

FALL 2018 14.01 PROBLEM SET 7 - Solutions

PROBLEM 1: TRUE OR FALSE (20 POINTS)

Determine whether the following statements are True or False. Explain your answer.

- (1) Consider the Cournot duopoly model with linear demand and asymmetric costs (that is, the cost functions are $C(q_1) = c_1q_1$ for firm 1 and $C(q_2) = c_2q_2$ for firm 2). In the equilibrium of the game,
- (a) (5 points) Claim 1: both firms will be producing strictly positive quantities.
Solution: *False, if the difference between productivities is sufficiently large, the most inefficient firm will be driven out of the market and produce zero.*
- (b) (5 points) Claim 2: the firm with the lowest cost will be producing a higher quantity than its competitor.
Solution: *True, the firm with the lowest cost will always produce a higher quantity than its competitor.*
- (2) (5 points) In a competitive labor market, the market labor supply curve is always upward sloping.
Solution: *False, if the income effect is sufficiently strong workers will supply less labor when the wage increases.*
- (3) (5 points) In the short run (capital is fixed), if the product produced by a firm becomes more valuable (i.e. its price increases), then the firm will respond by increasing its labor demand. Solution: *True, the firm's optimality condition is given by $pMPL = w$. For this condition to hold after an increase in p , MPL must decrease, which means that the firm must increase its labor demand.*

PROBLEM 2: LABOR DEMAND (30 POINTS)

Suppose that a representative, perfectly competitive, firm in the market has production function

$$F(K, L) = K^{\frac{1}{2}}L^{\frac{1}{2}}$$

the price of the firm's product is equal to p , the price per unit of capital is r , and the cost per unit of labor is w .

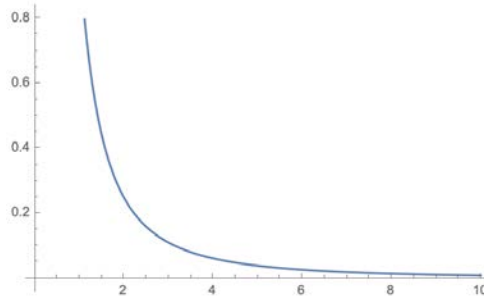
- (1) (10 points) Consider the short run case when capital is fixed at some level \bar{K} .
- (a) What is the firm's choice of labor as a function of r, w, \bar{K}, p ?
Solution: *Since the level of capital is fixed, the firm's short run optimality condition is given by setting the marginal revenue product of labor equal to*

the wage

$$\begin{aligned} w &= p \cdot MPL \\ &= \frac{1}{2} \left(\frac{\bar{K}}{L} \right)^{\frac{1}{2}} \\ \implies L &= \frac{p^2 K}{4w^2} \end{aligned}$$

- (b) Taking parameters r, \bar{K}, p as fixed, graph the firm's labor demand as a function of w .

Solution:



- (c) Now, taking w, \bar{K}, r as fixed, graph the firm's supply curve as a function of p . How does the firm supply curve change when w increases?

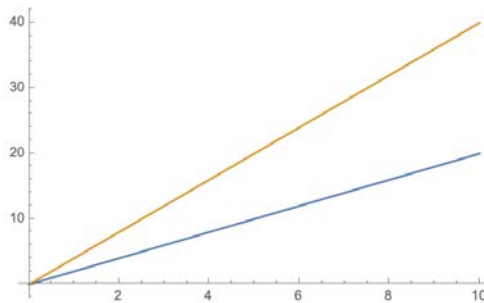
Solution: The firm's short run supply is given by $p = MC(q)$. Note that for any given level of quantity q , we have $L(q) = \frac{q^2}{\bar{K}}$ so that

$$VC(q) = wL(q) = \frac{wq^2}{\bar{K}} \implies MC(q) = \frac{2qw}{\bar{K}}$$

so that the firm supply curve is given by

$$p = \frac{2wq}{\bar{K}}$$

Note that here, it is always the case that $p \geq AVC(q) = \frac{wq}{\bar{K}}$. When w increases, the firm's supply curve rotates inward so that at every price p , a lower quantity is produced.



- (2) (8 points) Suppose that $\bar{K} = 180, r = 4$ and the demand for the firm's product is given by $Q^D = 10 - p$.

- (a) Solve for the equilibrium quantity and price in this market as a function of w . How does the wage affect the equilibrium price and quantity?

Solution: At equilibrium, quantity demanded for the product must equal quantity supplied. Therefore, we have from the firm's supply curve

$$\begin{aligned} p &= \frac{2wQ^s}{180} \\ &= \frac{2w(10-p)}{180} \\ \implies p &= \frac{10w}{90+w} \end{aligned}$$

Substituting this into the demand function, we have

$$Q = \frac{900}{90+w}$$

It is clear that an increase in wages decreases the equilibrium quantity. To see the effect on price, we take the derivative with respect to w :

$$\frac{dp}{dw} = \frac{(90+w)(10) - 10w}{(90+w)^2} = \frac{900}{(90+w)^2} > 0$$

so that increases in wage paid to workers increases the price of the product.

- (b) Find the elasticity of labor demand with respect to the wage in this market.
Solution: To find the elasticity of labor with respect to wage, note that we can write

$$\begin{aligned} L(q(w)) &= \frac{q(w)^2}{\bar{K}} \\ \implies \varepsilon_{L,w} &= \frac{w}{L} \cdot \frac{dL}{dw} \cdot \frac{dq}{dw} \\ &= -\frac{w}{\frac{q(w)^2}{\bar{K}}} \cdot \frac{2q(w)}{\bar{K}} \cdot \frac{900}{(90+w)^2} \\ &= \frac{-2w}{(90+w)} \end{aligned}$$

- (3) (8 points) Now consider the (perfectly competitive) long run, with the same demand curve $Q^D = 10 - p$ for the firm's product and price of capital is still $r = 4$, but the firm may freely adjust capital.

- (a) Now solve for the equilibrium price and quantity in the long run as a function of wage. How does wage affect the equilibrium price and quantity?

Solution: In the long run, the firm's optimality condition is given by

$$r = pMPK, \quad w = pMPL \quad \implies \frac{w}{r} = \frac{MPL}{MPK} \implies rK = wL$$

For any given level of quantity q , we have

$$L(q) = \left(\frac{r}{w}\right)^{\frac{1}{2}} q, \quad K(q) = \left(\frac{w}{r}\right)^{\frac{1}{2}} q$$

so that

$$TC(q) = 2q(wr)^{\frac{1}{2}}, \quad MC(q) = 2(wr)^{\frac{1}{2}}$$

By firm optimization, we know that firms' supply curve is given by

$$p = MC(q) = 2(wr)^{\frac{1}{2}}$$

Combining with the demand curve $P = 10 - Q$, we see that

$$10 - Q = 2(wr)^{\frac{1}{2}} \implies Q = 10 - 2(wr)^{\frac{1}{2}}$$

at $r = 4$, we have that $p = 4\sqrt{w}$, $Q = 10 - 4\sqrt{w}$. Here, increasing wages again increases the price and lowers the quantity. Additionally because quantity cannot be negative, we see that in the long run $w \leq 6.25$. To find the elasticity of wages in the long run, note that

$$L(w) = \frac{2}{\sqrt{w}} (10 - 4\sqrt{w}) = \frac{20}{\sqrt{w}} - 8$$

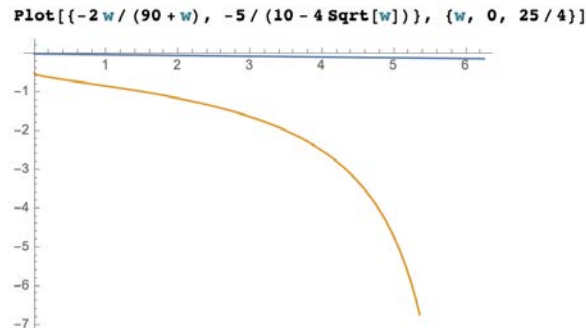
- (b) Find the elasticity of labor demand with respect to wage in the long run.

Solution: So the long run elasticity of labor demand with respect to wage is

$$\varepsilon_{L,w} = \frac{w}{L} \frac{dL}{dw} = \frac{w}{\frac{20}{\sqrt{w}} - 8} \left(-10w^{-\frac{3}{2}} \right) = \frac{-5}{10 - 4\sqrt{w}}$$

- (4) (4 points) Compare the wage elasticity of labor demand in the short run and long run. Explain why they are different.

Solution: In the long run, labor demand is much more elastic (the elasticity is more negative) because the firm can adjust its capital holdings. When wages increase, firms can switch to using machines rather than workers. Plotting the two elasticities, we can see that for every w , the short run elasticity is higher than the long run elasticity.



If we take the ratio of the short run elasticity to the long run elasticity (for $w \in [0, 6.25]$), we see that the ratio is always less than 1 and goes to 0 as w approaches 6.25. This means that the long run elasticity becomes infinitely more elastic than the short run for high wages.

PROBLEM 3: TAXES AND THE LABOR MARKET (20 POINTS)

Suppose a worker has preferences over consumption and leisure that can be represented by the following utility function:

$$U = \ln(c) + \ln(l)$$

There are 16 hours per day available for leisure (l) and work (L). The hourly wage is w , and assume that the price of each unit of consumption is \$1. The only source of income for this worker is the wage.

- (1) (4 points) Write down the worker's budget constraint in terms of c and L .

Solution:

$$c = wL$$

- (2) (6 points) Find the optimal consumption and work as a function of w .

Solution (Note to graders: students are not required to set up the maximization problem and derive the FOCs, they can directly use the tangency condition):

$$\begin{aligned} \max_{c,L} \ln(c) + \ln(16 - L) \\ \text{s.t } c = wL \end{aligned}$$

FOC:

$$\begin{aligned} w &= \frac{c}{16 - L} \\ c &= w(16 - L) \end{aligned}$$

From the budget constraint:

$$\begin{aligned} w(16 - L) &= wL \\ L &= 8 \\ c &= 8w \end{aligned}$$

- (3) (6 points) Now suppose there are 100 workers identical to the one we analyzed. There are also 100 firms, each one with a production function $y = \sqrt{L}$. Suppose that the price of the firms' output is \$1 per unit. Find the labor supply and labor demand curves, and use them to find the equilibrium wage and labor.

Solution: Labor supply is

$$\begin{aligned} L^S &= 100 \times 8 \\ &= 800 \end{aligned}$$

Labor demand from each firm is

$$\begin{aligned} \max_L \sqrt{L} - wL \\ w &= \frac{1}{2\sqrt{L}} \\ L_i^D &= \frac{1}{4w^2} \end{aligned}$$

To obtain the market demand we add across firms

$$L^D = \frac{25}{w^2}$$

To obtain the equilibrium we intersect demand and supply:

$$\begin{aligned} \frac{25}{w^2} &= 800 \\ w &= \frac{1}{2^{\frac{5}{2}}} \end{aligned}$$

- (4) (4 points) Suppose that the government sets a proportional tax on wages, so for every \$1 paid by the firm, the worker receives $\$(1 - \tau)$. Find the new equilibrium of the labor market

- (a) Write down the new budget constraint for the worker and find the optimal consumption and labor.

Solution: $c = w(1 - \tau)L$

- (b) Find the demand for labor by each firm as a function of w .

Solution: Let's denote w^G the gross wage, and w^N the net wage. The relation between them is

$$w^N = w^G (1 - \tau)$$

. Firms only care about the gross wage, so their demand will be the same we found before

$$L_i^D (w^G) = \frac{25}{(w^G)^2}$$

- (c) Find the new equilibrium wage and labor of the labor market.

Solution: Workers have a perfectly inelastic demand, so they will supply 800 units of labor regardless of the tax. Then, the gross wage will be the same as we found before:

$$\frac{25}{(w^G)^2} = 800 \Rightarrow w^G = \frac{1}{2^{\frac{5}{2}}}$$

The net wage is given by

$$w^N = \frac{1 - \tau}{2^{\frac{5}{2}}}$$

The equilibrium quantity will still be $L = 800$.

PROBLEM 4: FIRM & LABOR MARKET EQUILIBRIUM (30 POINTS)

Suppose that there are two types of workers in the economy: domestic workers and immigrant workers. All workers have preferences over consumption (x) and labor (ℓ) that can be represented by the following utility function:

$$U = \ln(x) - \frac{\ell^{1+\varepsilon}}{1+\varepsilon}$$

A representative, perfectly competitive firm produces the consumption good x according to the production function

$$F(L_I, L_D) = 120(L_I^r + L_D^r)^{\frac{1}{2r}}$$

where L_I is the total immigrant labor used and L_D is the total domestic labor. The price of a unit of the consumption good is equal to p , and immigrant and domestic workers are paid wages w_I and w_D respectively. The only source of income for a worker is his wage.

- (1) (6 points) Solve for the optimal labor choice of domestic and immigrant workers, taking as given the wage and the price of the consumption good.

Solution: The household budget constraint is given by $px \leq w\ell$. We can write the household's optimization problem as

$$\max_{\ell} \ln \left(\frac{w\ell}{p} \right) - \frac{\ell^{1+\varepsilon}}{1+\varepsilon}$$

Taking the derivative with respect to ℓ and setting equal to zero, we get $\ell = 1$.

- (2) (6 points) Suppose that immigrant and domestic labor are perfect substitutes ($r = 1$), and the economy has 4 immigrant workers and 12 domestic laborers. Find the equilibrium wage paid to workers. (In equilibrium, the total amount of labor supplied by workers must be equal to the total amount of labor used by the firm).

Solution: Since immigrant and domestic workers are perfect substitutes, they must be paid the same wage if the firm is to use both types of labor. Total labor supply is 16 (4 units from immigrants, 12 from domestic workers) and the firm's marginal product of labor is

$$MPL = 60L^{-\frac{1}{2}}, \quad L = L_I + L_D$$

The equilibrium wage therefore must be $w = w_I = w_D = 15$.

- (3) (6 points) Suppose there is an influx of immigrants so that the total number of immigrant workers increases to 13. What is the equilibrium wage now for domestic workers and has the wage increased or decreased? Explain why this has happened.

Solution: Now, total labor supply is 25 and the firm's marginal product of labor is

$$MPL = 60L^{-\frac{1}{2}}, \quad L = L_I + L_D$$

The equilibrium wage therefore must be $w = w_I = w_D = 12$. The wage of domestic workers decreases because the immigrants are substitutes for domestic workers, and the increase in the total number of workers decreases the firm's marginal labor revenue.

- (4) (6 points) Now consider the case when immigrant labor and domestic labor are complementary ($r = -1$) and suppose that there are 4 immigrant workers and 12 domestic laborers. What is the equilibrium wage now for immigrant and domestic workers?

Solution: Since immigrants and domestic workers are no longer perfect substitutes, it is not necessarily the case that $w_I = w_D$. In fact, wages are going to be such that

$$w_I = F_{L_I}(L_I, L_D), \quad w_D = F_{L_D}(L_I, L_D)$$

Taking the derivative and setting $L_I = 4, L_D = 12$, we get

$$w_I \approx 19.5, \quad w_D \approx 2.16$$

- (5) (6 points) Again, suppose that the total number of immigrant workers increases to 13. What is the equilibrium wage for domestic and immigrant workers? How does this compare to your answer in part (4)? Explain why your answer is different from part (3).

Solution: As before, wages are going to be such that

$$w_I = F_{L_I}(L_I, L_D), \quad w_D = F_{L_D}(L_I, L_D)$$

Taking the derivative and setting $L_I = 13, L_D = 12$, we get

$$w_I \approx 5.53, \quad w_D \approx 6.49$$

The wages for immigrants have decreased because all immigrant labor is the same and having more immigrants decreases the marginal product of immigrants. However, because immigrants and domestic workers are complements, more immigrants increases the marginal product of domestic workers and therefore the wage paid to domestic workers. Whether having more immigrants helps or hurts domestic workers fundamentally depends on whether immigrant labor is complementary to labor provided by domestic workers.

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