $t = t_0$? (Does it rise, fall, or is it ambiguous?) Give the intuition. (Explain the associated substitution and wealth (or income) effects.) Finally, how does capital and output evolve over time following the tax cut?   `15 points`

Question 1 (Discrete Time)

Consider the neoclassical growth model, in *discrete* time. Assume that there is no population growth and no exogenous technological change; and normalize the size of the population and labor to 1. We examine *decentralized* competitive allocations.

We introduce a government in the model. The only role of the government is to redistribute income (there is no government spending). The government imposes a proportional tax on household income and uses the collected taxes to finance a lump-sum transfer to each household. Denote the time-invariant tax rate with $\tau \in [0, 1)$ and the transfer with T_t.

The budget of the representative household is thus

$$c_t + i_t = (1 - \tau) \left(r_t k_t + w_t \right) + T_t, \tag{4}$$

where c_t denotes consumption, i_t investment, k_t capital, r_t the interest rate, and w_t the wage rate. The capital stock accumulates according to

$$k_{t+1} - k_t = i_t - \delta k_t, \tag{5}$$

where $\delta \in (0,1)$. The household's lifetime utility is given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c) = \frac{c^{1-1/\theta}}{1 - 1/\theta}, \tag{6}$$

where β is the discount factor ($\beta = \frac{1}{1+\rho}$, where $\rho > 0$ is the discount rate) and $\theta > 0$ is the elasticity of intertemporal substitution. The household maximizes (6) subject to (4) and (5), taking w_t and r_t, as well as T_t and τ, as given.

Firms are perfectly competitive and maximize profits. The technology is Cobb-Douglas. Output at time t is thus

$$y_t = f(k_t) = k_t^\alpha$$

where k_t denotes the capital stock, y_t denotes output or income, and $\alpha \in (0,1)$. Note that, in equilibrium,

$$y_t = r_t k_t + w_t.$$

Finally, the government budget requires that the tax revenue covers the cost of the transfer, so that

$$T_t = \tau (r_t k_t + w_t) = \tau y_t.$$

(a) Derive the Euler condition that characterizes the optimal consumption behavior of households? How does consumption growth depends on the tax rate τ? Interpret.   `15 points`

(b) What is r_t in equilibrium? Does a higher k_t increases or reduces r_t? Interpret.   `10 points`

(c) Consider now the general equilibrium. Write down the system of two first-order difference equations that characterizes the dynamics of consumption and capital. (*Remark:* These two conditions are the Euler condition and the resource constraints, expressed in such a way that the system depends only on $c_t, c_{t+1}, k_t, k_{t+1}$ and the exogenous parameters, namely $\tau, \theta, \alpha, \delta$, and β or ρ.)   `10 points`

(d) Solve for the steady-state value of capital. How does it depend on τ? Interpret the effect of the tax rate on capital accumulation.   `10 points`

(e) Now consider the phase diagram, as if time were continuous. Draw the $\dot{c} = 0$ and $\dot{k} = 0$ loci together with the saddle path for two cases: $\tau = \tau_{high}$ and $\tau = \tau_{low}$, where $\tau_{low} < \tau_{high}$.

Suppose the economy has been forever in the steady state with $\tau = \tau_{high} > 0$. Suddenly and unexpectedly, at some time $t = t_0$, the government reduces the tax to τ_{low}. Following the tax cut, τ is expected to stay at τ_{low} forever. How does consumption respond at $t = t_0$? (Does it rise, fall, or is it ambiguous?) Give the intuition. (Explain the associated substitution and wealth (or income) effects.) Finally, how does capital and output evolve over time following the tax cut?
15 points

Question 2

(a) Can the Solow growth model help us explain the cross-country differences in income levels? Can it also help us explain the cross-country differences in saving rates? 10 points

(b) In the context of the Ramsey model, can cross-country differences in tax policies help explain the cross-country differences in income levels and/or saving rates? (Hint: The results of Question 1 should help you answer this question.) 10 points

(c) Consider the Ramsey model with exogenously fixed labor. Briefly describe the response of aggregate consumption, investment, and output to a permanent increase in the level of TFP. Briefly give the intuition. 20 points

Good Luck!