

**Problem Set 5: Due November 12th in Class**  
**Econ 14.04**

Note: Due to Veterans day this problem set is due Monday instead of Friday. however, a new problem set will be distributed on Wednesday the 7th and due Friday the 16th so plan your time accordingly.

1. Recall that price elasticity is defined as the percent change in quantity given a percent change in price. There are two ways that we can write this - one using demand functions  $D(p)$  and one using inverse demand functions  $P(q) = D^{-1}(q)$  :

$$\varepsilon = -\frac{\frac{D'(p)}{D(p)}}{\frac{1}{p}} = -\frac{\frac{1}{q}}{\frac{P'(q)}{P(q)}}$$

- (a) Start with a monopolists problem with using the demand function:

$$\max_p pD(p) - c(D(p))$$

Show that:

$$\frac{p - c'(q)}{p} = \frac{1}{\varepsilon}$$

- (b) Start with the monopolists problem using the inversed dmeand function:

$$\max_q P(q)q - c(q)$$

Show that:

$$\frac{p - c'(q)}{p} = \frac{1}{\varepsilon}$$

2. Suppose that we have an inverse demand function  $P(q) = 1 - q$

- (a) What is the the elasticity of the demand function at the following points

1.  $(p, q) = (0, 1)$
2.  $(p, q) = (.5, .5)$
3.  $(p, q) = (1, 0)$

- (b) In case (a), Why would we never expect to see a monopolist produce more than .5 units of a good?

- (c) The market demand curve for heroin is said to be highly inelastic. heroin supply is also said to be monopolized by the Mafia, which we assume to be interested in maximizing profits. Are these two statements consistent?

3. A monopolist has a cost function of  $c(y) = y$  so that its marginal costs are constant at \$1 per unit. It faces the following demand curve:

$$D(p) = \begin{cases} 0 & p > 20 \\ 100/p & p \leq 20 \end{cases}$$

- (a) What is the profit-maximizing choice of output?

- (b) If the government could set a price ceiling on this monopolist in order to force it to act as a competitor, what price should they set?

- (c) What output would the monopolist produce if forced to behave as a competitor?

4. A monopolist has a cost function of  $c(y) = cy$  so that its marginal costs are constant at \$c per unit. The monopolist is operating at an output level where  $|\varepsilon| = 3$ . The government imposes a quantity tax of \$6 per unit of output.

- (a) If the monopolist is facing linear demand curve, how much does the price rise?

- (b) If the monopolist has constant elasticity, how much does the price rise?
5. \*An economy has two kinds of consumers and two goods. Type A consumers have utility function  $U_A(x_1, x_2) = 4x_1 - \frac{x_1^2}{2} + x_2$  and Type B consumers have utility function  $U_B(x_1, x_2) = 2x_1 - \frac{x_1^2}{2} + x_2$ . Consumers can only consume nonnegative quantities. The price of good 2 is 1 and all consumers have incomes of 100. There are  $N$  type A consumers and  $N$  type B consumers.
- (a) Suppose that a monopolist can produce good 1 at a constant unit cost of  $c$  per unit and cannot engage in any kind of price discrimination. Find its optimal choice of price and quantity. For what values of  $c$  will it be true that it chooses to sell to both types of consumers?
- (b) Suppose that the monopolist uses a "two-part tariff" where a consumer must pay a lump sum  $k$  in order to be able to buy anything at all. A person who has paid the lump sum  $k$  can buy as much as he likes at a price  $p < 4$ . What is the highest amount  $k$  that a type A is willing to pay for the privilege of buying at price  $p$ ? If a type A does pay the lump sum to buy at price  $p$ , how many units will he demand? Describe the function that determines demand for good 1 by type A consumers as a function of  $p$  and  $k$ . What is the demand function for good 1 by type B consumers? Now describe the function that determines total good 1 by all consumers as a function of  $p$  and  $k$ .
- (c) If the economy consisted only of  $N$  type A consumers and no type B consumers, what would be the profit maximizing choices of  $p$  and  $k$ ?
- (d) If  $c < 1$ , find the values of  $p$  and  $k$  that maximize the monopolist's profits subject to the constraint that both types of consumers buy from it.
6. \* (Optional) Suppose that a new technology is created that allows for owners of a video game to copy and resell digital material for free. We suppose that a bootlegger has some probability of getting caught  $\pi(x)$  of getting caught in which case the bootlegger has to pay a fine  $F$  and give up the revenues received from the copy. The probability  $\pi(x)$  is an increasing function of  $x$ : the more copies you make, the more likely you are to get caught. We assume  $\pi(x)$  is differentiable. A bootlegger attempts to choose  $x$  as to maximize profits:

$$x = \arg \max_x [1 - \pi(x)]px - \pi(x)F$$

We assume bootleggers enter the market up until profits are zero. Thus:

$$[1 - \pi(x)]px - \pi(x)F = 0$$

- (a) Show that the scale of operation  $x^*$  is independent of the size of the fine  $F$
- (b) Show that the price charged by a bootlegger is:

$$p^* = \frac{\pi(x^*)}{[1 - \pi(x^*)]x^*} F$$

- (c) The developer of a game faces a fixed cost  $K$  and variable costs  $c(q) = 0$  to produce his good. In order for the product to be produced at all, the developer must be able to cover its cost. Thus:

$$pD(p) \geq K$$

Show that

$$\pi F \geq (1 - \pi) \frac{x^*}{D(p^*)} K$$

- (d) Based on your answer for part (c), why would the punishment of bootlegging need to be greater in China than in the US?