Exercises

Problems on geostrophy and hydrostaticity are pretty much standard in any text on dynamic meteorology. Exercises 4.1–6 here are from Houghton (1977); Exercise 4.7 is from Holton (1979).

4.1 What is the pressure gradient required at the earth's surface at $45°$ latitude to maintain a geostrophic wind velocity of 30 ms^{-1}? What does this suggest about typical pressure gradients?

4.2 The atmospheric surface pressure at radius r_0 from the center of a tornado rotating with constant angular velocity ω is p_0. Show that the surface pressure at the center of the tornado is

$$p_0 \exp\left(-\frac{\omega^2 r_0^2}{2RT}\right)$$

when the temperature is assumed constant.

4.3 Figure 4.4a is a cross section of part of a front. At a certain level, the air possesses temperatures T_2 and T_1 $(T_2 > T_1)$, respectively, on the two sides of the front, which has a slope of α relative to the horizontal. Apply the hydrostatic

equation and the geostrophic wind equation to the region AB, together with the condition that the pressure must be continuous across the frontal surface. Show that in equilibrium the components of velocity v_2 and v_1 in the y–direction on the two sides of the front satisfy the relation

$$(T_1 - T_2)g \tan\alpha = (v_2 T_1 - v_1 T_2)f.$$

If $T_2 - T_1 = 3K$ and $v_1 - v_2 = 10\text{ms}^{-1}$, find α. The slopes of typical fronts vary from around 1/50 to 1/300. Note that if $f = 0$, that is, the earth were not rotating, the sloping surface could not be in equilibrium.

4.4 From the sense of the velocity change given by the above equation, show that the kink in the isobars is always cyclonic (*viz.* Figure 4.4b).
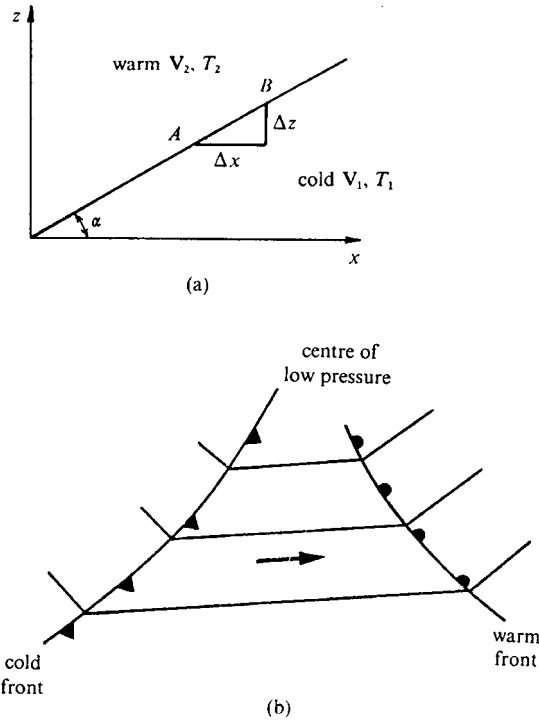
Figure 4.4: (a) Cross section through a front. (b) Warm sector of a typical depression showing warm and cold fronts. After Houghton (1977).

4.5   The wind at the surface is from the west. At cloud level it is from the south. Do you expect the temperature to rise or fall?

4.6   If the pole is 40K colder than the equator and the surface wind is zero, what wind would you expect at the 200mb pressure level? Compare the temperature and wind fields in Chapter 5, and show that they are *roughly* consistent with your result.

4.7   The planet Venus rotates about its axis so slowly that to a reasonable approximation the Coriolis parameter may be set equal to zero. For steady, frictionless motion parallel to latitude circles Equation 4.3 reduces to

$$\frac{u^2 \tan \phi}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial y}.$$

By transforming this expression to isobaric coordinates show that the *thermal wind* equation in this case can be expressed in the form

$$\omega_r^2(p_1) - \omega_r^2(p_0) = \frac{-R\ln(p_0/p_1)}{(a\sin\phi\,\cos\phi)}\frac{\partial <T>}{\partial y},$$

where R is the gas constant, a is the radius of the planet, and $\omega_r \equiv u/(a\cos\phi)$ is the relative angular velocity. How must $<T>$ (the vertically averaged temperature) vary with respect to latitude in order that ω_r be a function only of height? If the zonal velocity at $\simeq 60$km height above the equator ($p_1 = 2.9 \times 10^5$ Pa) is 100ms^{-1} and the zonal velocity vanishes at the surface of the planet ($p_0 = 9.5 \times 10^6$ Pa) what is the vertically averaged temperature difference between the equator and pole assuming that ω_r depends only on height? The planetary radius is $a = 6100$ km, and the gas constant is $R = 187$J kg^{-1} K^{-1}.