Preferences: $\mathbf{x} \succ \mathbf{y}$

Complete, Reflexive
Transitive, Continuous

Utility Function

U(**x**) > U(**y**) iff $\mathbf{x} \succ$ y

Consumption Set: {**x**,**y**,**z**,}

Budget Set

$B(p,m) = \{\mathbf{x} \in X \mid p \cdot x \leq m\}$

Budget Exhaustion
Weak Monotinicity + Strong Convexity

Consumer Maximization Problem

$\text{Max}_{x_1, x_2} U(x_1, x_2)$

Subject to: $p_1 x_1 + p_2 x_2 = m$

Monotonic Transformations

Utility Function (Ordinal)

$U(x) = \min_p V(p,m) \text{ st } x \cdot p = m$

Utility Maximization

$V(p,m) = \max_x U(x) \; x \cdot p = m$

Marshallian Demand Function

$$x_i(p,m) = \frac{\dfrac{\partial v(p,m)}{\partial p_i}}{\dfrac{\partial v(p,m)}{\partial m}}$$

$V(p,m)$: Homogenous of Degree 0,
Continuous, non increasing in P

Expenditure Minimization

$e(p,u) = \min \; x \cdot p \text{ st } U(x) > u$

Hicksian Demand Function $h_i(p,u) = \dfrac{\partial e(p,u)}{\partial p_i}$

$e(p,u)$: Homogeneous of degree 1 in p
Continuous, nondecreasing in p

$h_i(p,u)$: Homogeneous of degree 0 in p

$$e(p, v(p,m)) = m$$
$$v(p, e(p,u)) = u$$
$$x_i(p, e(p,u)) = h_i(p,u)$$
$$h_i(p, v(p,m)) = x_i(p,m)$$

Matrix Symmetry

$$\begin{bmatrix} \frac{\partial e(p,u)}{\partial p_1 \partial p_1} & \frac{\partial e(p,u)}{\partial p_1 \partial p_2} \\ \frac{\partial e(p,u)}{\partial p_2 \partial p_1} & \frac{\partial e(p,u)}{\partial p_2 \partial p_2} \end{bmatrix} \textit{is Neg SemiDef}:$$

$$\begin{bmatrix} \frac{\partial e(p,u)}{\partial p_1 \partial p_1} & \frac{\partial e(p,u)}{\partial p_1 \partial p_2} \\ \frac{\partial e(p,u)}{\partial p_2 \partial p_1} & \frac{\partial e(p,u)}{\partial p_2 \partial p_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial h_1(p,u)}{\partial p_1} & \frac{\partial h_1(p,u)}{\partial p_2} \\ \frac{\partial h_1(p,u)}{\partial p_2} & \frac{\partial h_2(p,u)}{\partial p_2} \end{bmatrix} \rightarrow \frac{\partial h_1(p,u)}{\partial p_1} \leq 0, \frac{\partial h_1(p,u)}{\partial p_2} = \frac{\partial h_1(p,u)}{\partial p_2}$$

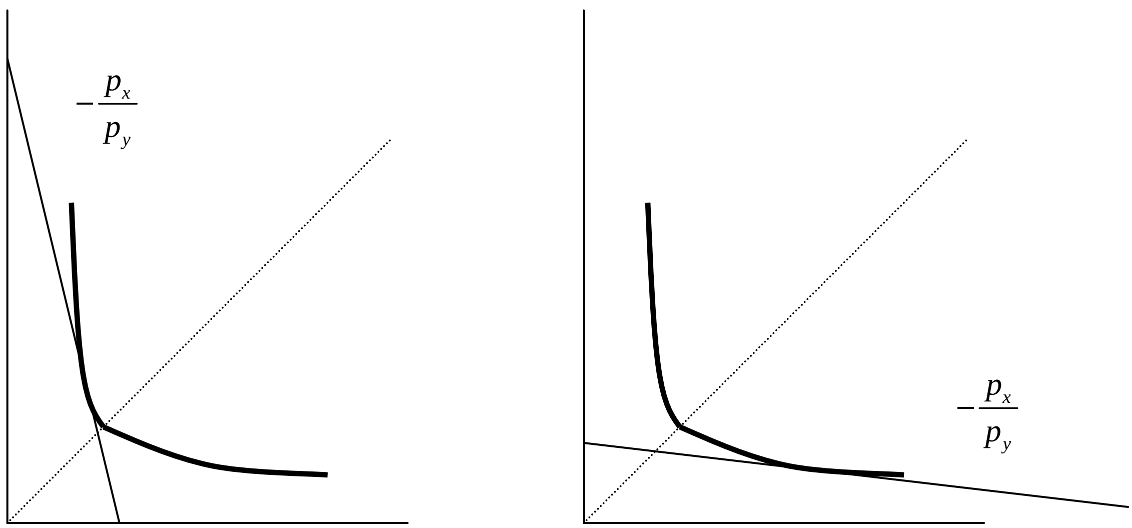
SlutskyMatrix

$$\frac{\partial h_i(p,u)}{\partial p_j} = \frac{\partial x_i(p,e(p,m))}{\partial p_j} + \frac{x_i(p,e(p,u))}{\partial e(p,u)} \frac{\partial e(p,u)}{\partial p_i}$$

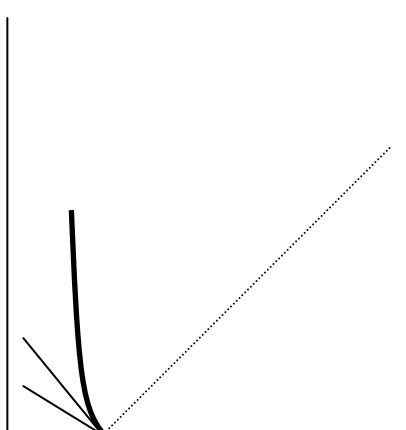
so

$$\frac{\partial x_i(p,m)}{\partial p_j} = \frac{\partial h_i(p,u)}{\partial p_j} - \frac{\partial x_i(p,m)}{\partial m} x_i$$

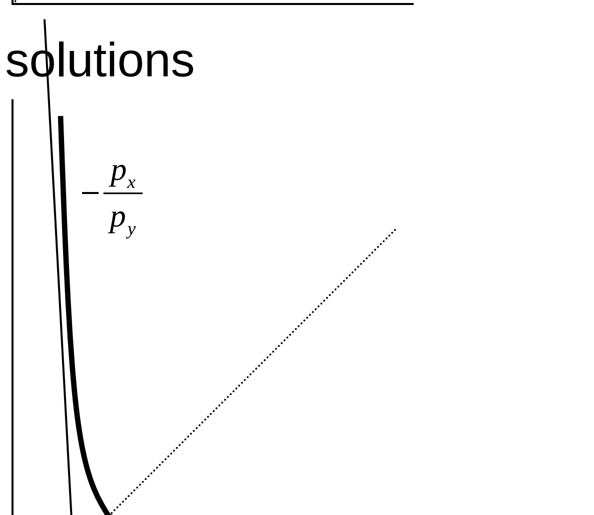
Looking for Corners



Possible Interior solutions



Multiple Price Vectors
at the Kink x=y



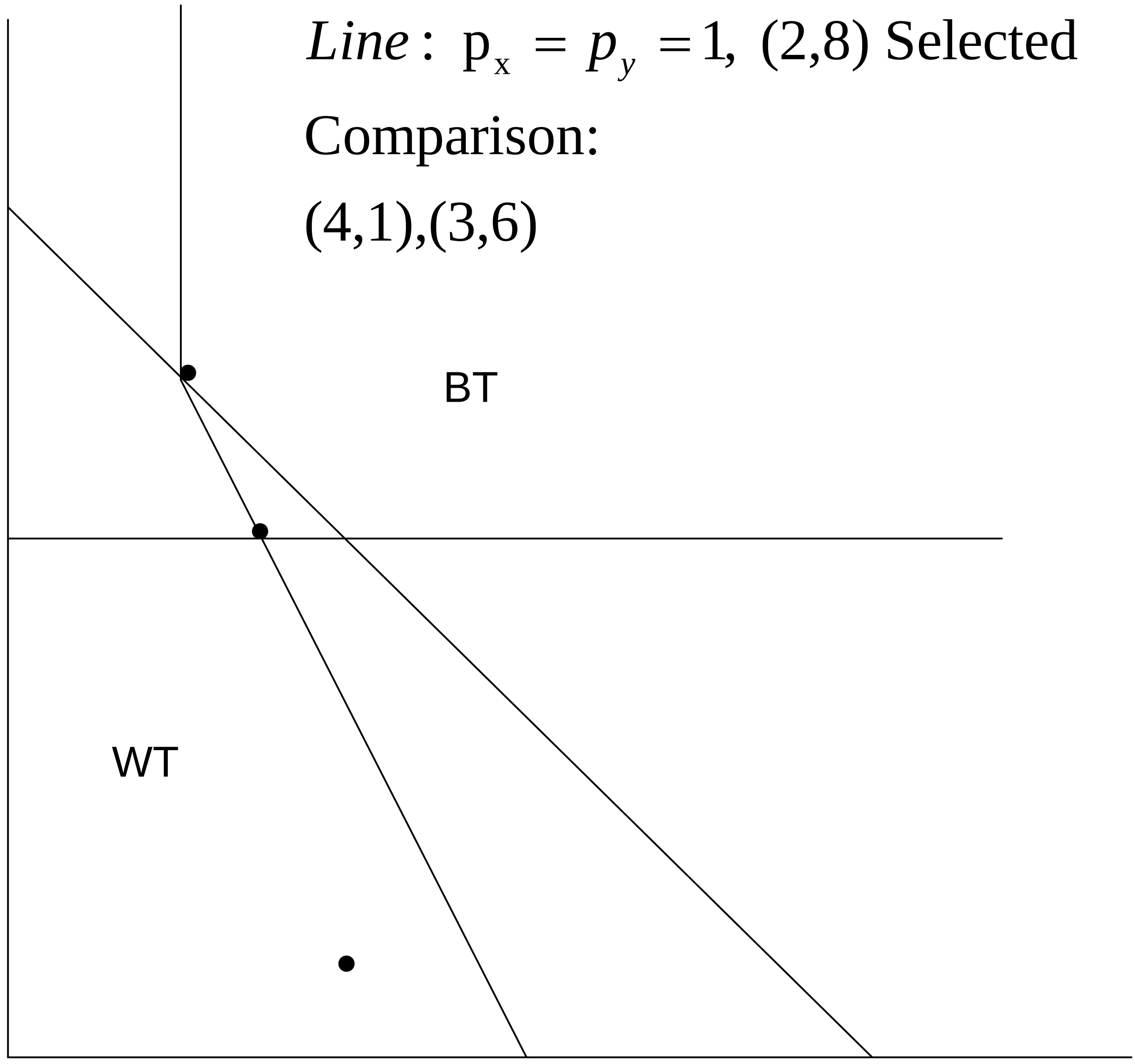
No Corner Solutions

WAPM and GARP

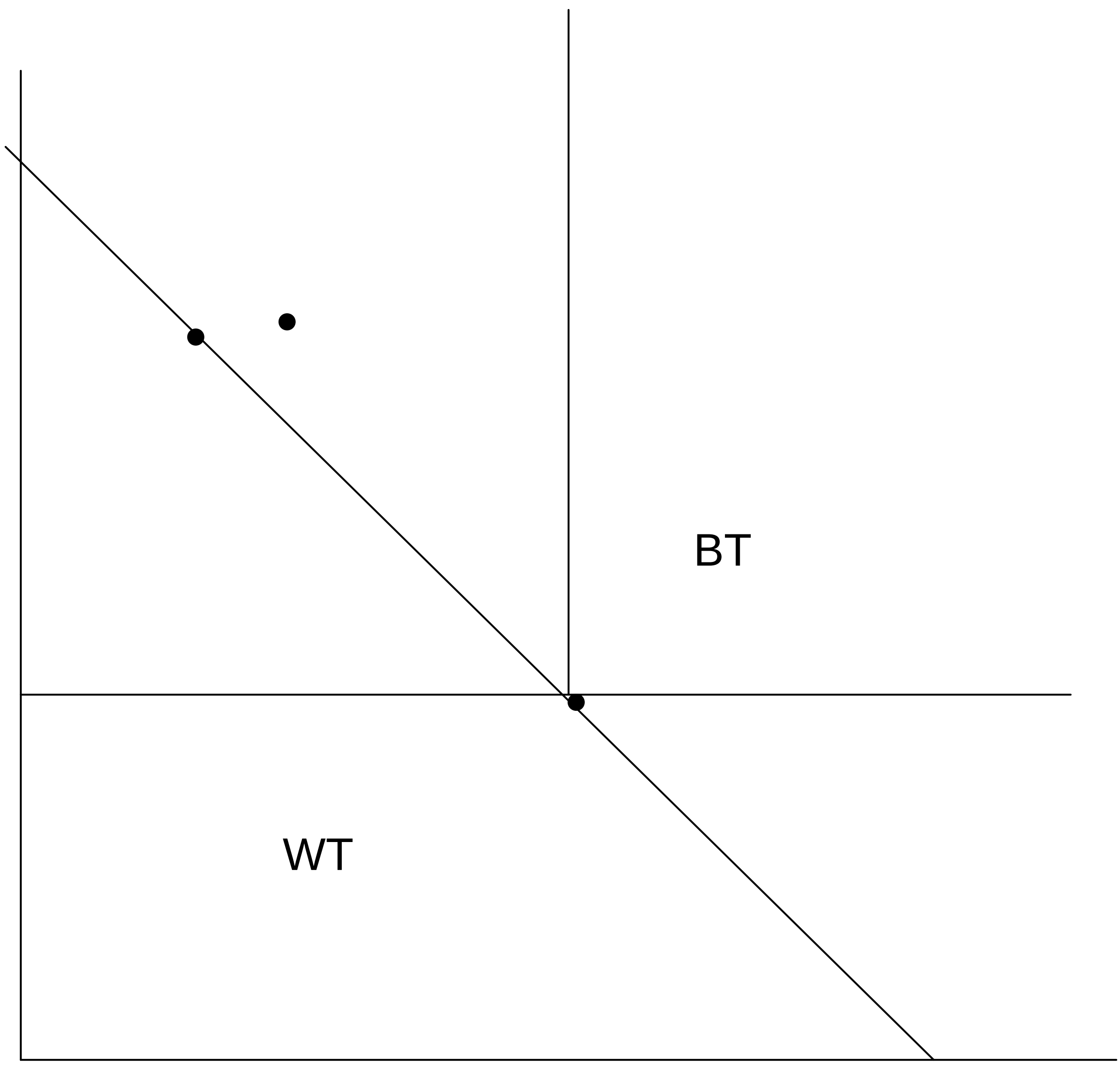
$Line: \ p_x = p_y = 1,\ (2,8)$ Selected

Comparison:

(4,1),(3,6)



WAPM and GARP



$Line:\ \mathrm{p}_x = p_y = 1,$ (2,8) Selected

Comparison:

(6,4),(3,8)

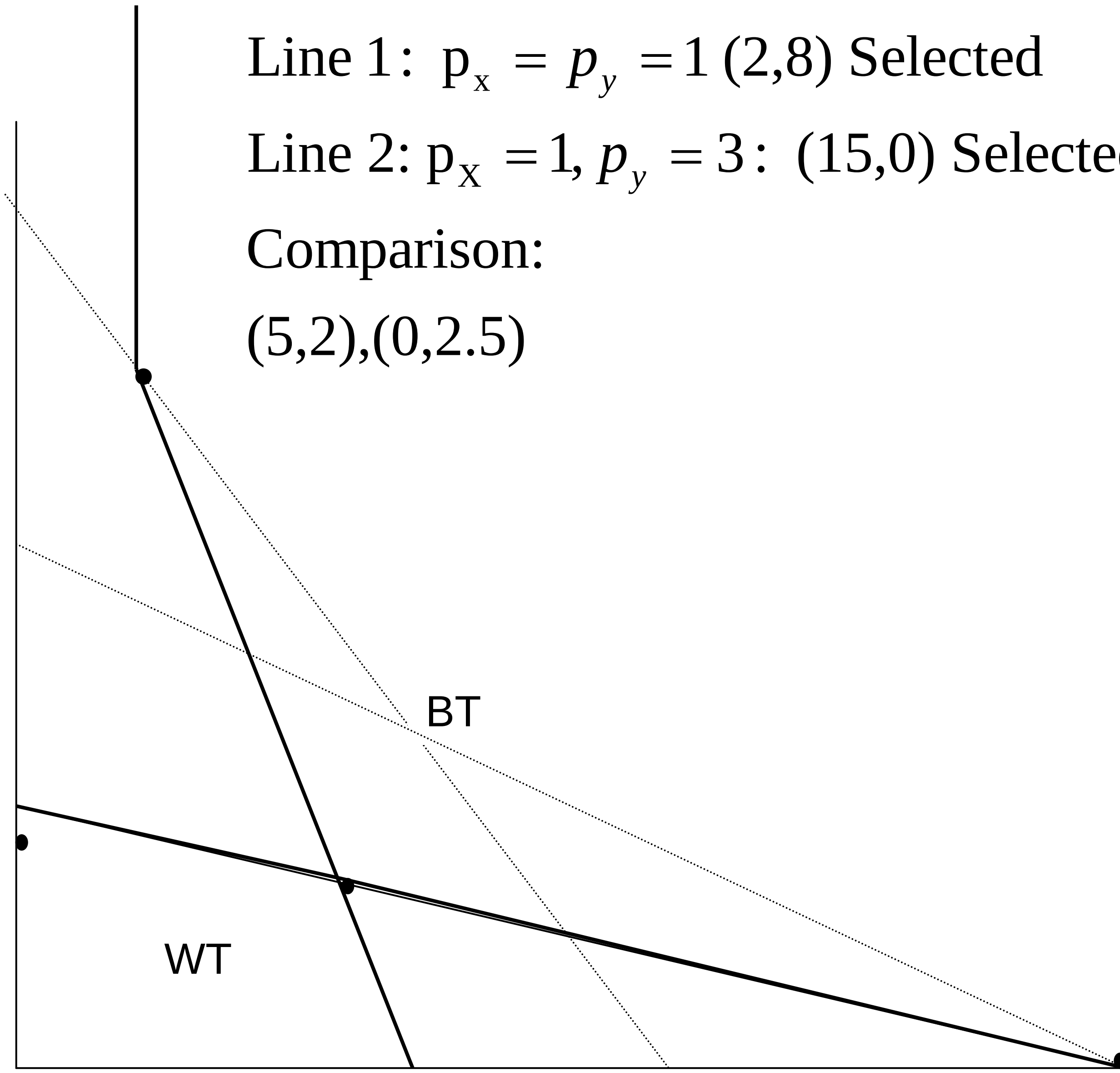
WAPM and GARP

Line 1: $\mathrm{p_x} = p_y = 1$ (2,8) Selected

Line 2: $\mathrm{p_X} = 1, p_y = 3$: (15,0) Selected

Comparison:

(5,2),(0,2.5)



Uncertainty Axioms

Continuity :

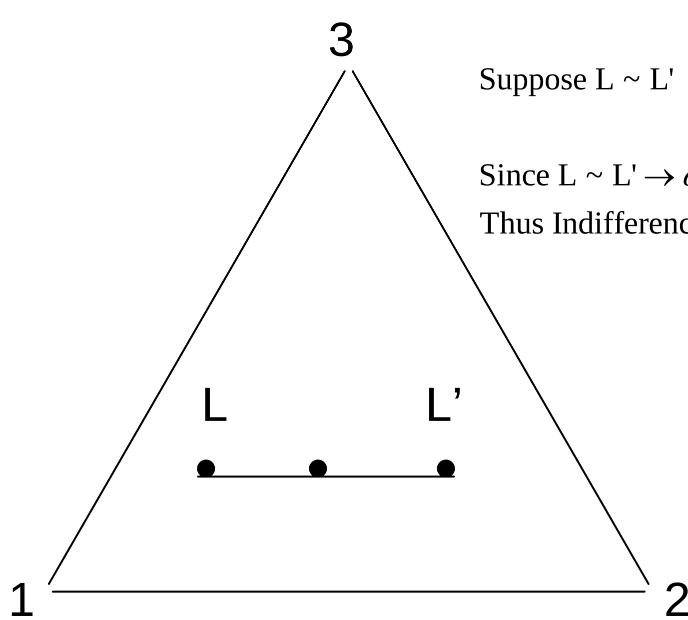
$\{\alpha \in [0,1]: \alpha L + (1-\alpha)L' \succsim L''\} \subset [0,1]$

$\{\alpha \in [0,1]: \alpha L + (1-\alpha)L' \precsim L''\} \subset [0,1]$

are closed

Independence axiom

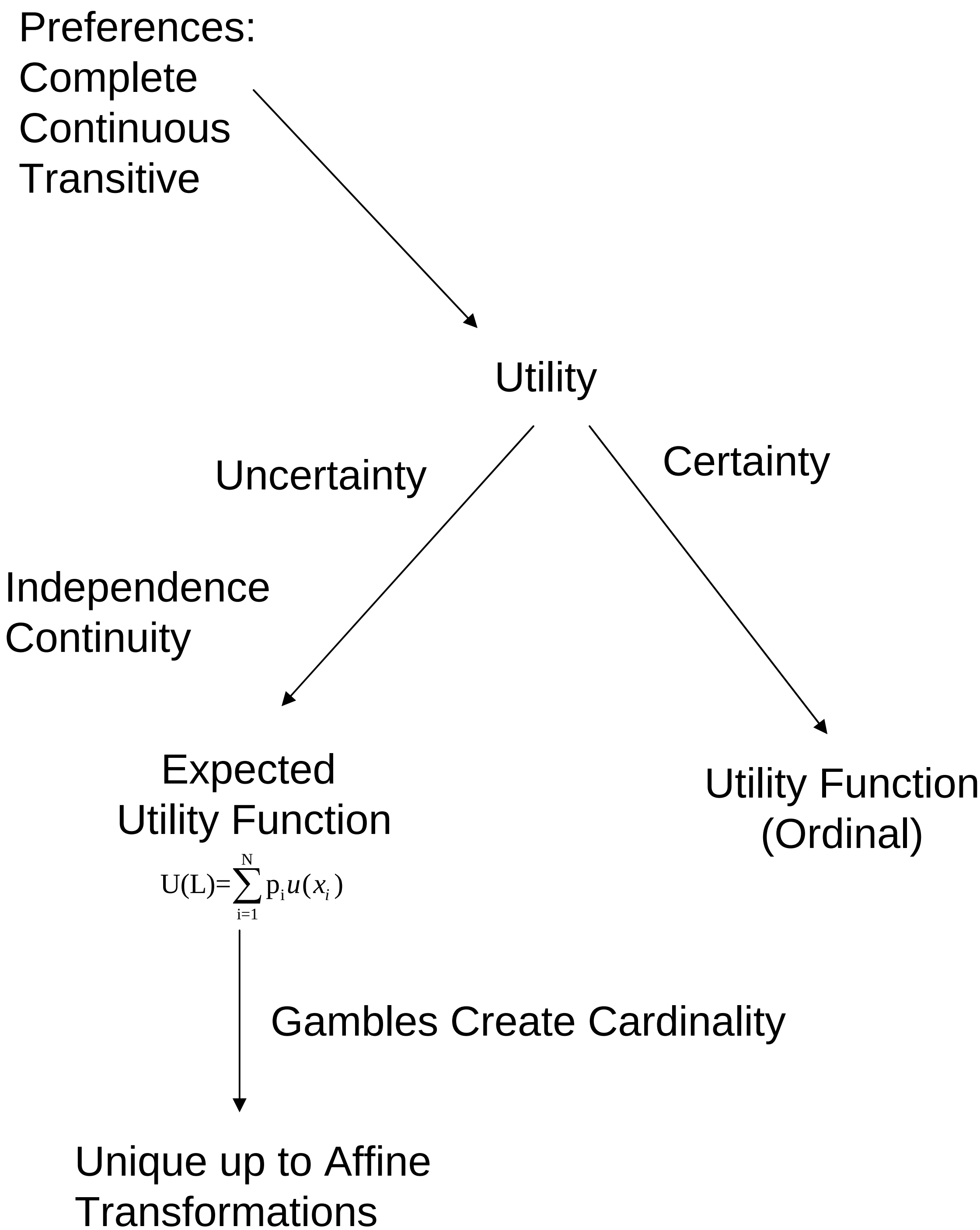
if L $\succsim L'$ *then* $\alpha L + (1-\alpha)L'' \succsim \alpha L' + (1-\alpha)L''$



Suppose L ~ L'

Since L ~ L' $\rightarrow$ αL ~ αL' $\rightarrow$ αL+(1-α)L' ~ αL'+(1-α)L'=L'

Thus Indifference over probabilities must be straight lines


Preferences:
Complete
Continuous
Transitive
Utility
Uncertainty
Certainty
Independence
Continuity
Expected
Utility Function
$U(L)=\sum_{i=1}^{N} p_i u(x_i)$
Utility Function
(Ordinal)
Gambles Create Cardinality
Unique up to Affine
Transformations

Risk Aversion

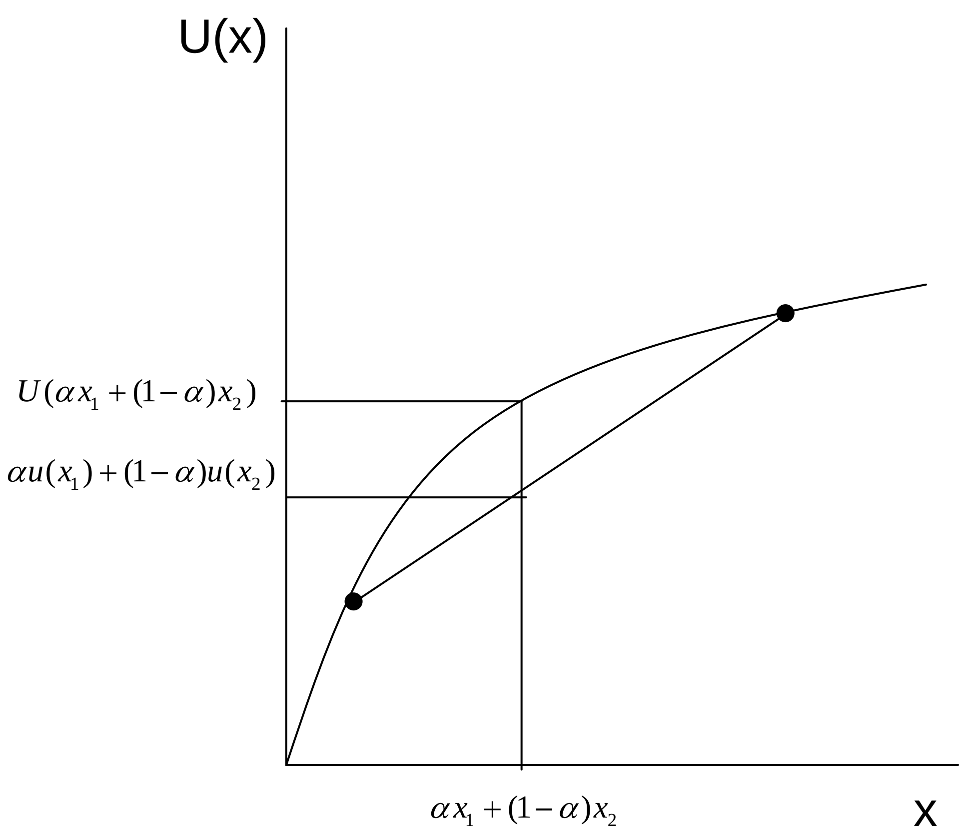
Let α be the probability of outcome x_1

$$E[U(L)] = \alpha u(x_1) + 1 - \alpha u(x_2)$$

$$U(E(x)) = u(\alpha x_1 + (1-\alpha)x_2)$$

$\rightarrow E[U(x)] < U(E(x))$ when u(x) is concave

(Jensen's Inequality)



Some Useful Definitions

Certainty Equivalent:

The certainty equivalent of any simple gamble g over wealth levels is an amount of wealth CE, offered with certainty, such that u(g)=u(CE).

Risk Premium:

The amount of wealth, P, such that u(g) ≡ u(E(g)-P)

Note that P ≡ E(g)-CE

Example:

u(w) = ln(w)

Consider a 50/50 bet of winning and losing wealth h:

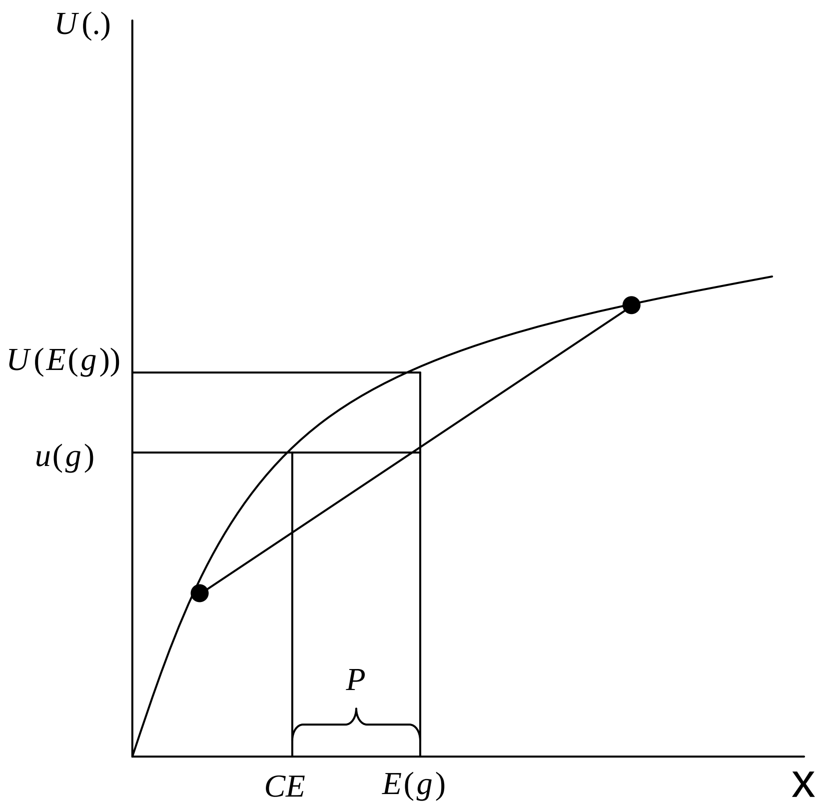
$$u(g)=\frac{1}{2}\ln(w_0+h)+\frac{1}{2}\ln(w_0-h)=\ln(w_0{}^2-h^2)^{\frac{1}{2}}$$

Thus :

$$CE \equiv (w_0{}^2-h^2)^{\frac{1}{2}}$$

$$P \equiv w_0-(w_0{}^2-h^2)^{\frac{1}{2}}$$

Risk Aversion



Production

Technology
f(x)

Profit Maximization

Profit Function $\pi(p,w) = \max_x \; pf(x) - wx$

Input Demand Function $x(p,w)$

Supply Function $y(p,w)$

$\pi(p,w)$:
- Nondecreasing in p
- Nondecreasing in p
- Homogeneous of Degree 1
- Continuous and Convex in p

Cost minimization

Cost Function $\text{c}(w,y) = \min_x \; wx$

$SubjectTo: \; f(x) = y$

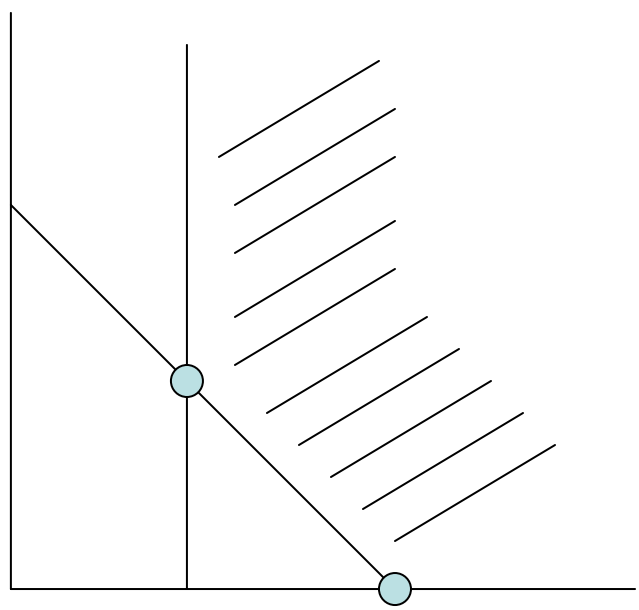
Conditional Demand Function $x(w,y)$

$c(w,y)$:
- Homogeneous of degree 1
- Nondecreasing in ω
- Concave in w

Solve $\max_{\text{y}} \; py - c(w,y)$

$$\max_{x_1, x_2} \; p\min(2x_1, x_1 + x_2) - w_1 x_1 - w_2 x_2)$$

1) Draw an Isoquant:



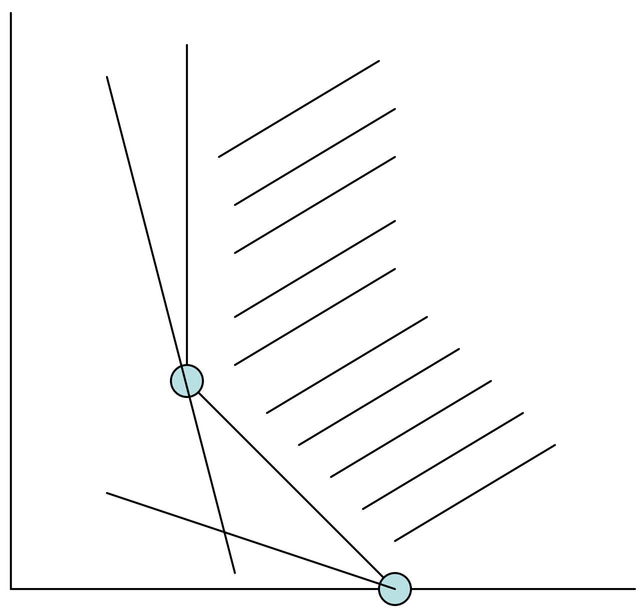
2) Think about production function:

$f(tx_1, tx_2) = \min(2tx, tx_1 + tx_2) = tf(x_1, x_2)$

Homogeneous of degree 1 $\rightarrow$ CRTS $\rightarrow$ $c(w_1, w_2, y) = c(w_1, w_2)y$

$$\max_{x_1, x_2} \; p \min(2x_1, x_1 + x_2) - w_1 x_1 - w_2 x_2)$$

3) Think about corners



3) Write out possible outcomes:

a) Use all x_1 : Second part binds so $f(x_1, x_2) = x_1$

b) Use both: Both will bind so $2x_1 = x_1 + x_2 \rightarrow x_1 = x_2 \quad f(x_1, x_2) = 2x_1$

$$\max_{x_1, x_2} \; p \min(2x_1, x_1 + x_2) - w_1 x_1 - w_2 x_2)$$

4) Solve the easier problems:

When $w_1 < w_2 : x_1 = y$

When $w_1 > w_2 : x_1 = x_2 = \frac{y}{2}$

$$c(w_1, w_2, y) = \begin{cases} w_1 y & w_1 < w_2 \\ \dfrac{(w_1 + w_2)y}{2} & w_1 \geq w_2 \end{cases}$$

$$or \;\; c(w_1, w_2, y) = \frac{\min(w_1, w_2) + w_1}{2} y$$