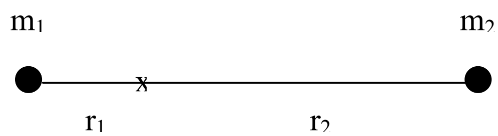
Tidal forces

Before we look at free waves on the earth, let's first examine one class of motion that is directly forced: astronomic tides. Here we will briefly consider some of the tidal generating forces for 2-body systems.



Consider the two masses in the diagram above. Each of these two masses, denoted by m_1 & m_2, rotate about one another and under the action of gravity are in a stable binary orbit. The center of rotation is on a line between the two masses at a position denoted by "x". The force balance on these two 'point' masses is given as:

$$F_g = \frac{Gm_1m_2}{r^2},$$

$$\frac{m_1v_1^2}{r_1} = \frac{m_2v_2^2}{r_2} = F_g$$

The first is the gravitational force of attraction (G is a universal constant) and the second is the orbital balance reflecting a balance between centrifugal force and gravity, which must be in balance for a stable orbit. The total distance between the two masses having velocities v is denoted by r. If the rotation rate is given by ω, then the above can be rewritten as

$$\frac{m_1v_1^2}{r_1} = m_1\omega^2 r_1, \ \frac{m_2v_2^2}{r_2} = m_2\omega^2 r_2 = F_g$$

$$m_1r_1 = m_2r_2$$

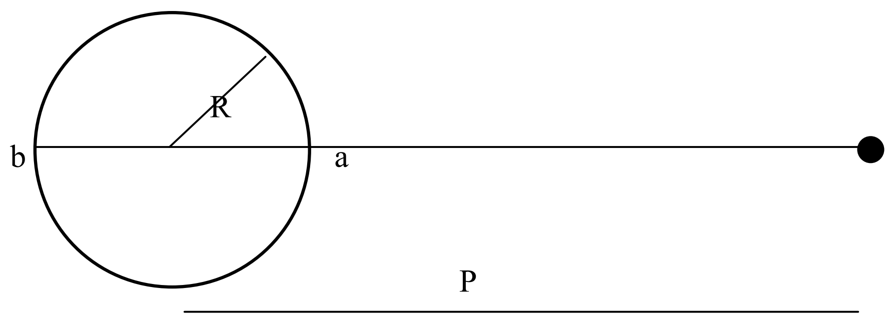
The second allows us to calculate the center of mass, where the center of rotation lies knowing the masses. For the case of the earth and moon, we have the following:

$$m_e = 5.983 * 10^{24} kg$$

$$m_m = 1.228 * 10^{-2} m_e$$

$$r_{em} = 3.8 * 10^8 m$$

With these we can calculate that the distance from the center of the earth to the center of rotation of the earth-moon binary system is 4500 km, within the earth (since the earth's mean radius is 6371 km). Now we are interested in the details of the forces relative to the center of mass of the earth due to another 'mass', be it the moon or the earth. So consider the picture below:



The distance from the center of the earth (on the left) to the 'other' mass is *P (~60R)*. At the points *a, b*, we are interested in the gravitational forces on particles *of unit mass* due to the other mass (of mass *m*). We also note that the two are in orbital balance, so that the centrifugal force, F_c, for every particle on the earth is prescribed by the orbital balance given above for point masses. Thus the force balance per unit mass for particles directly under the other mass but on either side of the earth is as follows:

$$F_a = \frac{Gm}{(P-R)^2}$$

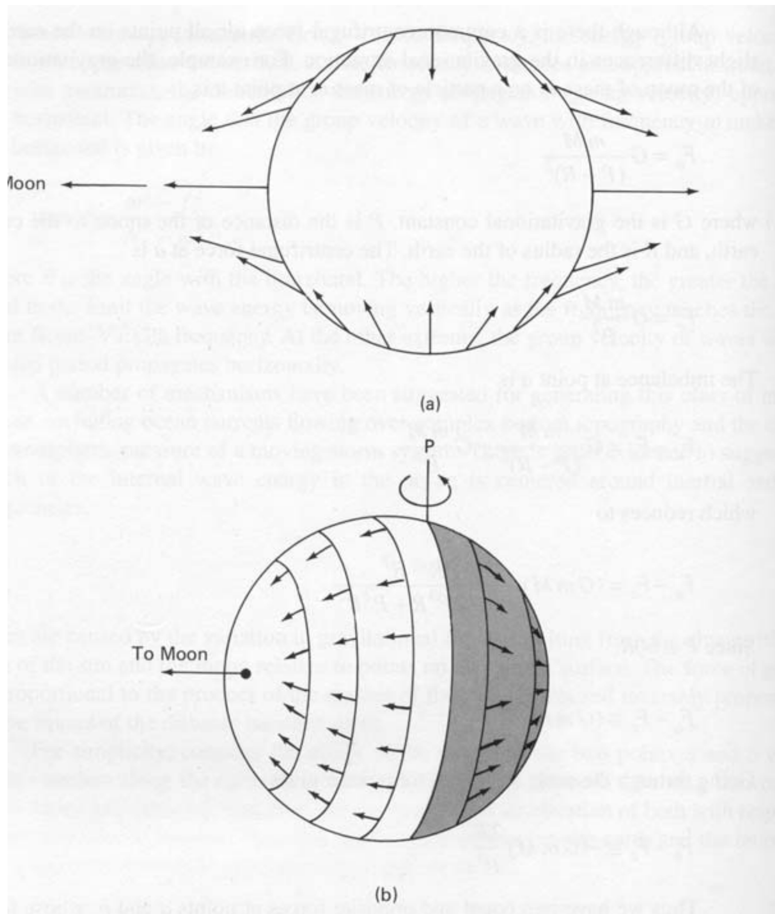
$$F_b = \frac{Gm}{(P+R)^2}$$

$$F_c = \frac{Gm}{P^2}$$

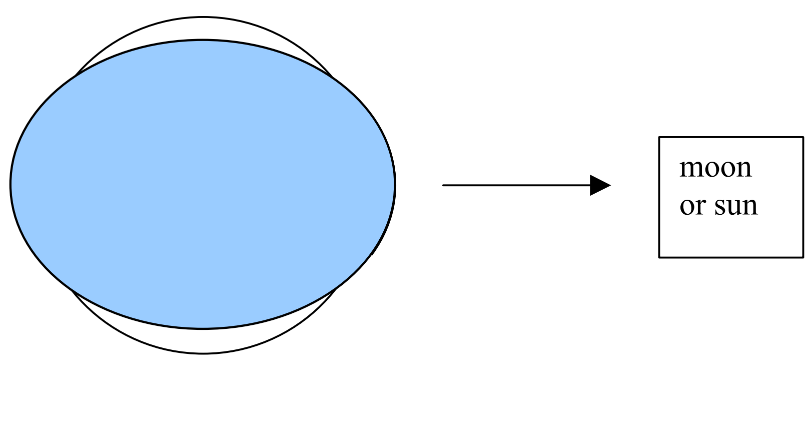
$$F_a - F_c = \frac{Gm}{(P-R)^2} - \frac{Gm}{P^2} \approx 2\frac{GmR}{P^3}$$

$$F_b - F_c = \frac{Gm}{(P+R)^2} - \frac{Gm}{P^2} \approx -2\frac{GmR}{P^3}$$

The force ‘balance’ between gravity and centrifugal forces will be unbalanced except at the center of mass of the earth and will lead to resulting force that is to the right at point *a* and to the left at point *b*. These forces relative to the center of mass are given for these two points above: $F_{a,b} - F_c$. These forces are equal and opposite, are proportional to the mass of the external body, and depend on the inverse cube of the distance. The net effect on the surface of the earth is shown in the figure below. This is taken from the book by Knauss and, to my dismay, has the moon on the left, not the right as plotted above.

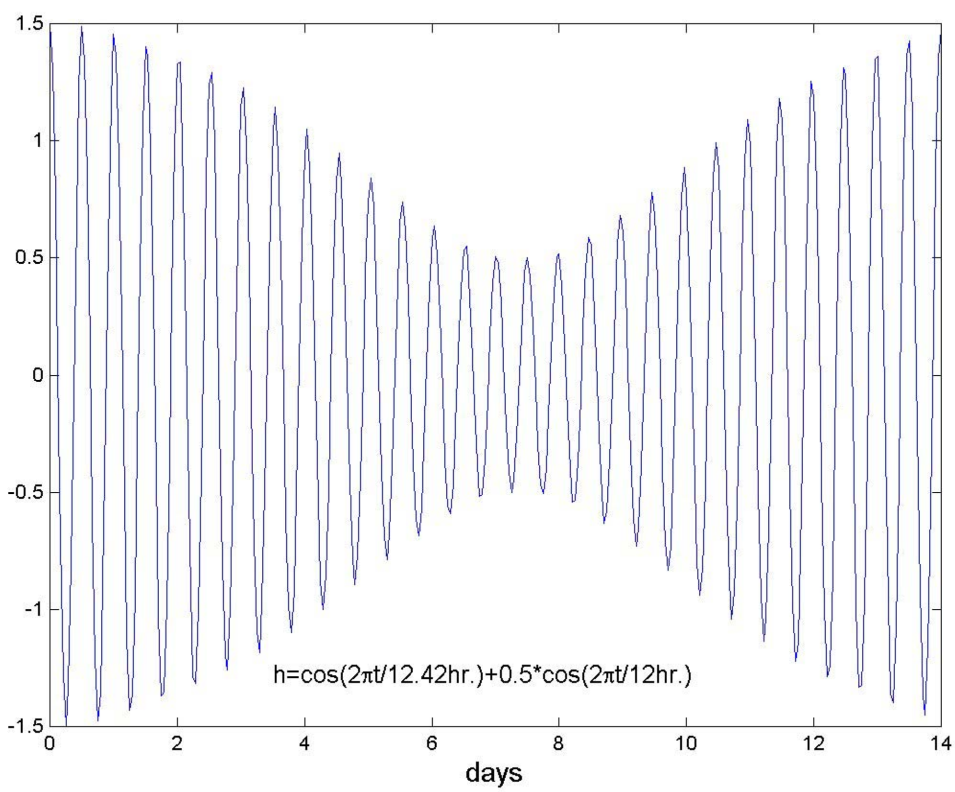


Although the sun has a mass that is larger than the moon by a factor of 2.5×10^7, because the distance between the earth/sun is about 400 times that of the earth/moon, the effective tidal forces are larger for the moon than the sun by about a factor of 2. The tidal forces are weak compared to local gravity (1 part in 9 million) but the horizontal components of the force will drive pressure gradients that will force fluid to move laterally. The force imbalance will cause, on a water covered globe, the surface to deform into an ellipsoidal shape which, under a theory of equilibrium tides, should cause a bulge on either side of the earth of about 55 cm due to the lunar forcing. Recall that the earth rotates about an axis which is 23.5



degrees from the plane of its rotation about the sun. The moon rotates around the earth (in 27.32 days) in an orbit that is tilted 5 degrees from the ecliptic plane: thus the declination of the moon can vary between 0 to 28 degrees over time. If, for example, the moon is directly above the equator, then north is up in the above figure and while the earth rotates, there will be two high equilibrium tides per lunar day with a period of 12.42 hours: a lunar semidiurnal tide. If, however, the moon is at its maximum declination and found 'above' a latitude of 28 degrees, there will be only one high tide per lunar day (24.84 hrs.) at the latitude of the sub-lunar point: a lunar diurnal tide.

The solar and lunar tides can reinforce one another during periods of a new or full moon. This is when the sun and moon are 'in line'. In the figure below, I have plotted two sinusoidal fluctuations having amplitudes of 1 (0.5) meter and periods of 12.42 (12) hrs, representing the semidiurnal tides of lunar (solar) origin. The pattern has a maximum or spring tide when the two are in phase or anti-phase and a weak or neap tide when they are 90 or 270 degrees out of phase.



At a latitude of 45N/S, the speed at which the equilibrium tide propagates as the earth revolves is quite large, over 300 m s^{-1}. This is faster than the phase speed of a surface gravity wave. Friction will cause the earth to drag the tidal bulge in the direction of the rotation causing a torque that acts to slow down the rotation of the earth at a rate of ca. 2.3 ms/century. A further interesting fact is that the solid earth has tides as well: equilibrium tidal displacements of the solid earth can actually reduce the fluid tides by about 30%.

Since the earth is not water covered, the equilibrium tide is not ‘observed’. Instead, the detail and shape of the continental boundaries play a major role in determining the actual tidal response to forcing. So does the wavelike nature of the long surface waves. In many parts of the ocean, forcing is within the frequency band where freely propagating waves can exist and thus the resulting patterns if tidal flow are related to natural standing modes of long wavelength surface waves and can vary considerably with the frequency characteristics of the various tidal components.

An example of a global tidal model for the M_2 (lunar semidiurnal: S_2 would be the corresponding solar semidiurnal) tide will be shown in class. The Topex-Poseidon altimeter has been successfully used to improve global models of tides beyond previous studies like the one shown.

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We will now investigate some aspects of free waves: waves which naturally occur on and below the ocean surface.