Problem Set 3
14.06 2004

Question 1: **Numerical Dynamic Programming. Submit your code and the requested plots.**

Consider the planner's problem in a basic ramsey model. There is no population growth. The planner's objective is to maximize

$$\sum_{t=0}^{\infty} \beta^t \ln (C_t)$$

capital evolves according to

$$K_{t+1} = (1-\delta) K_t + I_t \tag{1}$$

and the resource constraint of the economy requires that

$$C_t + I_t \leq Y_t = K_t^{\alpha} \tag{2}$$

You can assume (without loss of generality) that the resource constraint will always bind with equality in the planner's solution. Our task is to use the method of numerical dynamic programming to characterize the planner's solution to this problem. Observe that K_t is the state variable and the control variables are C_t and I_t. The bellman equaltion can then be written as

$$V(K_t) = \max_{C_t, I_t} \{\ln (C_t) + \beta V (K_{t+1})\} \tag{3}$$

subject to (1) and (2).

Use the following paramater values:

$$\alpha = 0.33, \beta = 0.95, \delta = 0.05$$

a) First, show (on paper) that the bellman equation can be written as

$$V(K_t) = \max_{K_{t+1}} \{\ln ((1-\delta) K_t + K_t^{\alpha} - K_{t+1}) + \beta V (K_{t+1})\}.$$

b) Using numerical dynamic programming, solve for the value function. You are looking to find a vector of values which give the value of V at different levels of K_t. Plot and print the value function (make sure you plot V against K_t). HINTS: Solve this by understanding and editing the example MATLAB code that I have posted on the Pset page. Now the state variable is capital instead of wealth. The range of the state variable should be strictly positive (pick the minimum to be very slightly above zero to avoid finding the origin as a solution). The most important thing that you need to change is the line where I evaluate the value function since the resource constraints here are different to those for the consumer's problem. Start with a coarse grid then when you have the program working run it with a finer grid.

c) Plot and print the policy function. For the problem, as we have re-formulated it, this means finding the function $K_{t+1}(K_t)$. Again make sure you get the axis right. [HINT: This should be easy since the code you have is already written calculate a policy function of $W_{t+1}(W_t)$.] Check that the policy function crosses the 45 degree line - if it does not then you should expand the range over which you evaualte the state variable. Explain why.

d) From your plot in part c) approximate what the steady state level of capital is.

e) Using the policy function in $K_{t+1}(K_t)$ combined with (1) and (2) find the optimal policy functions for consumption $C_t(K_t)$ and investment $I_t(K_t)$. Using these, approximate the steady state level of consumption and investment.

f) If you have made it this far you have already characterized the saddle path of the system! What is it? Suppose that the discount factor decreases from 0.95 to 0.85. How does this affect the saddle path? Show this by plotting the two saddle paths and comparing. How does the steady state level of capital and consumption change? [HINT: answer this by re-running your program with the new paramater value].