12.540 Principles of the Global Positioning System
Lecture 08

Prof. Thomas Herring

http://geoweb.mit.edu/~tah/12.540

Summary

- Review:
  - Examined methods for measuring distances
  - Examined GPS codes that allow a type of distance measurement and phase to be measured
- Today:
  - Examine how the range measurements are defined and used
  - Use of carrier phase measurements
  - Examine RINEX format and look at some "raw" data

Pseudorange measurements

- When a GPS receiver measures the time offset it needs to apply to its replica of the code to reach maximum correlation with received signal, what is it measuring?
- It is measuring the time difference between when a signal was transmitted (based on satellite clock) and when it was received (based on receiver clock).
- If the satellite and receiver clocks were synchronized, this would be a measure of range
- Since they are not synchronized, it is called "pseudorange"

Basic measurement types

- Pseudorange:

$$P_k^p = (t_k - t^p) \cdot c$$

Where P^p_k is the pseudorange between receiver k and satellite p; t_k is the receiver clock time, t^p is the satellite transmit time; and c is the speed of light

This expression can be related to the true range by introducing corrections to the clock times

$$t_k = \tau_k + \Delta t_k \qquad t^p = \tau^p + \Delta t^p$$

τ_k and τ^p are true times; Δt_k and Δt^p are clock corrections

Basic measurement types

- Substituting into the equation of the pseudorange yields

$$P_k^p = \left[(\tau_k - \tau^p) + (\Delta t_k - \Delta t^p)\right] \cdot c$$

$$P_k^p = \rho_k^p + (\Delta t_k - \Delta t^p) \cdot c + \underbrace{I_k^p}_{\substack{\text{Ionspheric}\\ \text{delay}}} + \underbrace{A_k^p}_{\substack{\text{Atmospheric}\\ \text{delay}}}$$

- ρ_k^p is true range, and the ionospheric and atmospheric terms are introduced because the propagation velocity is not c.

Basic measurement types

- The equation for the pseudorange uses the true range and corrections applied for propagation delays because the propagation velocity is not the in-vacuum value, c, 2.99792458×10^8 m/s
- To convert times to distance c is used and then corrections applied for the actual velocity not equaling c. In RINEX data files, pseudorange is given in distance units.
- The true range is related to the positions of the ground receiver and satellite.
- Also need to account for noise in measurements

Pseudorange noise

- Pseudorange noise (random and not so random errors in measurements) contributions:
  - **Correlation function width:** The width of the correlation is inversely proportional to the bandwidth of the signal. Therefore the 1MHz bandwidth of C/A produces a peak 1 μsec wide (300m) compared to the P(Y) code 10MHz bandwidth which produces 0.1 μsec peak (30 m) Rough rule is that peak of correlation function can be determined to 1% of width (with care). Therefore 3 m for C/A code and 0.3 m for P(Y) code.

Pseudorange noise

- More noise sources
  - **Thermal noise:** Effects of other random radio noise in the GPS bands
    Black body radiation: $I=2kT/\lambda^2$ where I is the specific intensity in, for example, watts/(m^2Hz ster), k is Boltzman's constant,1.380 x 10^{-23} watts/Hz/K and λ is wavelength. Depends on area of antenna, area of sky seen (ster=ster-radians), temperature T (Kelvin) and frequency. Since C/A code has narrower bandwidth, tracking it in theory has 10 times less thermal noise power (depends on tracking bandwidth) plus the factor of 2 more because of transmission power).
    Thermal noise is general smallest effect
  - **Multipath:** Reflected signals (discussed later)

Pseudorange noise

- The main noise sources are related to reflected signals and tracking approximations.
- High quality receiver: noise about 10 cm
- Low cost receiver ($200): noise is a few meters (depends on surroundings and antenna)
- In general: C/A code pseudoranges are of similar quality to P(Y) code ranges. C/A can use narrowband tracking which reduces amount of thermal noise
- Precise positioning (P-) code is not really the case.

Phase measurements

- Carrier phase measurements are similar to pseudorange in that they are the difference in phase between the transmitting and receiving oscillators. Integration of the oscillator frequency gives the clock time.
- Basic notion in carrier phase: $\phi = f\Delta t$ where ϕ is phase and f is frequency

Phase measurements

$$\phi_k^p(t_r) = \phi_k(t_r) - \phi_r^p(t_r) + N_k^p(1)$$

- The carrier phase is the difference between phase of receiver oscillator and signal received plus the number of cycles at the initial start of tracking
- The received phase is related to the transmitted phase and propagation time by

$$\phi_r^p(t_r) = \phi_t^p(t_t) = \phi_t^p(t_r - \rho_k^p / c) = \phi_t^p(t_r) - \dot{\phi}^p(t_r) \cdot \rho_k^p / c$$

Phase measurements

- The rate of change of phase is frequency. Notice that the phase difference changes as ρ/c changes. If clocks perfect and nothing moving then would be constant.
- Subtle effects in phase equation
  - Phase received at time t = phase transmitted at $t-\tau$ (riding the wave)
  - Transmitter phase referred to ground time (used later). Also possible to formulate as transmit time.

Phase measurements

- When phase is used it is converted to distance using the standard L1 and L2 frequencies and vacuum speed of light.
- Clock terms are introduced to account for difference between true frequencies and nominal frequencies. As with range ionospheric and atmospheric delays account for propagation velocity

Precision of phase measurements

- Nominally phase can be measured to 1% of wavelength (~2mm L1 and ~2.4 mm L2)
- Again effected by multipath, ionospheric delays (~30m), atmospheric delays (3-30m).
- Since phase is more precise than range, more effects need to be carefully accounted for with phase.
- Precise and consistent definition of time of events is one the most critical areas
- In general, phase can be treated like range measurement with unknown offset due to cycles and offsets of oscillator phases.

GPS Data file formats

- Receivers use there own propriety (binary) formats but programs convert these to standard format called Receiver Independent Exchange Format (RINEX)
- teqc available at http://www.unavco.org/facility/software/teqc/teqc.html is one of the most common
- The link to the RINEX format is:
- ftp://igscb.jpl.nasa.gov/igscb/data/format/rinex2.txt

Rinex header

```
 2.00           OBSERVATION DATA    G (GPS)
teqc  1998Jul1      Thomas Herring      20020117 06:28:28UTCPGM / RUN BY / DATE
Linux 2.0.30|PentPro|gcc|Linux|486/DX+
                                                    RINEX VERSION / TYPE
BIT 2 OF LLI FLAGS DATA COLLECTED UNDER A/S CONDITION       COMMENT
ETAB
tah                 MIT
7910                TRIMBLE 4000SSE                         REC # / TYPE / VERS
                                              NP 7.19; SP 3.04
7910                TRM22020.00+GP
 -2225431.6719 -4676995.2141  3711599.9580                  APPROX POSITION XYZ
                                                     OBSERVER / AGENCY
        1.0000        0.0000        0.0000
     1     1
                                                COMMENT
     7    L1    L2    C1    P2    P1    D1                  # / TYPES OF OBSERV
   15.0000
                                                  D2
 SNR is mapped to RINEX snr flag value [1-9]
                                                      WAVELENGTH FACT L1/2
  L1: 3 -> 1; 8 -> 5; 40 -> 9
  L2: 1 -> 1; 5 -> 5; 60 -> 9
                                          MARKER NAMEANTENNA: DELTA H/E/N
                                                ANT # / TYPE
  2002     1    16    18       15.000000
                                    49
                                                            END OF HEADER

                                                      COMMENT
                                                           TIME OF FIRST OBS
```



```
INTERVAL
   COMMENT
```

RINEX Data block

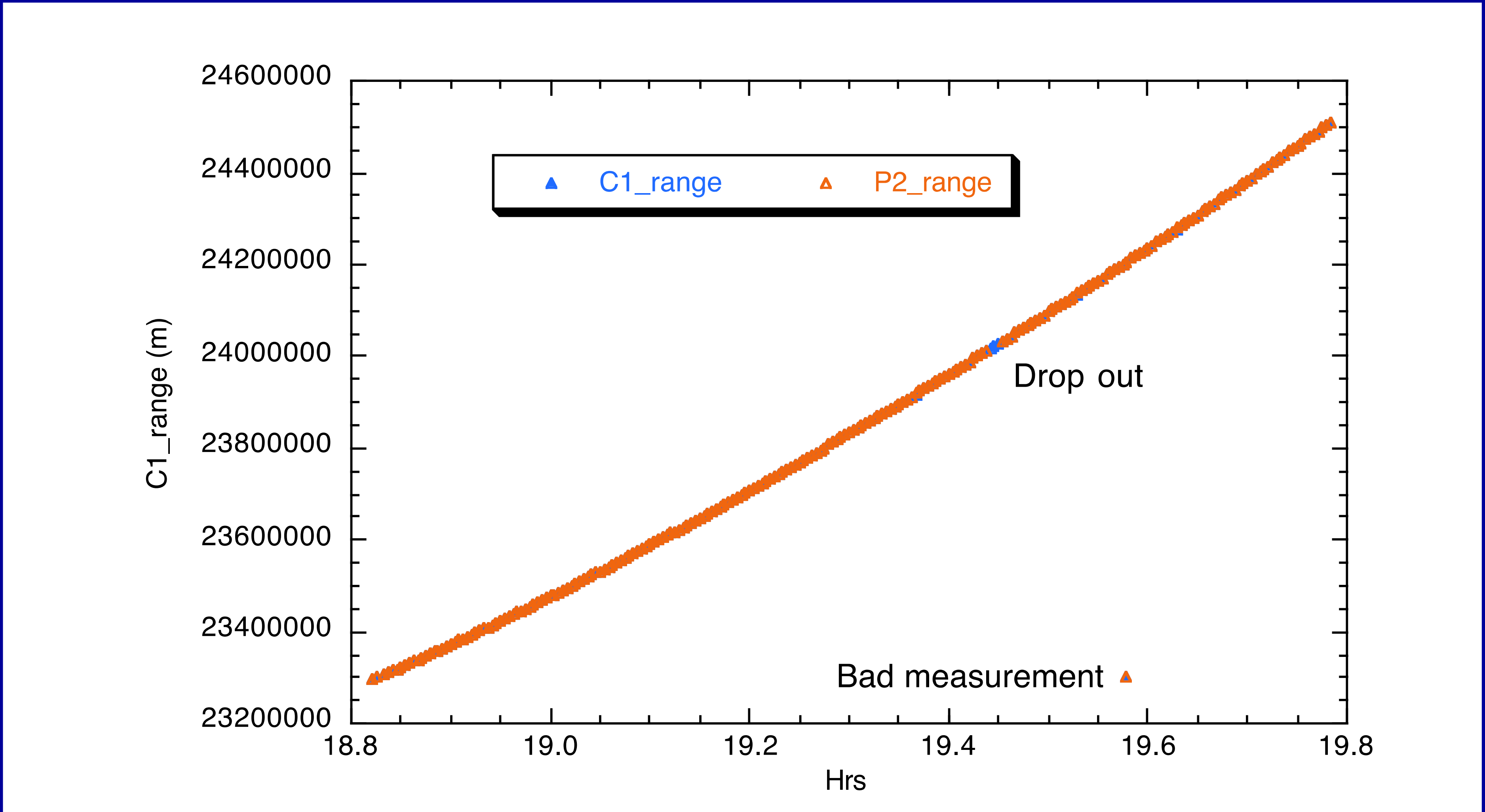
```
 2  1 16 18 49 15.0000000  0  6G 2G 7G11G26G27G28
    787986.44256     602246.12855  23296205.6024   23296215.6954
     -1344.9694       -1048.0284
  -2277471.81757  -1740781.13556  21398430.3444   21398436.5904
      2700.6094        2104.3714
  -1100283.16556   -822375.51955  23502290.7894   23502300.4844
      1062.9224         828.2514
  -1925082.16955  -1445658.56955  23293616.9844   23293626.4574
      2176.8284        1696.2304
   1016475.79056    786021.95356  21979554.0634   21979561.0984
     -1782.8124       -1389.2054
   -572573.66057   -446158.58357  20873925.7664   20873929.7624
       446.3594         347.8134
  2  1 16 18 49 30.0000000  0  6G 2G 7G11G26G27G28
```

- Phase in cycles, range in meters

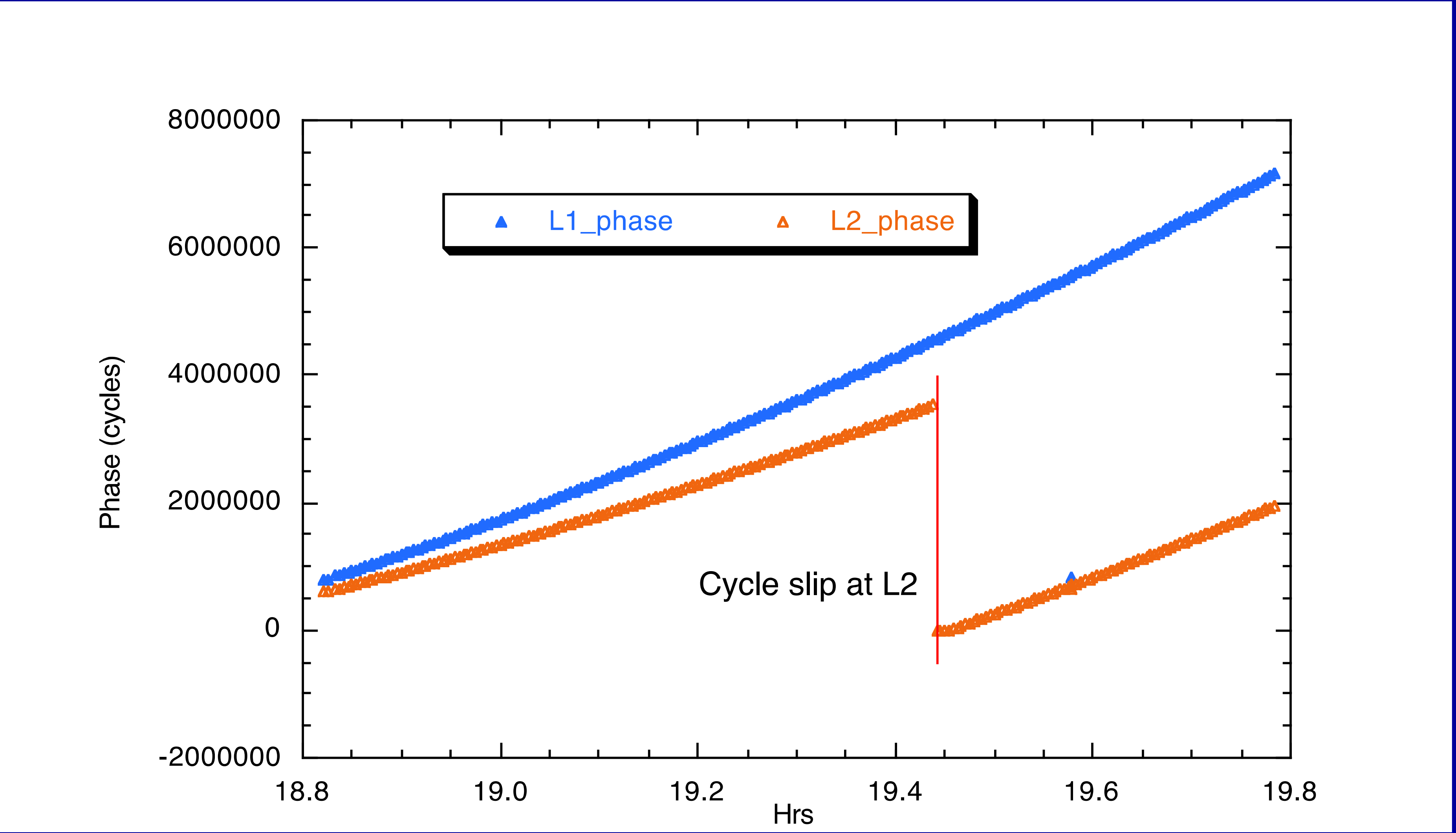
Examine Rinex file data

- Next set of plots will look at the contents of a rinex file.
- Examples for one satellite over about 1 hour interval:
  – Raw range data
  – Raw phase data
  – Differences between data

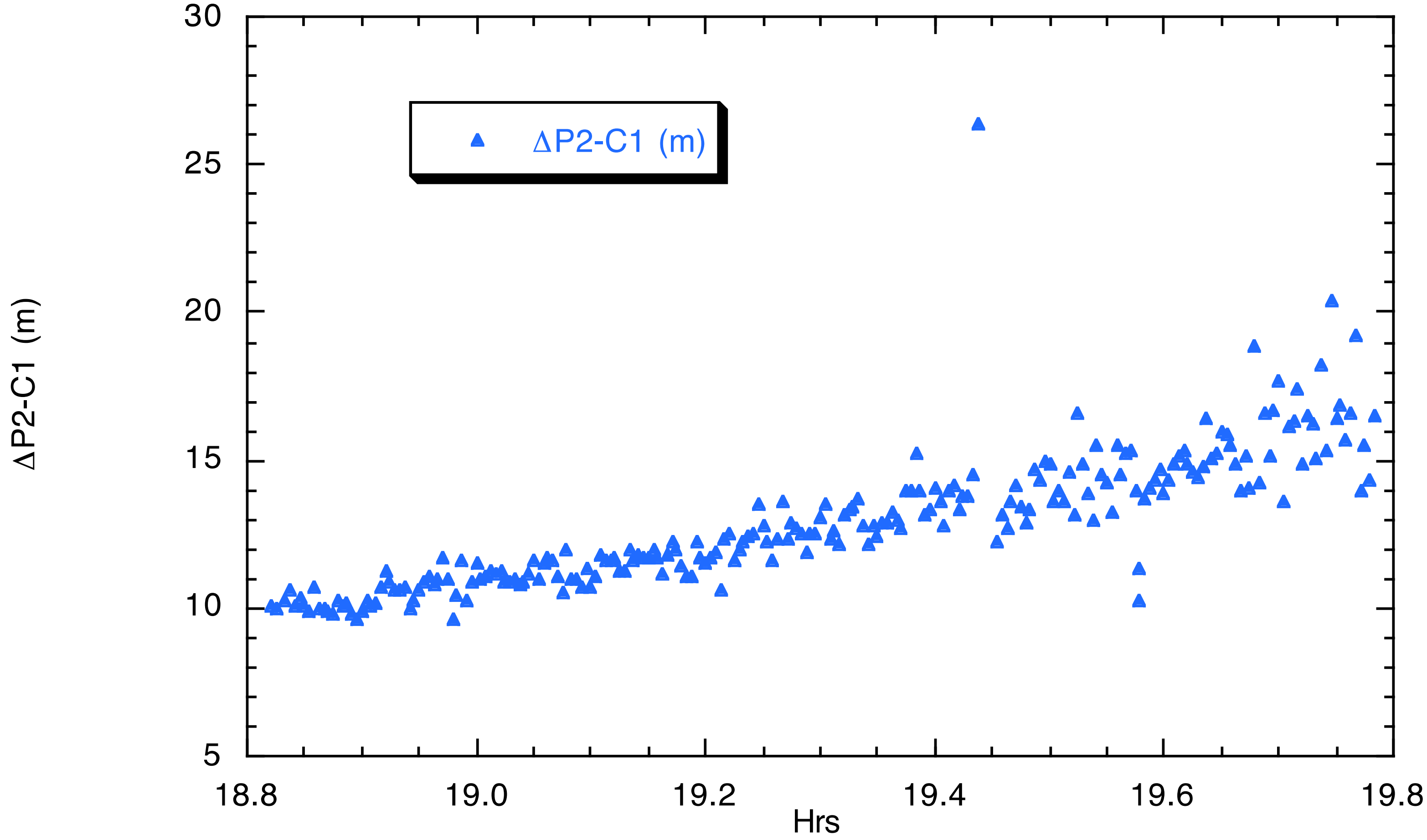
Raw range data



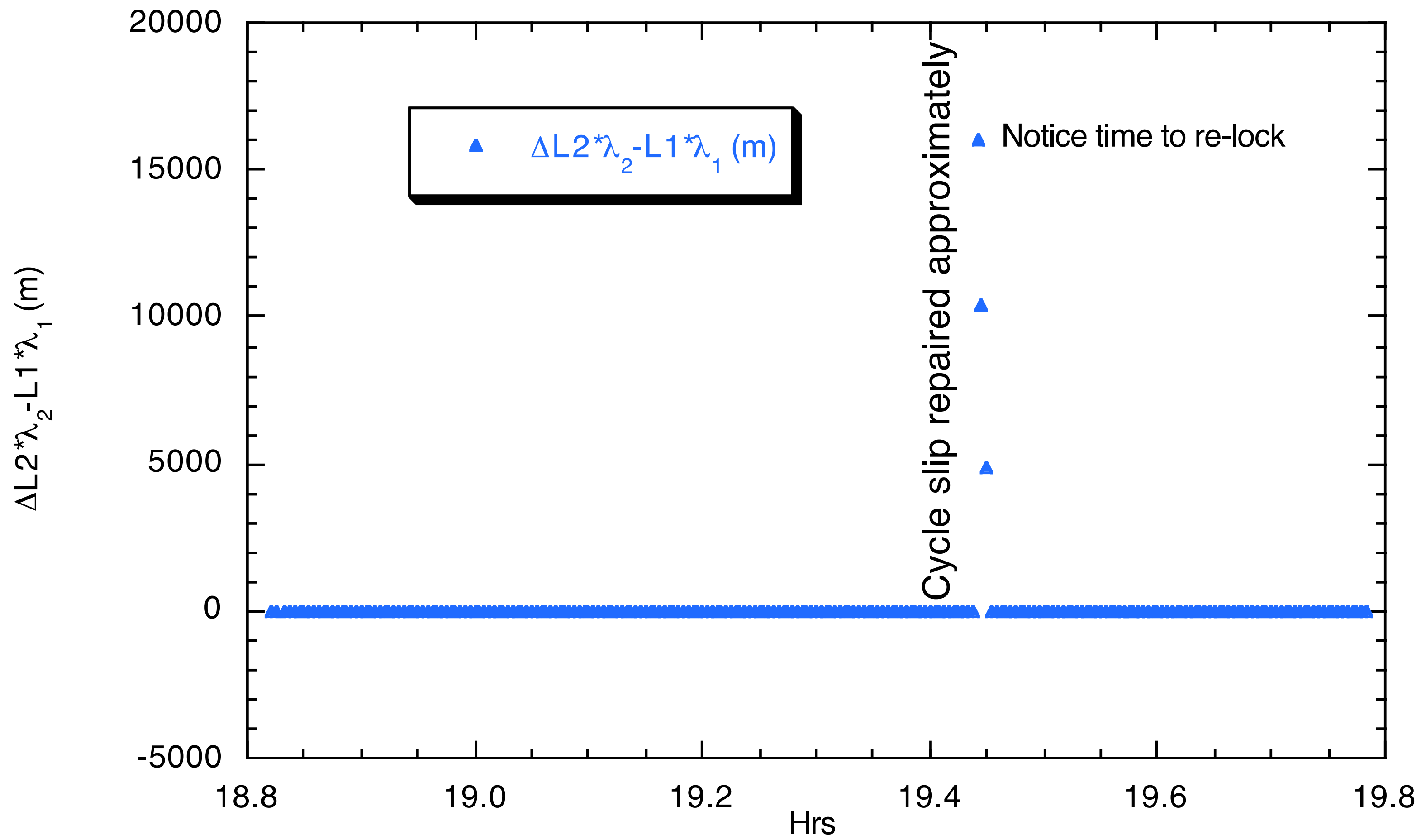
Raw phase data (Note: sign)



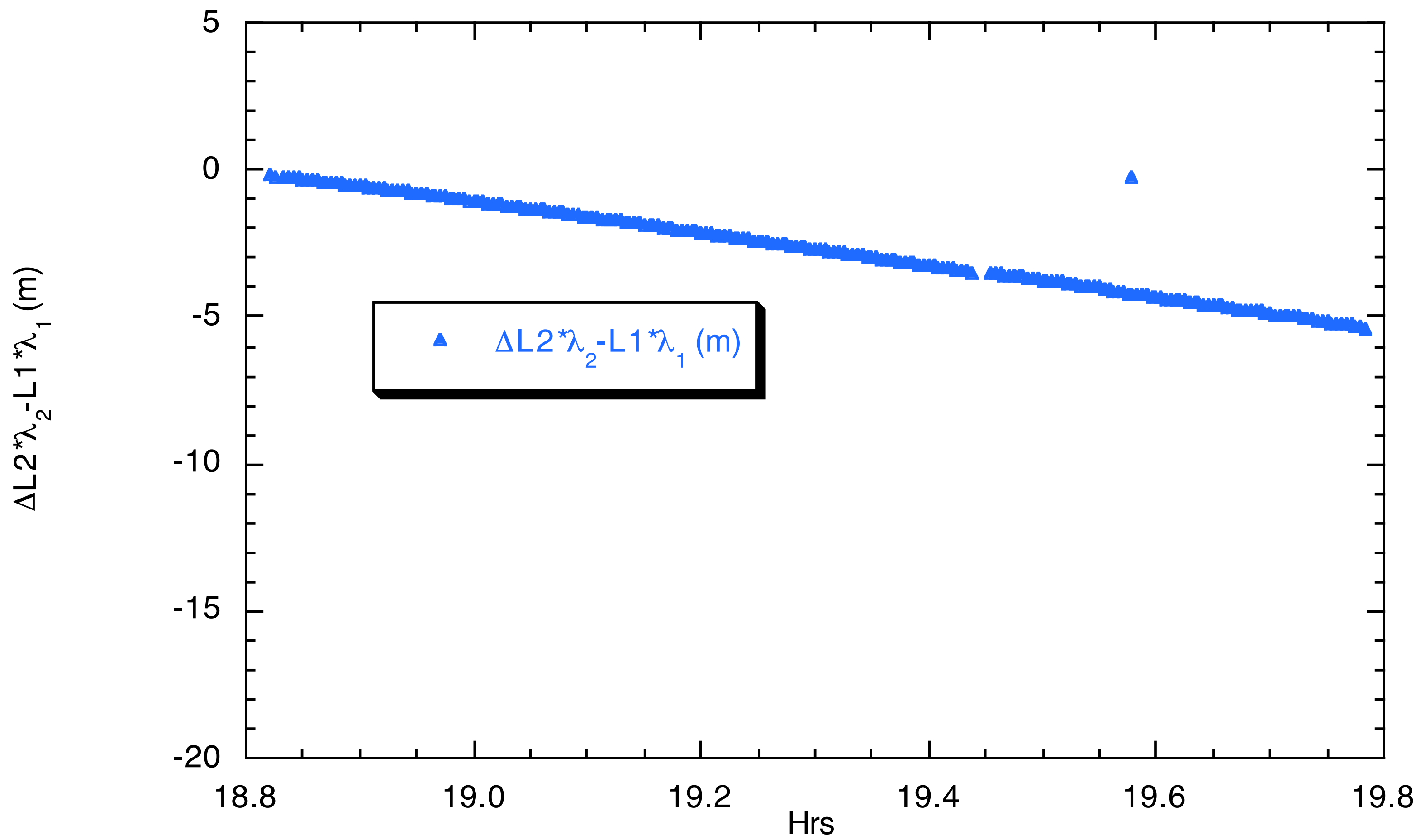
L2-L1 range differences



L2-L1 phase differences



Zoomed L2-L1 phase



Plot characteristics

- Data set plotted etab.plt.dat
- Notice phase difference is opposite sign to range difference (discuss more in propagation lectures)
- More manipulation can me made of data: How about C1-L1*λ

Summary

- Looked at definitions of data types
- Looked at data and its characteristics.
- Next class, we finish observables and will examine:
  - Combination of range and phase that tell us more things
  - How well with a simple model can we match the data shown.
  - Where do you get GPS data?