

MIT OpenCourseWare  
<http://ocw.mit.edu>

12.510 Introduction to Seismology  
Spring 2008

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

# 12.510 Introduction to Seismology

Date: 03/05/2008

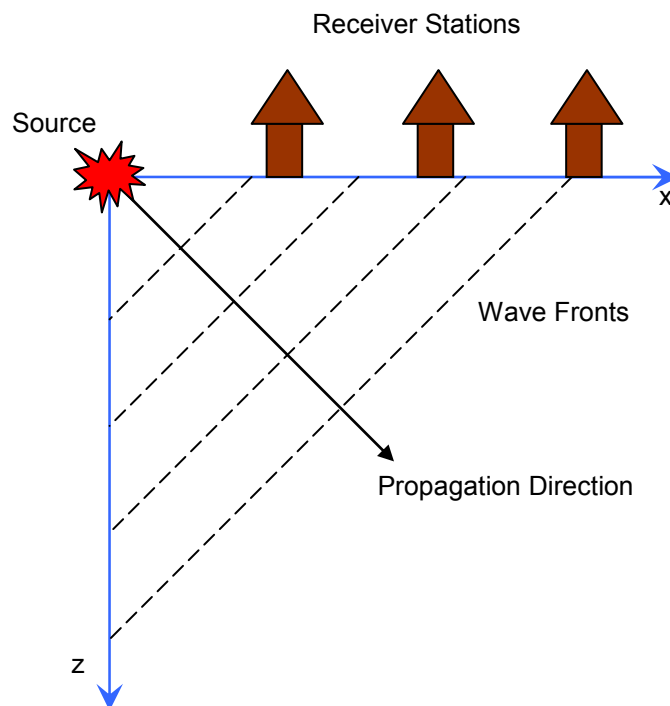
Topic: Travel-Time Curves

Organization:

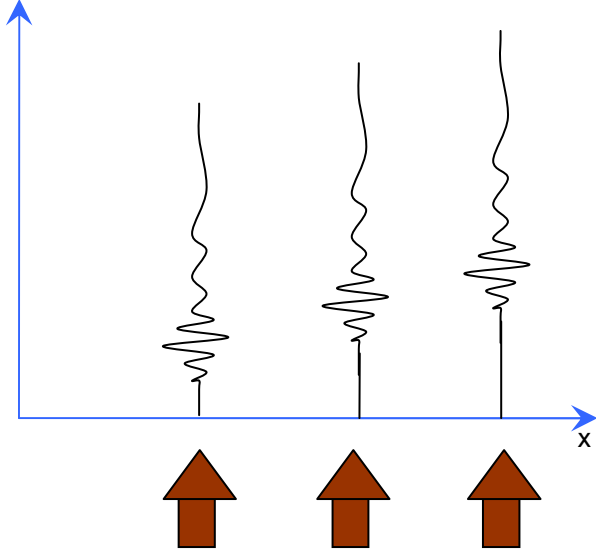
1. Basic Concepts
2. Travel-Time Curves for a Layer over a Half Space
3. Travel-Time Curves for More Complicated Layers

## I. Basic Concepts

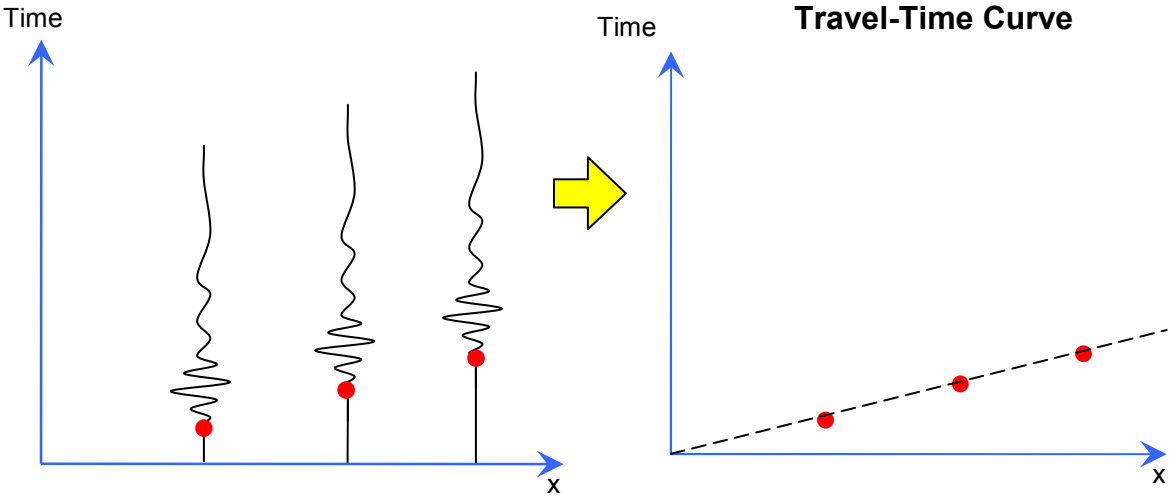
Consider the following picture, in which a number of receiver stations are positioned at certain distances away from a source:



The wave front will arrive first at the station closest to the source and later at subsequent stations. The seismograms recorded at each station may look something like this:



Connecting the same point on each seismogram leads to a travel-time curve:



## II. Travel-Time Curve for a Layer Over a Half Space

For a layer over a half space, there are three types of waves to consider:

- A. Direct waves
- B. Reflected waves
- C. Head waves

These waves are illustrated in the following picture:

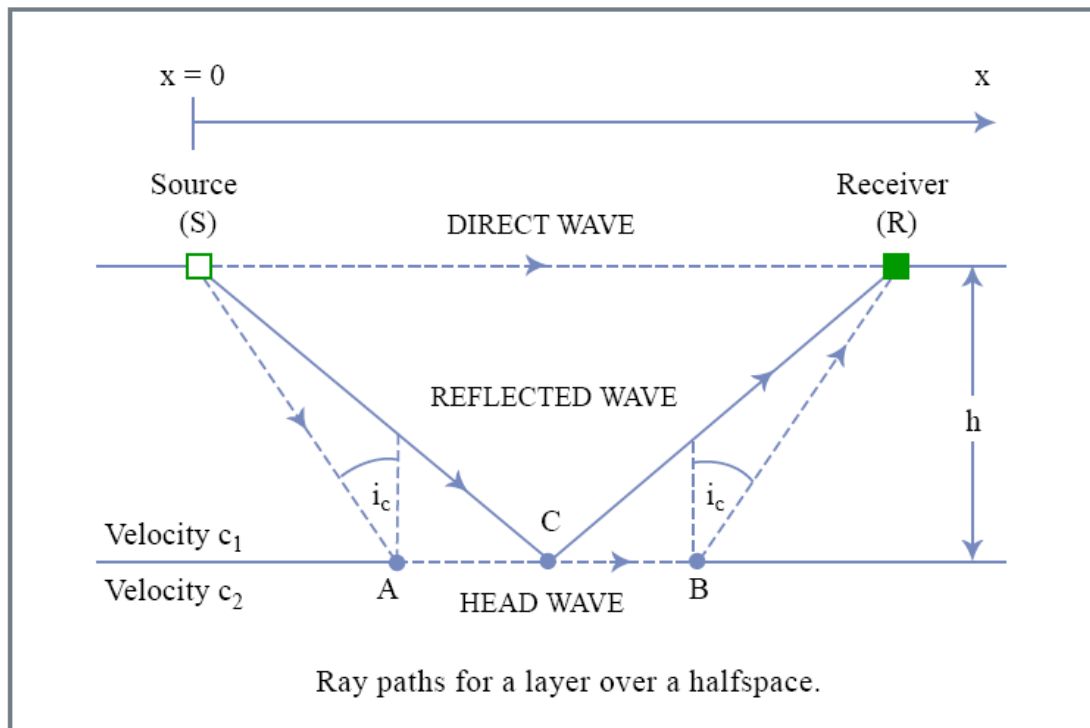


Figure by MIT OCW.

Adapted from Stein & Wysession (2003), *An Introduction to Seismology, Earthquakes, and Earth Structure*, fig. 3.2-1, p. 120, Blackwell Publishing

### A. Direct Waves

The travel time for a direct wave is easy to compute. It is simply the distance traveled divided by the wave speed in the layer:

$$t_{direct} = \frac{x}{c_1}$$

This simple relationship means that the travel-time curve is a straight line:

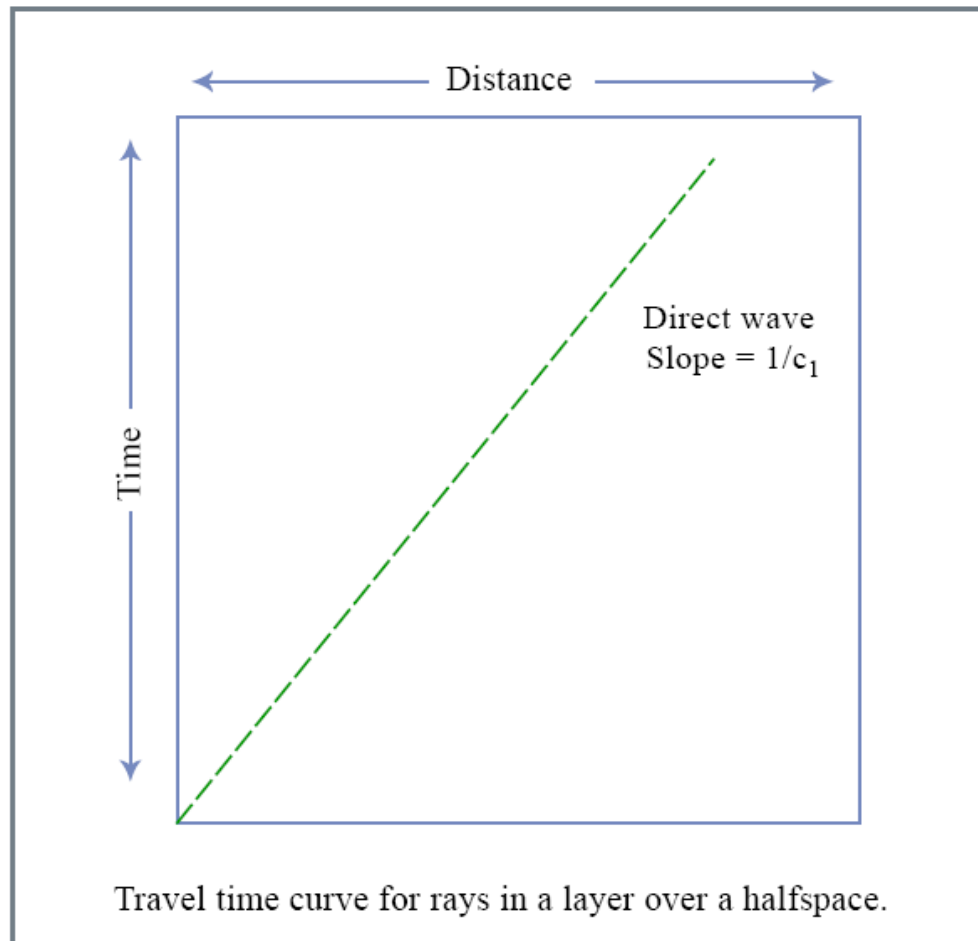


Figure by MIT OCW.

Adapted from Stein & Wysession (2003), *An Introduction to Seismology, Earthquakes, and Earth Structure*, fig 3.2-2, p. 121, Blackwell Publishing

## B. Reflected Waves

The travel time for a reflected wave is calculated from two parts. The first part is the travel time from the surface to the layer and the second part is the travel time from the layer back to the surface:

$$t_{\text{reflected}} = \frac{SC}{c_1} + \frac{CR}{c_1} = \sqrt{\left(\frac{1}{2}x\right)^2 + h^2} \left(\frac{2}{c_1}\right)$$

The travel-time curve for reflected waves is a hyperbola. The asymptote is the travel-time curve for direct waves. This is because at far distances the angle of reflection approaches  $90^\circ$  and the incident and reflected waves look more and more like direct waves.

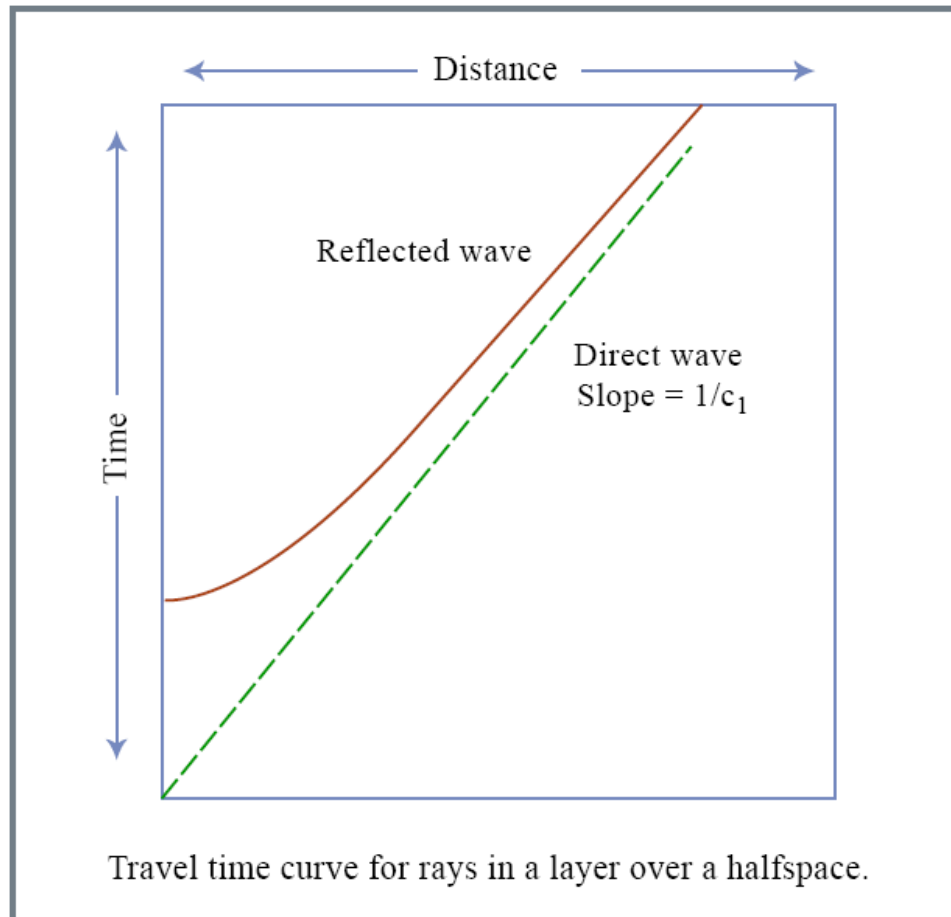


Figure by MIT OCW.

Adapted from Stein & Wysession (2003), *An Introduction to Seismology, Earthquakes, and Earth Structure*, fig. 3.2-2, p. 121, Blackwell Publishing

### C. Head Waves

Head waves occur when the incident angle is greater than the critical angle.

#### *Critical Angle*

The critical angle is the incident angle for which the refracted wave travels along the interface. This angle is determined by using Snell's Law and

noting that the refracted wave travels along the interface when its angle is  $\frac{\pi}{2}$ .

$$\frac{\sin i_c}{c_1} = \frac{\sin(\pi/2)}{c_2}$$

$$i_c = \sin^{-1}\left(\frac{c_1}{c_2}\right)$$

The travel time for a head wave is computed from three pieces:

$$t_{head} = \frac{SA}{c_1} + \frac{AB}{c_2} + \frac{BR}{c_1}$$

$$t_{head} = \frac{x}{c_2} + \frac{2h}{c_1} \sqrt{1 - \left(\frac{c_1}{c_2}\right)^2}$$

This equation has the form  $y=mx +b$ , so it gives a straight line. However, the line is not continuous since head waves do not develop until the critical angle is reached. As a result, the line actually starts at the critical distance  $x_c$ , which is given by:

$$x_c = 2h \tan i_c$$

The travel-time curve for head waves, in addition to the curves for direct and reflected waves, is shown below:

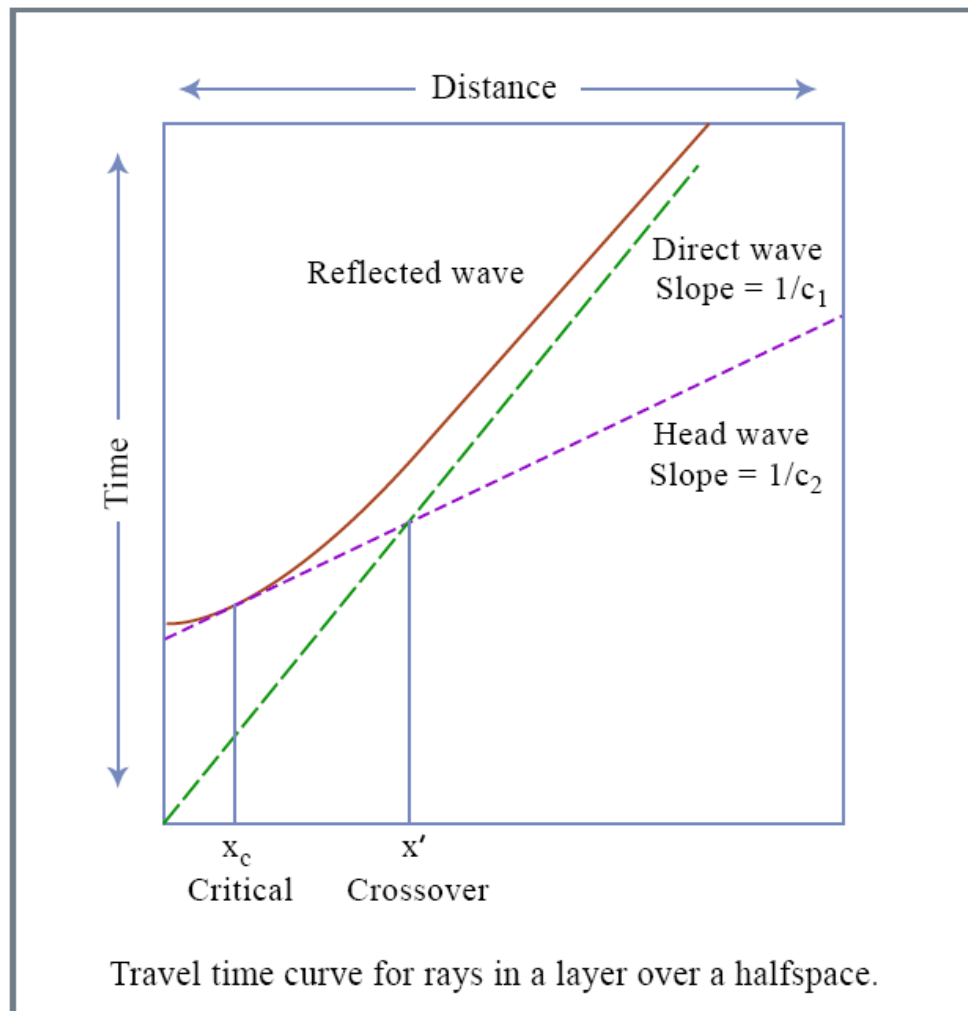


Figure by MIT OCW.

Adapted from Stein & Wysession (2003), *An Introduction to Seismology, Earthquakes, and Earth Structure*, fig. 3.2-2, p. 121, Blackwell Publishing

Note that at the critical distance the direct wave arrives before the head wave. However, since the velocity of the head wave  $c_2$  is larger than the velocity of the direct wave  $c_1$ , the slope of the head-wave curve is smaller and the two curves eventually cross. This distance, called the crossover distance, is given by:

$$x_d = 2h \left( \frac{c_2 + c_1}{c_2 - c_1} \right)^{1/2}$$



Beyond this distance, the head wave arrives before the direct wave.

### III. Travel-Time Curves for More Complicated Layers

If the layer is not flat or there are multiple layers, the travel-time curves become more complex.

#### *Dipping Layers*

A dipping layer changes the slope of the travel-time curve for head waves. Instead of having a slope given by the reciprocal of the velocity

$$slope = \frac{1}{c_2}$$

it will have a slope given by

$$slope = \frac{1}{c_a}$$

where  $c_a$  is the apparent velocity. This apparent velocity changes based on the dip direction. For a layer dipping downward at an angle  $\theta$  relative to the surface, it is given by:

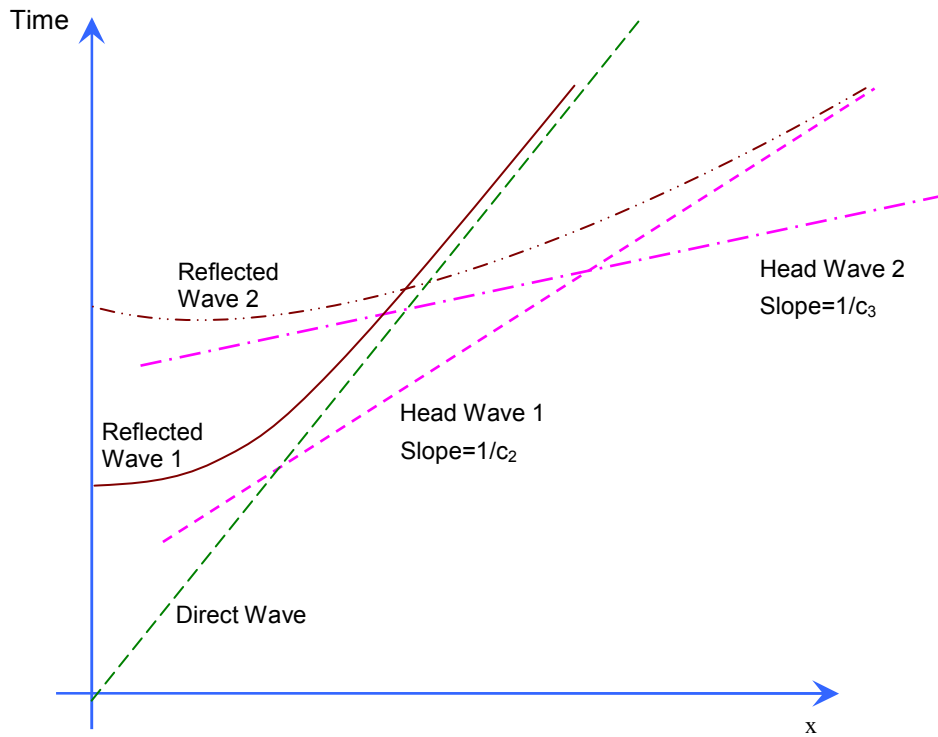
$$c_a = \frac{c_1}{\sin(i_c + \theta)}$$

For a layer dipping upward, it is given by:

$$c_a = \frac{c_1}{\sin(i_c - \theta)}$$

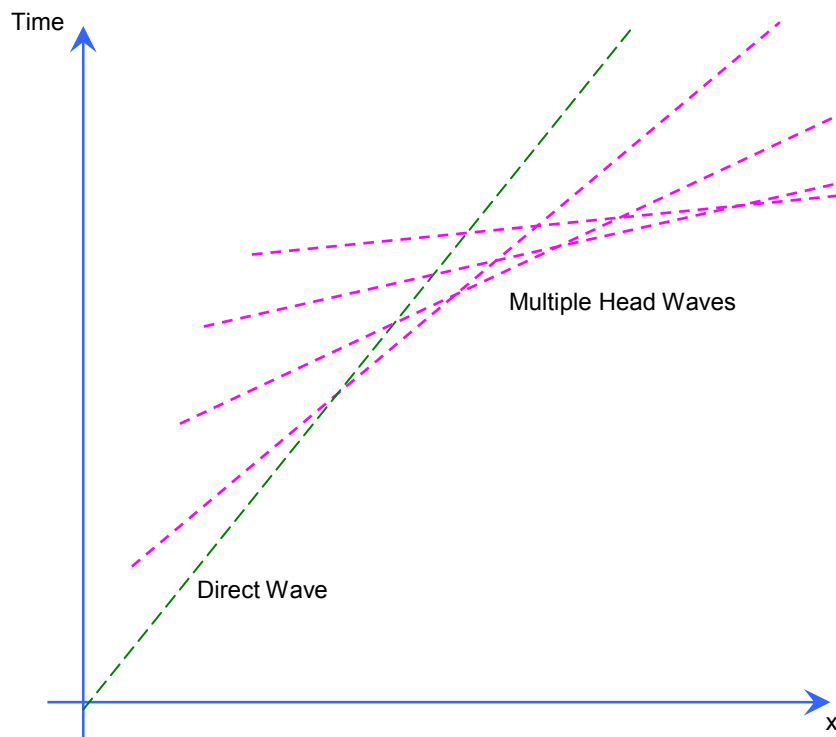
## Multiple Layers

When there are multiple layers, each layer generates its own reflected waves and head waves. For the case of two layers, the travel-time curves look like those below:



## *Continuously Changing Velocity*

The result for multiple layers can be extended to the case in which the velocity varies continuously with depth. This is done by letting the number of layers approach infinity and the layer thickness approach zero. On the travel-time diagram, this would correspond to plotting more and more curves for head waves and reflected waves. In the plot below, this is shown for only the head waves for simplicity:



Since the graphs become very complicated as more and more layers are added, only the first arrival times at a given location are usually plotted. This plot is shown below in red:

