Econ 14.04 Fall 2006
Assignment 3: Solutions

1. (a) Taking the derivative of x wrt to p$_j$ yields:

$$\frac{dx_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{dp_j} = \frac{\partial x_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{\partial p_j}\Big|_{\mathbf{p}\overline{\mathbf{x}}=constant} + \frac{\partial x_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{\partial \mathbf{p}\overline{\mathbf{x}}}\frac{\partial \mathbf{p}\overline{\mathbf{x}}}{p_j}$$

Since

$$\frac{\partial x_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{\partial p_j}\Big|_{\mathbf{p}\overline{\mathbf{x}}=\text{constant}} = \frac{dh_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{dp_j} - \frac{\partial x_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{\partial m}x_i$$

we have:

$$\frac{dx_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{dp_j} = \frac{\partial h_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{\partial p_j} + \frac{\partial x_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{\partial m}[\overline{x}_j - x_j]$$

(b) When $\overline{x}_j = x_j$, the last term drops out of the above formula. However we know:

$$\frac{\partial h_1}{\partial p_2} = \frac{\partial e}{\partial p_1 \partial p_2} = \frac{\partial e}{\partial p_2 \partial p_1} = \frac{\partial h_2}{\partial p_1}$$

Now since:

$$\frac{dx_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{dp_j} = \frac{\partial h_i(\mathbf{p},\mathbf{p}\overline{\mathbf{x}})}{\partial p_j}$$

Then the marshallian demand functions must have symmetric cross partials.

(a) The weak axiom of revealed preference is vioalted if

1) Bundle 1 is reveal preferred to bundle 2
2) Bundle 2 is reveal preferred to bundle 1.

For Bundle 1 to be reveal preferred to 2, it must be that bundle 2 was available. Thus:

$$100 \cdot 120 + 100y \leq 100 \cdot 100 + 100 \cdot 100$$

For Bundle 2 to be reveal preferred to 2, bundle 1 must be availabe when bundle 2 was chosen. Tis requires:

$$100 \cdot 100 + 80 \cdot 100 \leq 100 \cdot 120 + 80 \cdot y$$

These are both satisfied when $y \in [75, 80]$

(b) Only condition one must hold here, thus for $y < 80$

(c) Only condition two must hold here, thus $y > 75$

(d) One of the three conditions hold for any budget level

(e) Suppose that $y < 75$. Then:

$$100 \cdot 120 + 100y \leq 100 \cdot 100 + 100 \cdot 100$$

and

$$100 \cdot 100 + 80 \cdot 100 > 100 \cdot 120 + 80 \cdot y$$

Here real wealth decreased from year 1 to year 2 and the relative price of good 1 increases. However, the demand for good 2, y decreases because $y < 75 < 100$. This means that the wealth effect of good 1 must be negative. (It might be easier to think about this going the other way from year 2 -> year 1. Then income increases and the relative price of good 1 decreases but output decreases).

(f) If $y \in [80, 100]$:

$$100 \cdot 100 + 80 \cdot 100 \leq 100 \cdot 120 + 80 \cdot y$$

and

$$100 \cdot 120 + 100y > 100 \cdot 100 + 100 \cdot 100$$

Hence the real wealth increases from year 1 to year 2 and the relative price of good 2 decreases. But the demand for good 2, y, decreases since y < 100. This means that the wealth effect of good 2 must be negative and hence an inferior good.

(a) Marshallian demand functions are homogeneous of degree 0, thus:

$$\frac{tp_2}{tp_1 + tp_2}\frac{(tm)^\alpha}{tp_1} = \frac{p_2}{p_1 + p_2}\frac{t^\alpha m^\alpha}{tp_1} = \frac{p_2}{p_1 + p_2}\frac{(m)^\alpha}{p_1}$$

Thus $\alpha = 1$. The slutsky matrix is also symmetric. Taking the off terms of the slutsky matrix we have:

$$\frac{\partial x_1}{\partial p_2} = \frac{1}{p_1 + p_2}\frac{m}{p_1} - \frac{p_2}{(p_1 + p_2)^2}\frac{m}{p_1} + \frac{p_2}{p_1 + p_2}\frac{1}{p_1}x_2$$

Combining the first two terms yields:

$$\frac{\partial x_1}{\partial p_2} = \frac{p_1}{(p_1 + p_2)^2}\frac{m}{p_1} + \frac{p_2}{p_1 + p_2}\frac{1}{p_1}\frac{p_1}{p_1 + p_2}\frac{m}{p_2}$$

Which reduces to:

$$\frac{\partial x_1}{\partial p_2} = \frac{2m}{(p_1 + p_2)^2}$$

Similar calculation for $\frac{\partial x_2}{\partial p_1}$ yields:

$$\frac{\partial x_2}{\partial p_1} = \frac{\beta 2m}{(p_1 + p_2)^2}$$

Thus $\beta = 1$.

1. $h_1(\mathbf{p}, u) = \frac{p_2}{p_1}u$, thus:

$$\frac{\partial h_1}{\partial p_2} = \frac{u}{p_1} = \frac{\partial h_2}{\partial p_1}$$

Integrating up wrt to p_1 yields:

$$h_2 = \ln(p_1)u + g(p_2, u)$$

Our function must be homogenous of degree 0 (fyi: the derivative of a homogeneous of degree 1 function is homogeneous of degree 0), thus we know there is a complimentary term involving p_2 and u that will cause this. The only possible term for all price vectors is $-ln(p_2)u$. Thus we have:

$$h_2 = \ln(\frac{p_1}{p_2})u + g(u)$$

2. With our initial condition, we see that our hicksian demand function remains a constant reguardless of the u we chose. Thus $g(u) = 0$.

(a) By graphing, we know the x non-negativity constraint won't bind. Setting up the KT:

$$KT : \max_{x,y} \ln(x) + y - \lambda(m - p_x x - p_y y) - \mu y$$

Taking the FOC:

$$\begin{aligned} \frac{1}{x} - \lambda p_x &= 0 \\ 1 - \lambda p_y + \mu &= 0 \\ p_x x + p_y y &= m \\ y &\geq 0, \mu \geq 0, y\mu = 0 \end{aligned}$$

Suppose $\mu > 0, y = 0, x = \frac{m}{p_y} \rightarrow \frac{1}{m} = \lambda \rightarrow 1 \leq \frac{p_y}{m}$ Otherwise $\mu = 0$, $x = \frac{p_y}{p_x}$

$$\begin{aligned} x &= \begin{cases} \frac{m}{p_x} & m < p_y \\ \frac{p_y}{p_x} & Otherwise \end{cases} \\ y &= \begin{cases} 0 & m < p_y \\ \frac{m-p_y}{p_y} & Otherwise \end{cases} \end{aligned}$$

Plugging in for the indirect utility function yields:

$$v(p_1, p_2, m) = \begin{cases} \ln(\frac{m}{p_x}) & m < p_y \\ \ln(\frac{p_y}{p_x}) + \frac{m}{p_y} - 1 & Otherwise \end{cases}$$

The expenditure function replaces m with e(p,u) and v($p_1, p_2, e(p,u)) = u$ thus:

$$u = \begin{cases} \ln(\frac{e(p,u)}{p_x}) & e(p,u) < p_y \\ \ln(\frac{p_y}{p_x}) + \frac{e(p,u)}{p_y} - 1 & Otherwise \end{cases}$$

Rearranging yields:

$$e(p,u) = \begin{cases} p_x e^u & e^u < \frac{p_y}{p_x} \\ p_y[(u+1) - \ln(\frac{p_y}{p_x})] & Otherwise \end{cases}$$

The hicksian demand functions are found by differentiating $e(p,u)$ by p_x and p_y :

$$\begin{aligned} x^h(p,u) &= \begin{cases} e^u & e^u < \frac{p_y}{p_x} \\ \frac{p_y}{p_x} & Otherwise \end{cases} \\ y^h(p,u) &= \begin{cases} 0 & e^u < \frac{p_y}{p_x} \\ [(u+1) - \ln(\frac{p_y}{p_x})] - 1 & Otherwise \end{cases} \end{aligned}$$

1. When $m^i > p_y$, we know that everyone will consume $\frac{p_y}{p_x}$ of good 1 and the rest on good 2. Thus:

$$X(p_1, p_2, m^i) = \frac{5p_y}{p_x}$$
$$Y(p_1, p_2, m^i) = \sum_{i=1}^{5} \frac{m^i}{p_y} - 5$$

   Note that the log linear utility function when m is sufficienctly large has the gorman form so we can simply add the utility functions.

2. Here two agents don't have enough money to switch over to good b. Thus:

$$X(p_1, p_2, m^i) = \frac{m^1 + m^2}{p_x} + 3\frac{p_y}{p_x}$$
$$Y(p_1, p_2, m^i) = \frac{m^3 + m^4 + m^5}{p_y} - 3$$

(a) Taking the FOC we have:

$$-q(1-\pi)u'(w - \alpha q) + (1-q)\pi u'(w - \alpha q - D + \alpha) = 0$$

(b) The insurance is actuarily fair, thus the money brought in equals the money sent out:

$$\alpha q = (\pi)\alpha \rightarrow q = \pi$$

Substitution into the FOC yields:

$$u'(w - \alpha q) = u'(w - \alpha q - D + \alpha)$$

a Strictly concave utility function will have the same slope only at the same point, thus:

$$w - \alpha q = w - \alpha q - D + \alpha \rightarrow D = \alpha$$

1. Since $D = \alpha$, the utility is the same in the two states. Thus the agents utility is simply:

$$u(w - \alpha q)$$

   if he leaves the car running and

$$u(w - \alpha q) - e$$

   If he does not.

2. We need to find conditions under which the agent will prefer to turn his car off. This requires that the utility from turning off his car is greater than his utility of leaving it on:

$$[1-\pi(e)]\ln(w-\alpha q)+\pi(e)\ln(w-\alpha q-D+\alpha)-e \geq [1-\pi(0)]\ln(w-\alpha q)+\pi(0)\ln(w-\alpha q-D+\alpha)$$

After some subsitution, this equation reduces to:

$$[\pi(e) - \pi(0)][\ln(w - \alpha q) - \ln(w - \alpha q - D + \alpha)] \leq -e$$

From the properties of the natural log we get:

$$\ln\left[\frac{(w - \alpha q)}{(w - \alpha q - D + \alpha)}\right] \leq \frac{-e}{[\pi(e) - \pi(0)]}$$

Since the RHS is negative and ln(1)=0, we know $\left[\frac{(w-\alpha q)}{(w-\alpha q-D+\alpha)}\right] < 1 \rightarrow \alpha < D$