Problem Set #4

Course 14.06 – Intermediate Applied Macroeconomics

Distributed: April 7, 2004
Due: Thursday, April 14, 2004 [in class]

1. **Learning by Doing Model**

   This question is based on the Learning by Doing model discussed in the class notes and Romer's textbook (page 120). The economy is described as follows:

   $$u(c_t) = \frac{c_t^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}}$$

   $$Y_t = K_t^{\alpha}(h_t L_t)^{1-\alpha}, \qquad k_{t+1} = (1-\delta)k_t + i_t, \qquad h_t = bk_t, \ b > 0, \qquad L_t = L \ \forall t$$

   (a) Describe how human capital, as given by h_t, accumulates in this economy. Do firms or individuals directly invest in improving the level of human capital? Or, is it simply a side effect of physical capital accumulation? Given your answer, do you believe that the social planner and decentralized competitive equilibrium will coincide in this model?

   (b) Write out and solve the social planner's problem in this economy. *[Assume that the discount factor is $\beta \in (0,1)$]*

       a. What is the growth rate of consumption in the economy?
       b. What is the optimal savings rate? *(Assume a linear savings function as done in the class notes)*

   (c) Solve for the competitive equilibrium in this economy. (Solve the model in the same fashion as done in the first half of the course. For simplicity, assume that the labor supply of individuals is exogenous and equal to 1. Moreover, assume that the borrowing constraint is never binding.)

       a. Prove the household budget constraint can be written as:
   $$c_t + k_{t+1} + b_{t+1} = w_t h_t + (1 + r_t - \delta)k_t + (1 + R_t)b_t$$
           where b_t are government bonds that have return R_t
       b. What is the arbitrage condition between risk-free bonds and capital?
       c. Use the FOCs of the household to find the Euler condition.
       d. Using the firm's profit maximizing behavior, what is the equilibrium interest rate for physical capital and the equilibrium wage rate? (Remember, the firm takes human capital as exogenous).
       e. What is the growth rate of consumption?

   (d) Compare the growth rate of the social planner's equilibrium from Part (b) to that of the decentralized equilibrium in Part (c). How are they different, and why?

(e) Now suppose that the government subsidizes the private cost of capital for firms. In particular, assume that the private cost of capital for firms is now given by $(1-\tau)r_t$ and the government pays for this subsidy via lump sum taxes, T, on individuals. Resolve for the competitive equilibrium of this economy.

      a. What is the subsidy, τ, the government should set such that the competitive equilibrium growth rate of consumption coincides with the social planner's outcome we found in Part (a)?

      b. Why does the government want to subsidize investment in this model?

2. Taxes in the Human Capital Model

This question will add taxes to the human capital model discussed in the notes. Specifically, assume the following conditions for the economy:

$$u(c_t) = \frac{c_t^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}}$$

$$Y_t = BK_t^{\alpha}(h_t L_t)^{1-\beta}, \qquad B>0, \qquad L_t = L \;\; \forall t$$

$$k_{t+1} = (1-\delta)k_t + i_t^k$$

$$h_{t+1} = (1-\delta)h_t + i_t^h$$

(a) What must I assume about the variables α and β in order to have constant returns to scale (CRS) for physical capital and human capital? Why do we want to make this type of assumption?

For the remainder of this question, assume CRS as done in Part (a).

(b) Solve the social planner's problem in this economy.

    1. What is the optimal k/h ratio?
    2. What is the equilibrium growth rate of consumption?

(c) Solve for the decentralized competitive equilibrium: Assume that labor supply is exogenous, and that the borrowing constraint is never binding.

    1. Prove that the household budget constraint can be written as:
$$c_t + k_{t+1} + h_{t+1} + b_{t+1} = (1+w_t-\delta)h_t + (1+r_t-\delta)k_t + (1+R_t)b_t$$
    2. What is the arbitrage condition of the economy?
    3. Use the arbitrage condition to show that the household's budget constraint can be written as: $c_t + a_{t+1} = (1+R_t)a_t$, where $a_t = b_t + k_t + h_t$, and use the FOCs to find the Euler Condition.
    4. Using the firm's profit maximizing behavior, what are the equilibrium interest rate and wage rate (for human capital)?
    5. What is the optimal k/h ratio?
    6. What is the growth rate of consumption?

(d) Does the consumption growth in the competitive equilibrium from Part (c) match the social planner's optimal consumption growth rate you found in Part (b)? Why are they different or the same? How is this model different than the learning by doing model in Question #1?

(e) Assume the government decides to tax a firm's output in this economy. In particular, assume the government takes a fraction τ^y of all firms' output. The government then balances its budget by returning the tax as a lump sum payment T_t to individuals. Resolve for the decentralized equilibrium as done in Part (c).

      1. Is the optimal ratio of k/h affected? Why or why not?.
      2. How does the equilibrium return to physical capital change?
      3. How does the tax affect the growth of consumption?

(f) Now assume that the government taxes the private return of human capital for individuals. In other words, the private return is now given by $(1-\tau^h)w_t$ where $\tau^h > 0$. The government returns the tax as a lump sum subsidy to individuals. Resolve for the decentralized equilibrium.

      1. Is the optimal ratio of k/h affected now? Why or why not?
      2. How does the tax affect the equilibrium prices for capital and the growth of consumption?