12.520 Lecture Notes 21

Fluids

Fluids – no memory of shape – flow under applied tractions, body forces, stop flowing (don't reverse flow) when "driving forces" removed.

Newton's concept of viscosity – subject fluid to shearing

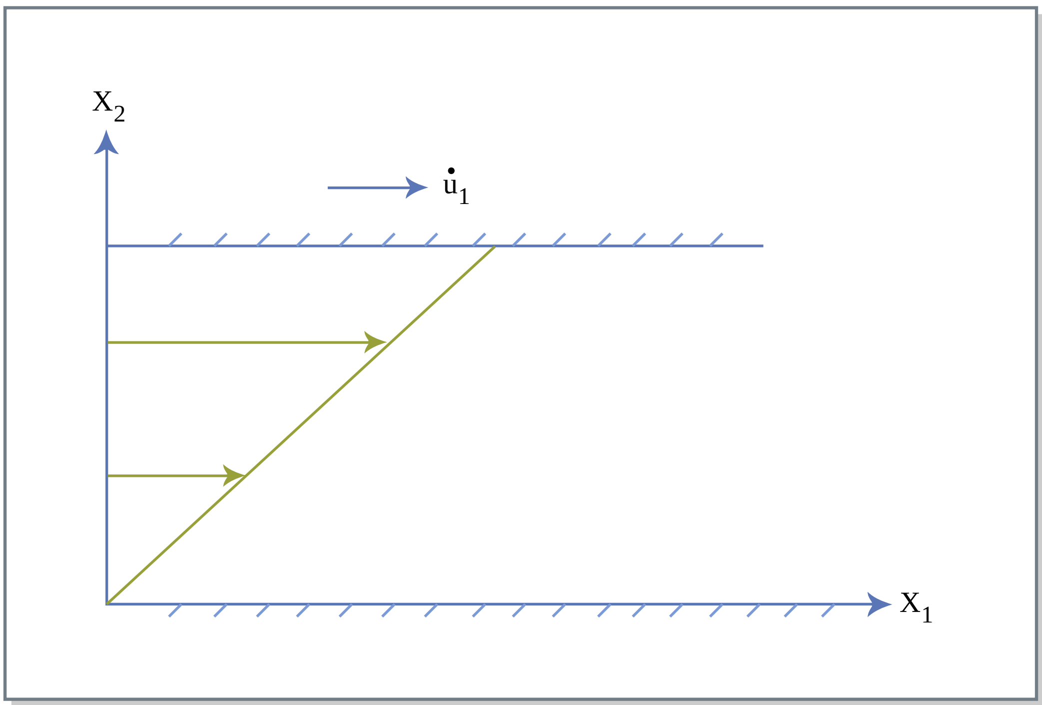

Figure 21.1
Figure by MIT OCW.

$$\sigma_{12} = \mu \frac{\partial u_1}{\partial x_2}$$

where μ is shear viscosity.

Substance	μ (Pa·sec) at 20°C	Poise (10 Pa·sec)
air	$2 \cdot 10^{-5}$	$2 \cdot 10^{-4}$
water	10^{-3}	10^{-2}
glycerine	1	10
ice (0°C)	10^{13}	10^{14}
glass	10^{17}	10^{18}
"Earth"	$10^{19} - 10^{21}$	$10^{20} - 10^{22}$

Physical cause – gasses – (vertical) motion of particles with different horizontal velocities.

- Fluids – elastic resistance to distortion of atomic "cages" as atoms and molecules "slide by".

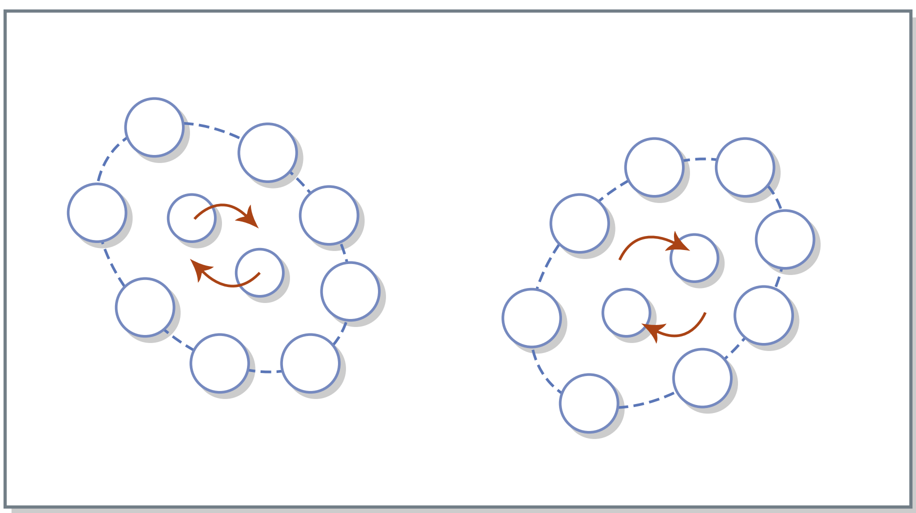

Figure 21.2
Figure by MIT OCW.

- Solids – diffusion of defects in the lattice (vacancies or interstitials); motion of dislocations in lattice structure.

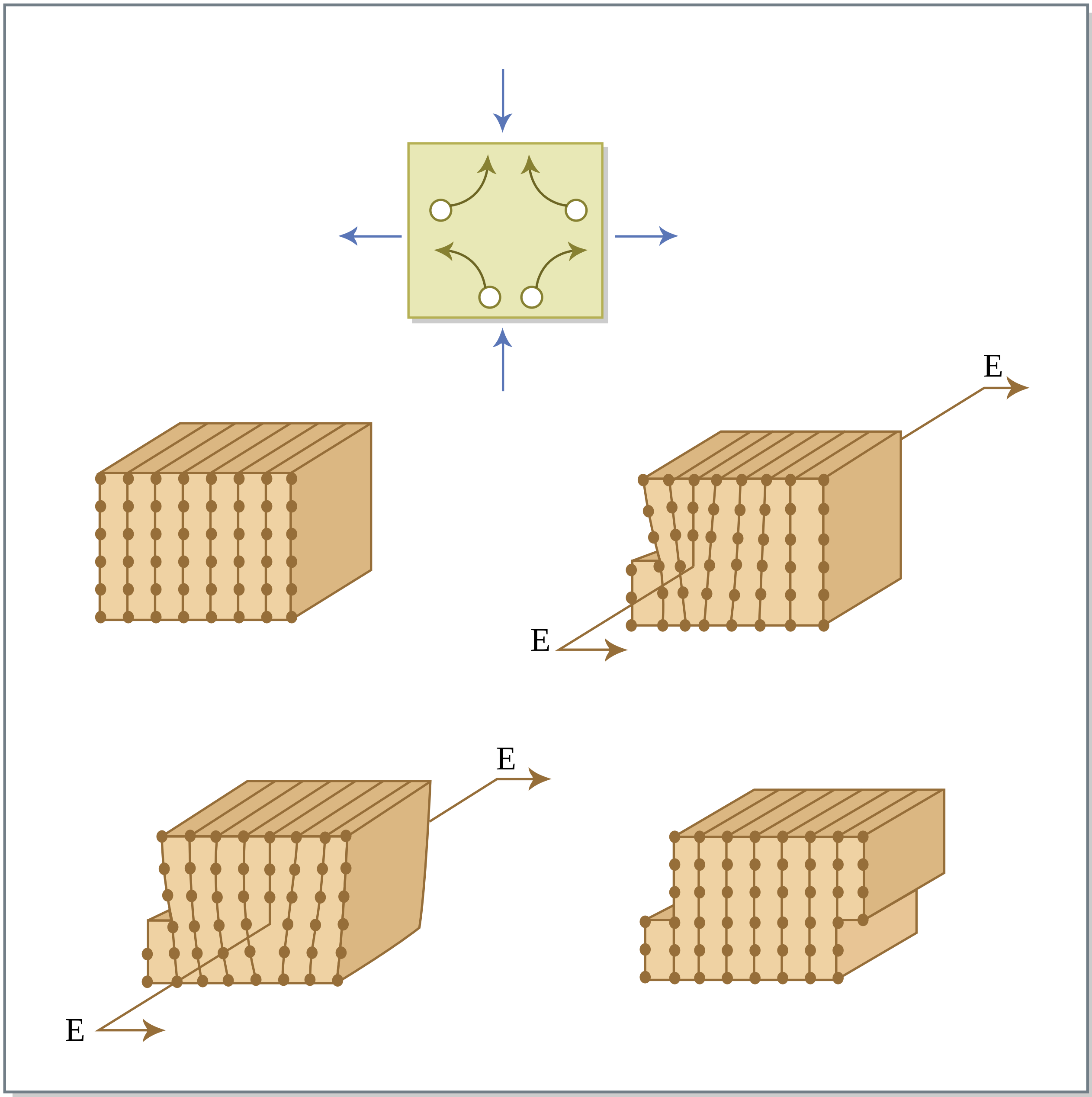

Figure 21.3

Figure by MIT OCW.

For a general stress-strain, for a Newtonian fluid

$$\sigma_{ij} = -p'\delta_{ij} + D_{ijkl}\varepsilon_{kl}$$

where D_{ijkl} is viscosity tensor and $\dot{\varepsilon}_{kl}$ is strain rate tensor.

For an isotropic fluid

$$\sigma_{ij} = -p'\delta_{ij} + \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}$$

For volumetric strain rate

$$\sigma_{kk} = -3p' + (3\lambda + 2\mu)\varepsilon_{kk}$$

Mean normal stress: $\dfrac{\sigma_{kk}}{3} = -p' + (\lambda + \dfrac{2}{3}\mu)\varepsilon_{kk}$

where $\lambda + \dfrac{2}{3}\mu$ is bulk viscosity.

For many applications, $\lambda + \dfrac{2}{3}\mu = 0$ (Stokes fluid)

$$\sigma_{ij} = -p'\delta_{ij} + 2\mu\varepsilon_{ij} - \frac{2}{3}\mu\varepsilon_{kk}\delta_{ij}$$

For many applications, $\dot{\varepsilon}_{kk} \simeq 0$

$$\sigma_{ij} = -p'\delta_{ij} + 2\mu\varepsilon_{ij}$$

Often η is used for viscosity

$$\sigma_{ij} = -p'\delta_{ij} + 2\eta\varepsilon_{ij}$$

Sometimes $\eta \to 0$ ("perfect fluid")

$\sigma_{ij} = -p'\delta_{ij}$