

Question 1: (a) Convert geodetic position 290 deg Long 42 deg latitude ellipsoidal height 0 m into Cartesian and geocentric coordinates.

(b) How far apart on the surface of the Earth (approximate radius 6371km) would be locations that have geocentric latitude 42 deg and geodetic latitude 42 deg. Give answer in km.

(c) Using the conversion between XYZ and Geodetic coordinates, determine if ellipsoid heights are measured along a straight line. (Orthometric heights are measured along a curved path that is always along the local direction of gravity).

(30 pts)

(a) This is simple problem using the formulas given in class (or most geodetic text books). The corresponding XYZ coordinates, using the WGS84 ellipsoid parameters $a=6378137.0$ m, $f=1/298.257222101$, yields

Cartesian XYZ coordinates 1623569.2675 4460719.9014 4245603.8360 m

(b) Converting these values back to geocentric latitude and longitude coordinates yields: Lat = 41.8087 deg, Long = -70.0 deg, radius = 6368606.6873 m. If the latitude types were confused, then the error in the latitude, expressed as a distance along the circular arc (using 6371 km as mean radius of the Earth), would be 21.3 km.

The figure below shows the error as function of latitude. Since longitude is the same between geodetic and geocentric coordinates, the error only depends on latitude.

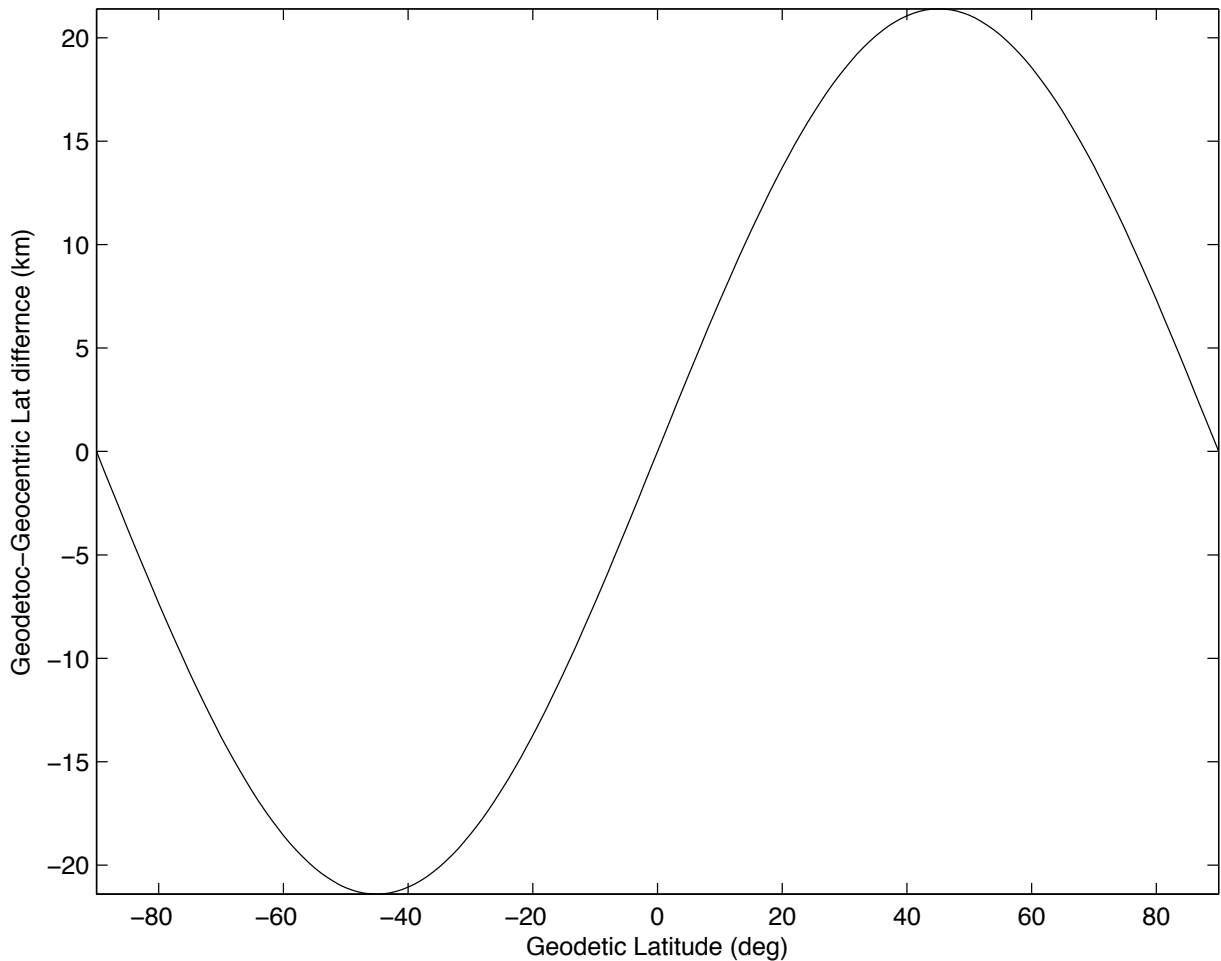
(c) This question involves constructing a vector normal to the ellipsoid and testing for points along this normal all have the same geodetic latitude and longitude and height which equals the amount displayed along the normal. The transformation vector is

$$dXYZ = [\cos(\text{lat}) \cdot \cos(\text{long}) \cos(\text{lat}) \cdot \sin(\text{long}) \sin(\text{lat}_i)] dh$$

Part c of the solution gives the results showing indeed that the height is along the normal to ellipsoid.

Out put of matlab codes

```
12.540 HW 01 Question 1
(a) Coordinates
GEODETTIC Lat, Long, Height      42.0000 290.0000 deg      0.0000 m
Cartesian XYZ coordinates      1623569.2675 -4460719.9014 4245603.8360 m
GEOCENTRIC Lat, Long, Radius    41.8087 290.0000 deg    6368606.6873 m
(b) Difference at latitude ( 42.0000 deg) 21.2718 km
(c) Difference with height
For height change 1000.0000 m,
Difference in lat 3.6637e-15 rad, height -3.9116e-08
+++++
```



Question 2: A broadcast ephemeris file for March 31 2010 is given below. All of the GPS satellites are given. For the set of PRN's 2 5 8 10 15 18 21 24 27 29 compute

- (1) A 3-D plot of the trajectories of the motions of the satellites in
 - (a) Inertial space. Produce plots for 2 orbits of the satellites. Comment on the how closely the satellite tracks overlap.
 - (b) Earth fixed frame. Plots for 2 orbits of the satellites.
- (2) A ground track plot (i.e., the track of the satellite position radially projected to the surface of the sphere). OPTIONAL: show projection onto an ellipsoid (Projection of ground normal to the satellite).
- (3) A sky map for geocentric location 288.90 deg longitude, 42.36 deg latitude for the period 2010 03 31 14 00 to 2010 04 01 14 00 GPST (i.e., 24-hours) . A sky map shows azimuth and zenith distance (90-elevation angle) as a radial plot with zenith distance scaled to a convenient distance unit. At the 15:15 GPST all satellites in the list should be visible.

(70 pts).

The solutions to this homework are implemented in both Fortran and Matlab. The Fortran solution also uses the Generic Mapping Toolbox (GMT) for the plots. The Matlab solution code is [HW01_2012.m](#)

(1.a) The broadcast ephemeris formulas can be used except that in computing the corrected longitude of the ascending node (page 100 of ICD200C), the value for the rotation rate of the Earth ($\dot{\Omega}_e$) is set to zero (thus removing the rotation of the Earth). Figure 1 shows one view of the inertial orbits. (Note: the Earth is shown only for scale. It will be rotating in this figure. However, the direction toward Greenwich on the figure does show the direction of the inertial X-axis). The dots are shown in the orbit every 5 minutes. The positions of each satellite are shown at 2010 03 31 14 00. Only the 10 satellites listed above are shown in the Figure 1a. These results are generated with the Fortran code. Figure 1b shows the same figure generated with Matlab.

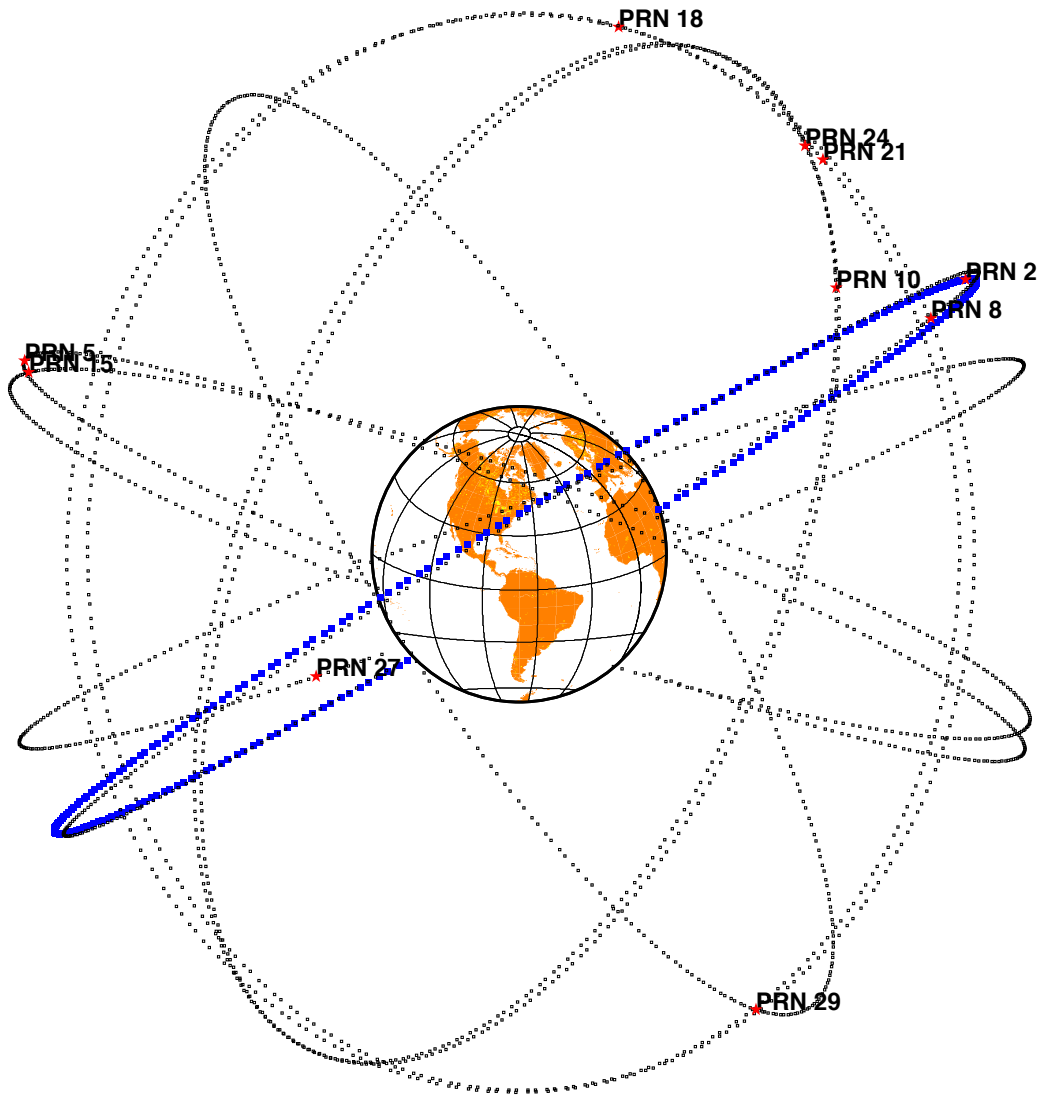


Figure 1a: Inertial view of orbits. Viewed from declination 34 degs and Right Ascension -245 degs. During GPS week 1519, the satellites were in the following planes:

Plane 00: 07, 08, 09, 24, 27, 31;

Plane 01: 05, 12, 16, 28, 30;

Plane 02: 03, 06, 17, 19, 29;

Plane 03: 02, 04, 11, 21, 24;

Plane 04: 10, 18, 20, 22, 32;

Plane 05: 13, 14, 15, 23, 26.

12.540 HW01: Q2.1(a)

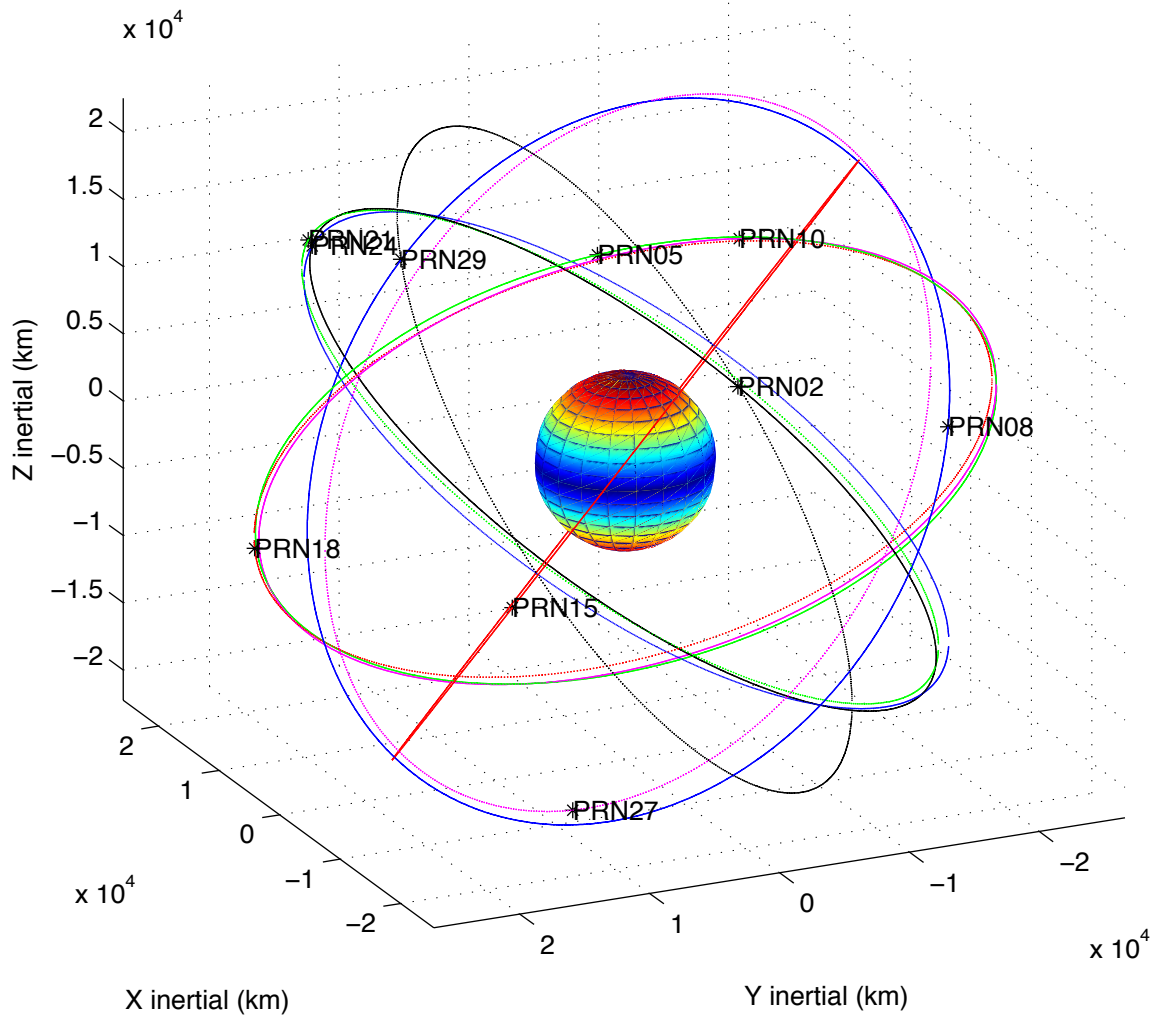


Figure 1b: Inertial plot generated with Matlab code. View angle is different from Figure 1a.

(1.b) Earth fixed orbits. These are generated using the formulas for the broadcast ephemeris. In this case, the formulas in the ICD200C document are used exactly as they appear. Results are shown in Figure 2. The positions are marked at 17:00 GPST. This time is 1 hour before the times on broadcast ephemeris messages (i.e., the tk used in calculating the orbital positions is - 3600 seconds). Getting the time correct will be important for the next homework exercise where the satellite positions will be used to compute theoretical range data to be compared to measured pseudo ranges.

In units of meters; the positions of the satellites are (X Y Z coordinates) at the first time are March 31, 2010 at 14:00 hrs GPS-time

Generating svx_mit00900.10n.t3d

EMJD	55286.58	PRN 02	XYZ	17858419.688	-14112320.575	13266513.491
EMJD	55286.58	PRN 03	XYZ	-9966634.523	23229994.378	7198261.237
EMJD	55286.58	PRN 04	XYZ	26175407.208	-5560362.718	-87234.876
EMJD	55286.58	PRN 05	XYZ	6675782.428	-14640726.590	21095927.777
EMJD	55286.58	PRN 06	XYZ	-13550107.018	20200354.134	10691028.119
EMJD	55286.58	PRN 07	XYZ	18556496.412	8085733.200	17271813.996
EMJD	55286.58	PRN 08	XYZ	25852890.988	924792.807	7123807.582
EMJD	55286.58	PRN 10	XYZ	13767035.201	-7261806.426	21364851.209
EMJD	55286.58	PRN 11	XYZ	4730448.580	16311748.690	-20651620.226
EMJD	55286.58	PRN 12	XYZ	-9627959.577	-22190643.257	-11084299.707
EMJD	55286.58	PRN 13	XYZ	7781433.187	15581789.413	19908897.551
EMJD	55286.58	PRN 14	XYZ	-16024500.145	3920233.808	-20651050.641
EMJD	55286.58	PRN 15	XYZ	6449232.512	-25639638.957	-1756552.807
EMJD	55286.58	PRN 16	XYZ	-12132395.552	8904717.408	21940837.783
EMJD	55286.58	PRN 17	XYZ	16412106.487	-3871371.531	-20370047.281
EMJD	55286.58	PRN 18	XYZ	-19764420.432	-17278020.606	-4930987.492
EMJD	55286.58	PRN 19	XYZ	-7597857.875	25041422.510	-3435832.254
EMJD	55286.58	PRN 20	XYZ	12365873.332	21943101.110	-8654221.970
EMJD	55286.58	PRN 21	XYZ	-22210633.095	-6114882.795	14063976.554
EMJD	55286.58	PRN 22	XYZ	-20649821.752	-7020558.159	-15054600.747
EMJD	55286.58	PRN 23	XYZ	2375861.408	22557935.154	13532626.808
EMJD	55286.58	PRN 24	XYZ	-21675898.965	-6460675.857	13833224.027
EMJD	55286.58	PRN 25	XYZ	7781433.187	15581789.413	19908897.551
EMJD	55286.58	PRN 26	XYZ	-6834742.214	-17254173.569	-19342002.145
EMJD	55286.58	PRN 27	XYZ	5195809.818	-15530151.505	-20195310.811
EMJD	55286.58	PRN 28	XYZ	20521379.875	10094899.480	-13275915.694
EMJD	55286.58	PRN 29	XYZ	-7867346.600	-17162815.239	18792191.143
EMJD	55286.58	PRN 30	XYZ	-17489142.555	-19937696.981	-2307620.478
EMJD	55286.58	PRN 31	XYZ	-25491009.349	7376627.752	2723976.601
EMJD	55286.58	PRN 32	XYZ	3250257.083	21589652.471	-15006294.901

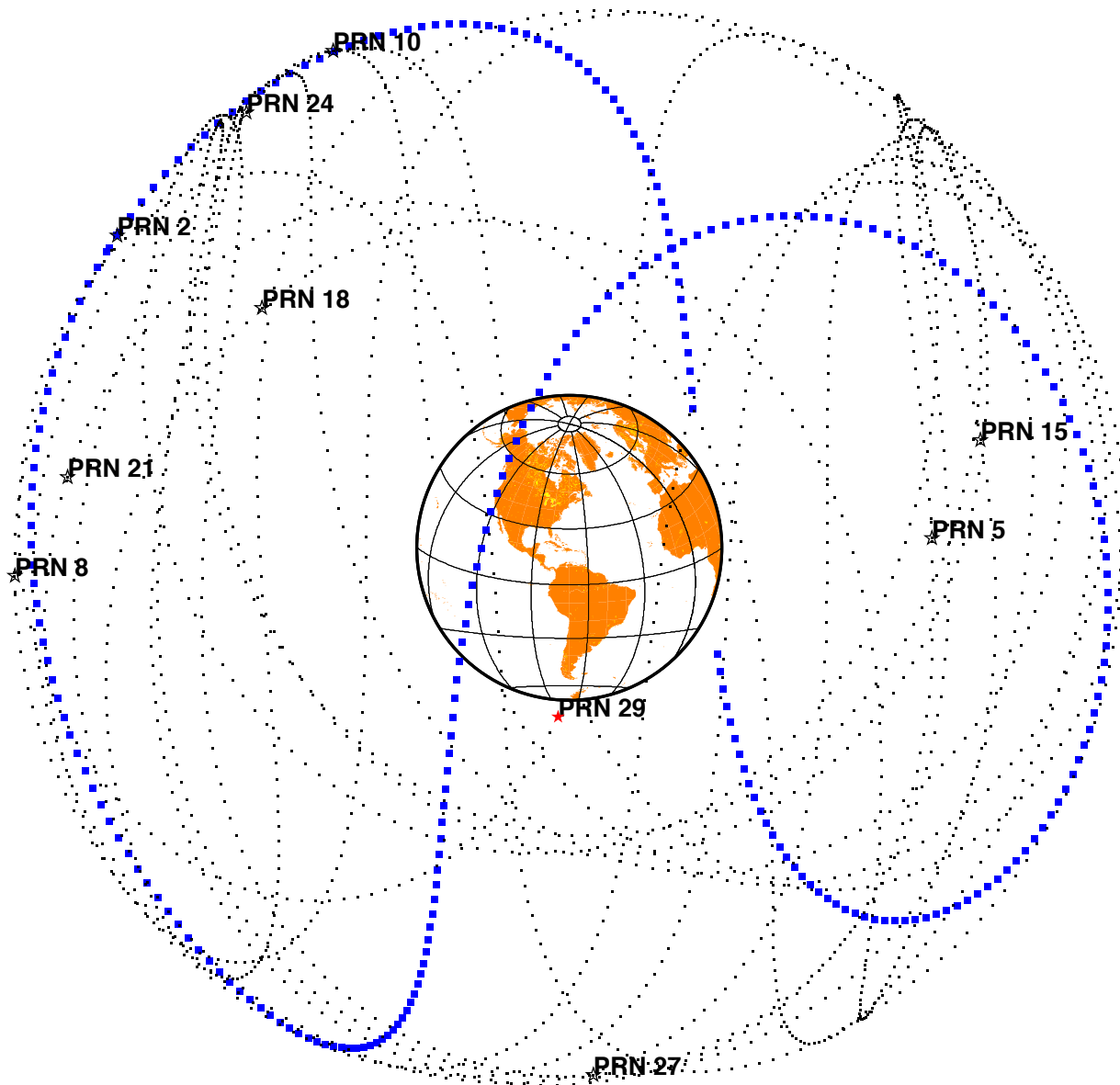


Figure 2a: Earth fixed coordinates of satellites from broadcast ephemeris.

12.540 HW01: Q2.1(b)

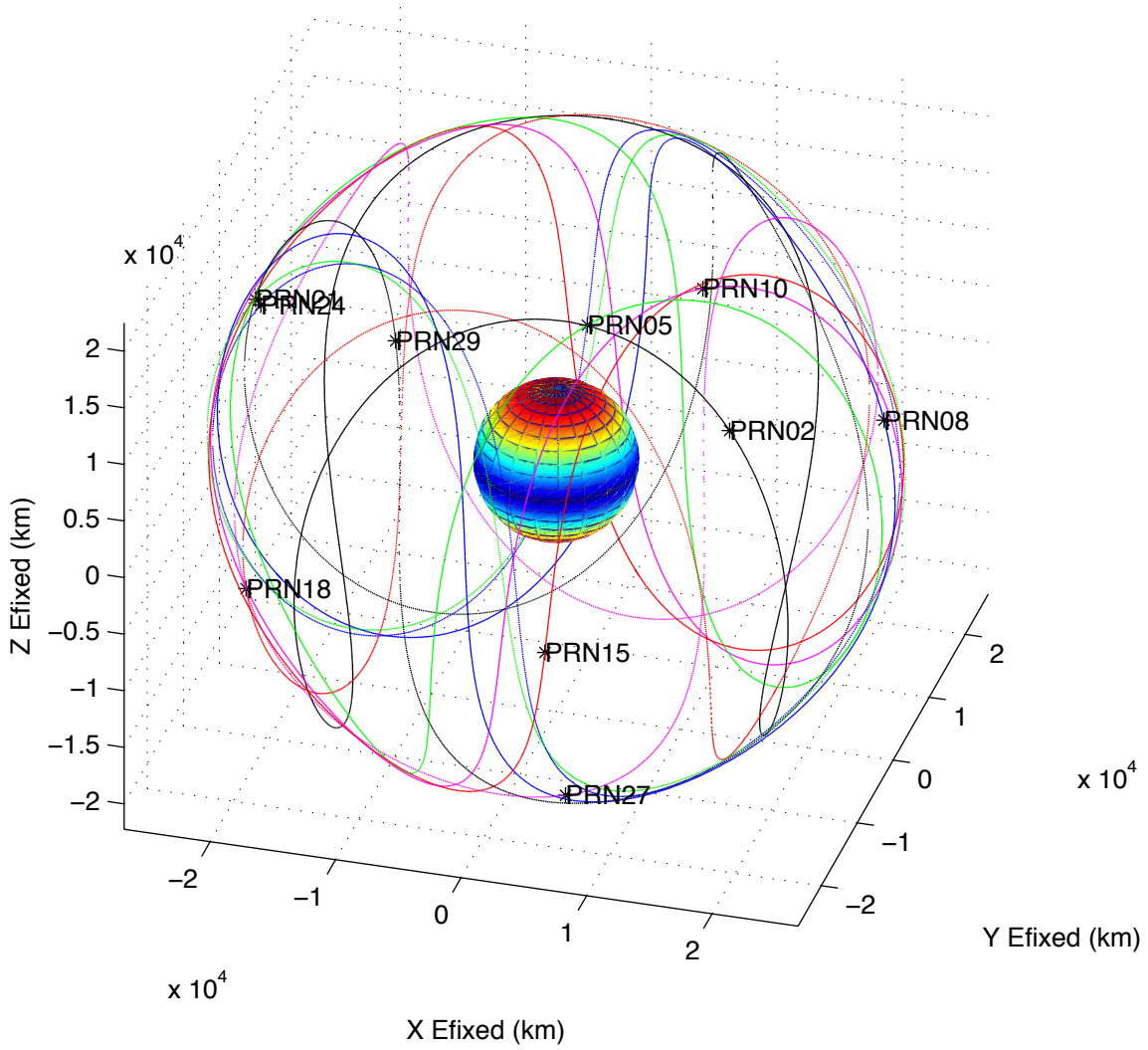


Figure 2b: Earth view from the matlab solution.

(2) Ground track: This is simply a matter of converting the XYZ Earth-fixed coordinates of the satellite to latitude (either geodetic or geocentric was OK) and longitude and plotting the results. The results are shown in Figure 3 with the stars showing the location on March 31, 2010 at 14:00 hrs GPS-time.

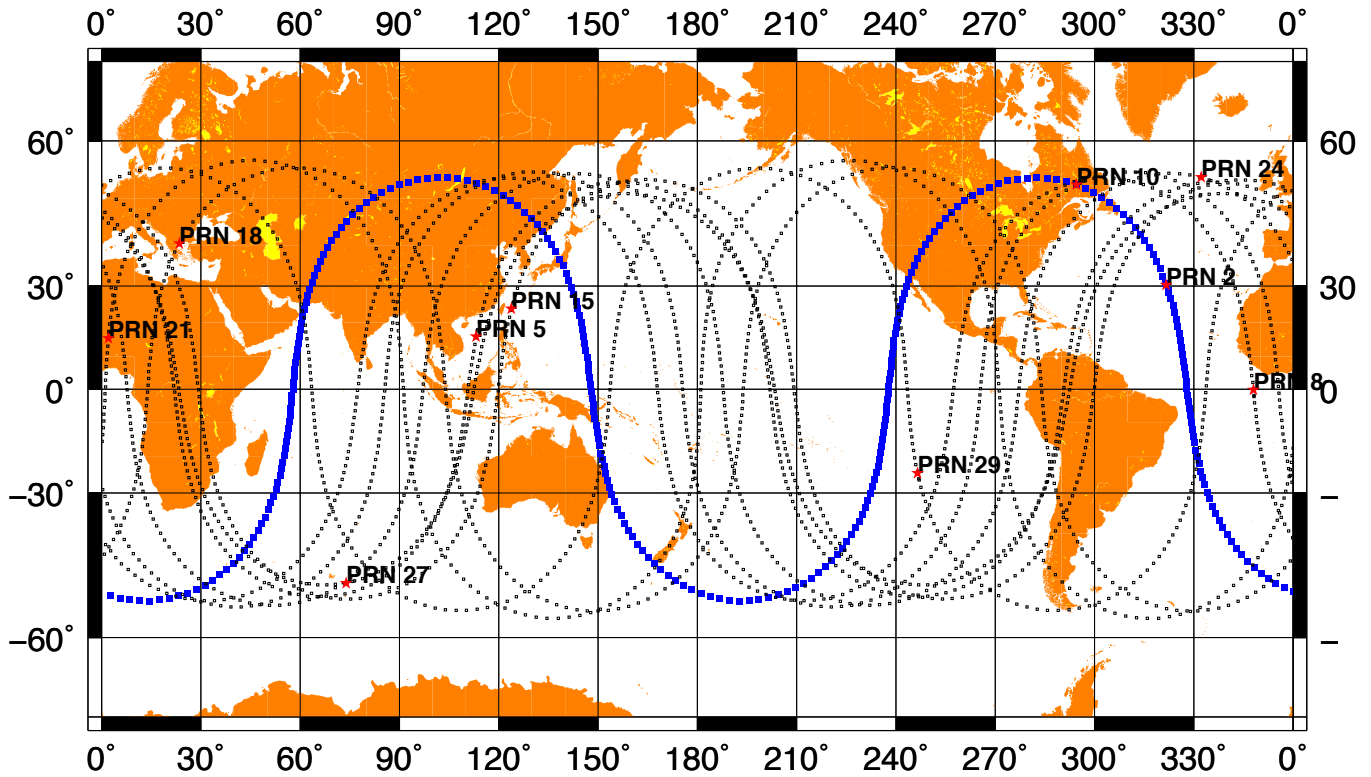


Figure 3a: Ground track from Fortran code.

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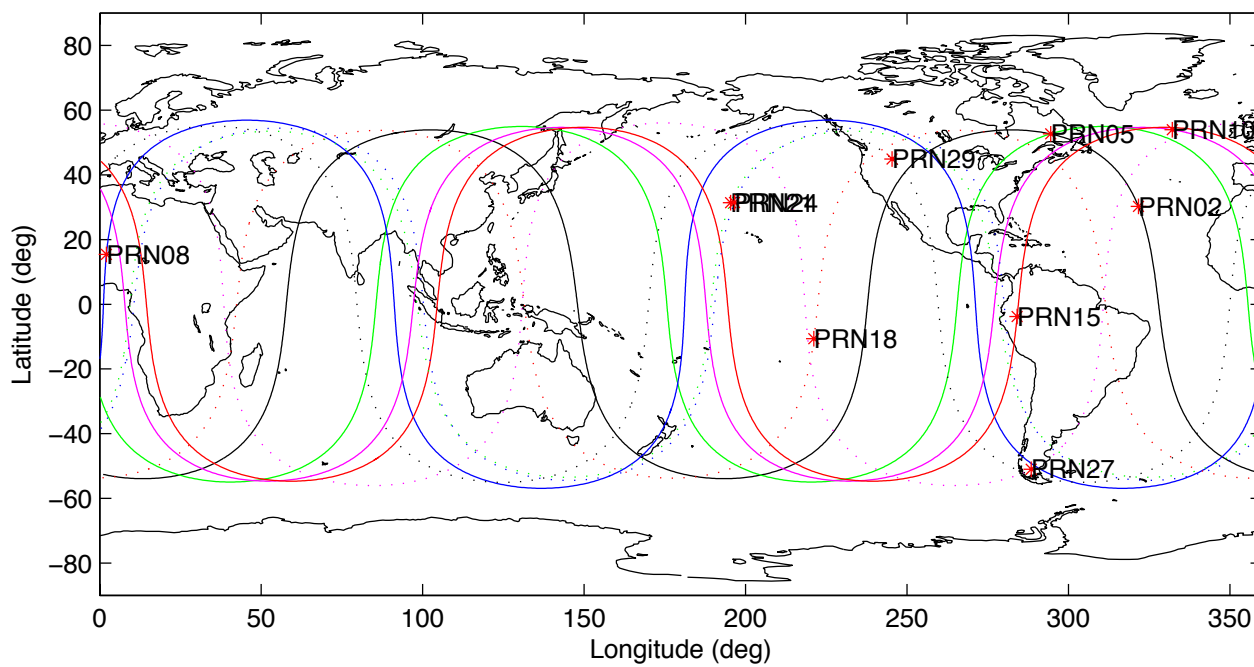


Figure 3b: Ground track from Matlab code.

(3) Sky plots of tracks of satellites for 288.90 deg longitude, 42.36 deg latitude. The basic method for solving this problem is take the ground-station to satellite unit-vector which is known in global XYZ coordinates and rotating it into a local North, West, Up frame. In this frame, the azimuth and elevation can be compute using the same formulae used compute latitude and longitude. Below I give Fortran code fragments that be used for the calculation. Rotation matrix from XYZ to NEU (In the definition below the E is the East coordinate and the local system is left-handed).

```

c      lat_i is the latitude (rads in Fortran), long is longitude. rot_mat is
c      matrix that rotates from XYZ to NEU.
c....  latitude -- north component
        rot_mat(1,1) = -sin(lat_i)*cos(long)
        rot_mat(1,2) = -sin(lat_i)*sin(long)
        rot_mat(1,3) =  cos(lat_i)
c
c....  longitude -- east component
        rot_mat(2,1) = -sin(long)
        rot_mat(2,2) =  cos(long)
        rot_mat(2,3) =  0.d0
c
c....  radius      -- up component
        rot_mat(3,1) = cos(lat_i) * cos(long)
        rot_mat(3,2) = cos(lat_i) * sin(long)
        rot_mat(3,3) = sin(lat_i)

```

The satellite-ground station vector, converted to unit vector is multiplied by rot_mat to get local topocentric coordinates. These are converted to azimuth and elevation angle using:

```

*      Compute the horizontal length of the unit vector.  unit_loc is the
*      station to satellite unit vector rotated into the local_frame, hlen is
*      horizontal component the vector.  Zen_dist is the angle from zenith to
*      satellite (rads), and azimuth is the azimuth from North (rad).
        hlen = sqrt(unit_loc(1)**2+unit_loc(2)**2)
        zen_dist = atan2(hlen, unit_loc(3))
        azimuth  = atan2(unit_loc(2),unit_loc(1))

```

Applying the above algorithm generates the sky plot shown below.

Note: North points to the right in this plot, and azimuth runs counter-clockwise (opposite of the normal method). These are "features" of the plotting package used.

Note: The plot has been generated for 1-solar day (that is the normal day). The satellites repeat the last few points because the orbital period of GPS is one sidereal day. The satellites return approximately 4 minutes earlier each day. (The matlab figures are generated at 1 minute spacing and hence azzd(1:2,1:5,sv) and azzd(1:2,1437:1441,sv) where sv is satellite number and the first 1:2 is for azimuth and zenith distance (NaN are used for zenith distances > 90 deg and thus below the horizon).

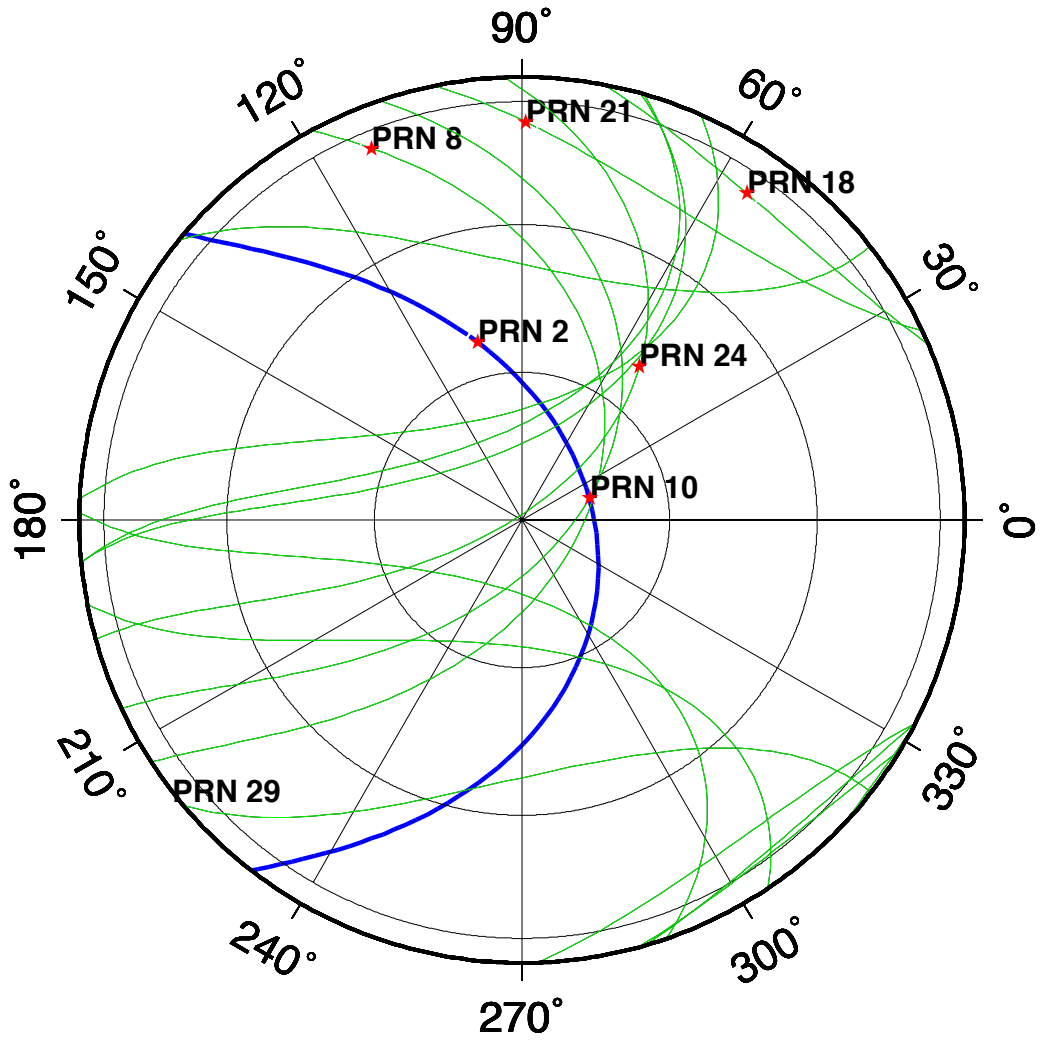


Figure 4a: Shows the sky-tracks for the first 9-satellites. Positions are marked at 14:00 GPST on March 31, 2010.

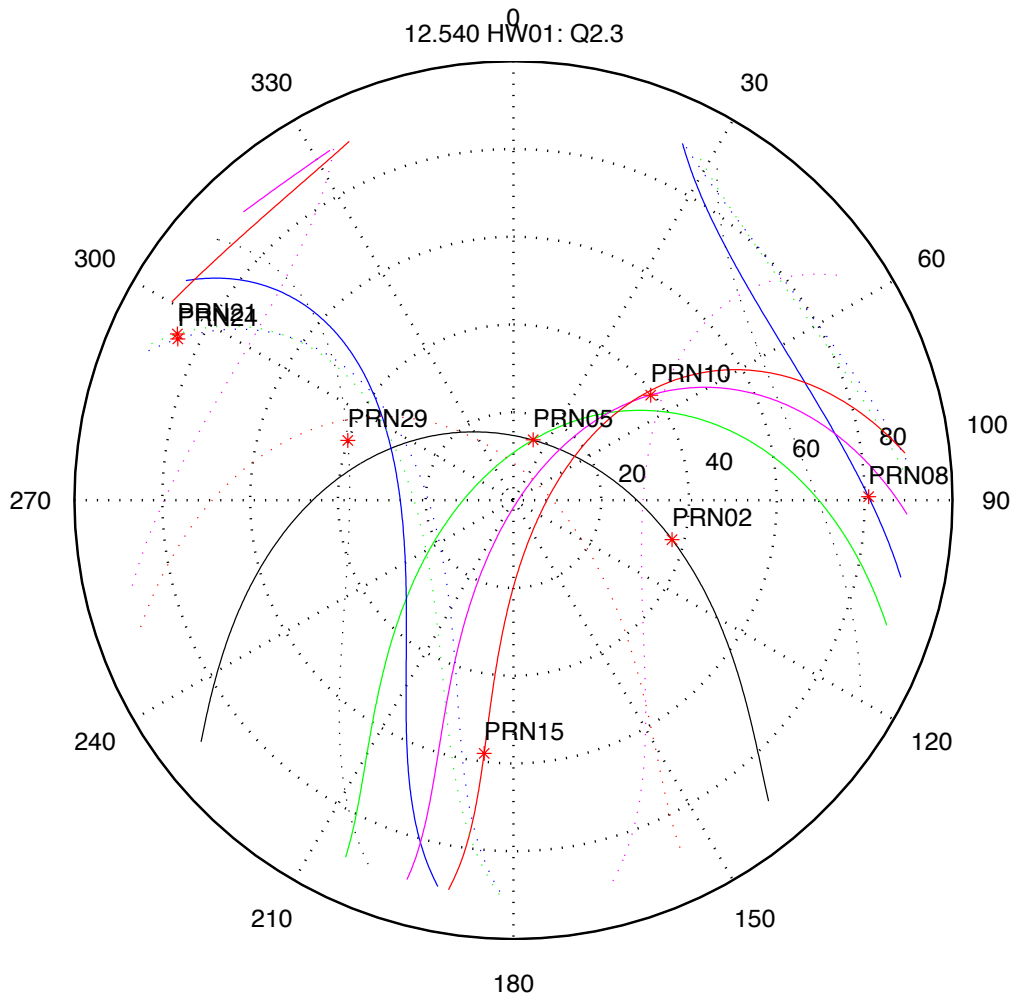


Figure 4b: Matlab version of same figure. Here we set the view so that North is up and the azimuth increases in a clockwise direction. (The scale here is to 100 degrees zenith distance compared with 90 in Figure 4a.)

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