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Presentation and discussion by Group 1 (Sandeep, Veronika, Rosalee, Laurel)

Paper:

Cooling of the Earth: A parameterized convection study of whole versus layered models

by

McNamara and Van Keken 2000

LAYERED CONVECTION MODEL

1) Geochemical Differences and distinct reservoirs in Earth's mantle:

MORB: → Geochemically homogenous on a global scale (consistent chemistry around the world);
→ depleted/enriched in incompatible elements;
→ samples the upper mantle

OIB: → Geochemically hetrogeneous;
→ less depleted in incompatible elements with respect to MORB;
→samples the deep reservoir and brought to the surface through mantle plumes

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2) Earthquakes in subduction zone
→ Continue down to mantle and stops at 670 Km
→ commonly compressional implying
=> slabs meeting with resistance to further subduction

3) Classical Layered Geochemical Model: boundary between upper (depleted) and
lower (enriched)layer at 670 km [Ringwoodite → Pv + (Mg,Fe)O]

Distribution of radioactive elements:

Present day heat flow = Secular cooling + Radiogenic heating
Deep lower mantle
In layered model: delay between heat generation and conduction through surface and the secular cooling of Earth extends the thermal evolution

WHOLE MANTLE CONVECTION MODEL

Geophysical Observations:

(a) Seismic tomographic models → evidences that the subducted slabs are able to penetrate lower mantle;

(b) mantle plumes may originate from the CM-boundary or at least below 670 km

Numerical Calculations: Presence of a thermal boundary layer at 670 km would produce a significant amount of topography and gravity anomalies

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Parameterized convection → insulating effect of such boundary layer tends to raise temperature of lower layer above melting temperatures, unless there is a strong depletion of radiogenic elements with respect to upper layer (contradiction to the main assumption of radioactive enrichment in the lower layer)

→ Approach: Use of parameterized convection models

To establish a relationship between layered model and seismic observations

→ Layered model was proposed that places boundary at 1600 km depth

(a) it may explain interpretations of some tomographic results

(b) can be consistent with the seismic heterogeneities below 1600 km

→ Requirement:

Substantial enrichment of radioactive elements in the lower layer to Account present day mantle heat loss of 40 TW

→ It satisfies:

(a) present day heat flow

(b) range of mantle viscosity

(c) average mantle temperature

→ Started with Whole mantle convection model and compared to the proposed model

→Fundamental parameters:

Boundary layer thickness $$\delta = D\left(\frac{Ra_c}{Ra}\right)^{\beta}$$

Rayleigh Number $$Ra = \frac{\alpha\rho g D^3 \triangle T}{\kappa\eta(T)}$$

Heat Flow $$q = \frac{K\triangle T_\delta}{D}\left(\frac{Ra}{Ra_c}\right)^{\beta}$$

Values of β and R_{ac} is uncertain….try to limit the of acceptable β in present study

Whole-Mantle Convection

- Heat flow: $$q = \frac{k\Delta T_\delta}{D}\left(\frac{Ra}{Ra_c}\right)^\beta$$
- Conservation of energy:

$$V\rho c_p \frac{dT}{dt} = q_{in} A_{bottom} - q_{out} A_{top} + Q_{produced} V$$

- where $Q_{total} = \sum Q_i$ and $Q_i = H_i R_i [U] \exp(\lambda_i (t_0 - t))$
- Viscosity: $$\eta = \eta_0 \exp\left(\frac{E}{RT}\right)$$

Numerical method

- Assumptions: qin = 0 (insulating layer),
  qmantle ~ 40 TW
- Input parameters: β, Rac, E
- β: 0.10 – 0.33
- R_{ac}: 1100 and 87.79
- E: 0 and 525 kJ/mol
- η_0 and [X] are varied until ηpd and qpd are reached
- Criterion for rejection: Tavg > 4000 K at any time or T_{pd} < 1500 K

Results

- Analytical expression:

$$T_{pd} = \left(\frac{q_{pd} D}{k} \left(\frac{\kappa \eta_{pd} Ra_c}{\alpha g \rho D^3} \right)^{\beta} \right)^{\frac{1}{1+\beta}} + T_s$$

- Main conclusion: wide range of parameters can satisfy T_{pd}

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Figure 1. (a) Temperature, (b) heat flow, and (c) heat production results from two whole-mantle parameterized convection calculations. Both use $Ra_c = 1100$, $\eta(T_{pd}) = 10^{21}$ Pa · s, and $\beta = 0.30$. One of the calculations utilizes an isoviscous rheology (E = 0 kJ/mol) and the other has a temperature-dependent one (E = 525 kJ/mol).

* T_{pd} and η_{pd} can be satisfied with different rheologies

* concentration of radioactive elements depends strongly on E

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- Low values of β and high values of η_{pd} lead to high mantle temperatures

Figure 2. Whole-mantle convection results for two different values of critical Rayleigh number, (a) Ra_c = 1100 and (b) Ra_c = 87.79, in which final temperature is plotted against β. The analytical solutions for the three constrained values of present-day viscosity, η_{pd} are plotted as curves. Symbols represent final temperatures obtained from the parameterized convection calculations. Triangles, squares, and circles represent results for $\eta_{pd} = 10^{20}$, 10^{21}, and 10^{22} Pa s, respectively.

Layered Mantle Convection

- 2 layers of convecting material; boundary at 1600 km depth
- 2 models tested (end-member situations):
  - (a) Bottom TBL of top layer (TBL between layers) has zero thickness
  - (b) Bottom and top thermal boundary layer (TBL) of the top convecting layer are symmetric (equal thickness)

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Figure 3. Schematic sketches of the thermal profiles under the two treatments of the lower thermal boundary layer of the top layer: (a) this thermal boundary layer is removed, and (b) this thermal boundary layer is equal in thickness to the upper boundary layer of the top layer. Q_s and Q_b are the heat flows at the surface and boundary between the layers, respectively. D_{top} and D_{bottom} are the thicknesses of the top and bottom layers, respectively. T_s, T_{top}, T_b, and T_{bottom} are the temperatures of the surface, top layer, boundary between the layers, and bottom layer, respectively.

- Model (a):
  - Mimics convection heated entirely from within the top convecting layer
  - Gives lower bound to thickness of TBL between layers
  - The energy balance equation is solved for each layer separately:

$$V \mathrm{r} c_p \frac{dT}{dt} = q_{\mathrm{in}} A_{\mathrm{bottom}} - q_{\mathrm{out}} A_{\mathrm{top}} + Q_{\mathrm{produced}} V \qquad (4)$$

$$q_{\mathrm{bot,out}} = q_{\mathrm{top,in}}. \qquad (16)$$

- Model (b):
  - Mimics isoviscous convection heated entirely from below
  - Gives an upper bound on the thickness of the TBL between the convecting layers
  - Energy balance equation is more complicated

$$q_{\mathrm{top,out}} = \frac{K(T_{\mathrm{top}} - T_s)}{D_{\mathrm{top}}} \left(\frac{Ra_{\mathrm{top}}}{Ra_c} \right)^{\beta}, \tag{10}$$

$$q_{\mathrm{top,in}} = \frac{K(T_b - T_{\mathrm{top}})}{D_{\mathrm{top}}} \left(\frac{Ra_{\mathrm{top}}}{Ra_c} \right)^{\beta}, \tag{11}$$

$$q_{\mathrm{bot,out}} = \frac{K(T_{\mathrm{bot}} - T_b)}{D_{\mathrm{bot}}} \left(\frac{Ra_{\mathrm{bot}}}{Ra_c} \right)^{\beta}, \tag{12}$$

$$q_{\mathrm{bot,in}} = 0, \tag{13}$$

$$Ra_{\mathrm{top}} = \frac{\alpha_{\mathrm{top}} g \rho_{\mathrm{top}} (T_b - T_s) D_{\mathrm{top}}^3}{\kappa_{\mathrm{top}} \eta(T_{\mathrm{top}})}, \tag{14}$$

$$Ra_{\mathrm{bot}} = \frac{\alpha_{\mathrm{bot}} g \rho_{\mathrm{bot}} (T_{\mathrm{bot}} - T_b) D_{\mathrm{bot}}^3}{\kappa_{\mathrm{bot}} \eta(T_{\mathrm{bot}})}. \tag{15}$$

$$q_{\mathrm{bot,out}} = q_{\mathrm{top,in}}. \tag{16}$$

Ways to address the heat flow from the core

- TBL between the core and mantle
- Core and mantle are thermally coupled (no TBL)
- Insulated CMB
  - The authors choose this method because heat flow from the core into the mantle is assumed small compared with heat production of the mantle.

Radioactive element ratios and concentrations

- Start with ratios from *Kellogg et al (1999):*
  - *U concentrations of 7 ppb in top layer, 25.6 ppb in bottom layer*
- *Observed heat flow requires higher concentration in lower layer; K/U and Th/U ratios are constant*

Layered convection results

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- Top layer has brief fast cooling, then long slow cool to present
- Bottom layer has brief initial heating, then long slow cool
- Top layer cools more rapidly initially
- Bottom layer undergoes more intense heating over longer time; temperature constraints not satisfied

Figure 4: (a) Vanishing lower TBL in top layer; (b) Symmetric TBLs in top layer

(a) Vanishing lower TBL in top layer

(b) Symmetric TBLs in top layer

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(a) Constraints are met for only a narrow range of parameter values

(b) Constraints are not met for any parameter values

Figure 5: Red circles indicate cases in which the temperature of the bottom layer exceeded 4000K at some point in its thermal history. Blue squares represent cases in which the top layer has a final temperature below 1500K. Green asterisks are for cases that satisfy the temperature constraints.

Discussion / Results

- whole-mantle convection satisfies the constraints for a fairly wide range of the parameters
- layered-mantle convection satisfies the constraints for a much smaller range of parameters
- furthermore, acceptable results are only observed for the layered case when the lower thermal boundary layer of the top layer is removed

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Discussion / Results

- models have assumed the same value of beta in both layers
- increasing beta reduces the insulating effect of boundary layers, and decreasing beta will do the reverse
- thus, if the top layer has a lower beta than the bottom layer, the parameter range that produces acceptable results will greatly increase
- however, the reverse is thought to be true, reducing the range of acceptable values even further
- **it is extremely difficult to reconcile layered mantle models with geophysical observations using a parameterized convection approach; thus, whole-mantle convection is favored**