12.510 Introduction to Seismology
Spring 2008

Problem Set 1 (Due on Wednesday, Feb. 28, 2007)

All the figures in this PS are taken from *Introduction to Seismology* [Peter M. Shearer, 1999].

(1) Let $U = 2\dfrac{x}{y} + 2\dfrac{xy^3}{z^2} + 3xz^4$ and $\vec{B} = \dfrac{z}{x}\hat{x} + 2y^3 z\hat{y} + 2\dfrac{y^3 z^3}{x^2}\hat{z}$

(a) Calculate the gradient of U.

(b) Calculate the divergence of $\vec{B}$.

(c) Calculate the Laplacian of U.

(d) Calculate the curl of the gradient of U.

(e) Calculate $\vec{B}\cdot(\vec{B}\times\nabla U)$.

(2) (a) Using $c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$, $\Delta = u_{k,k}$, $\varepsilon_{ij} = \dfrac{1}{2}(u_{i,j} + u_{j,i})$, and $\tau_{ij} = c_{ijkl}\varepsilon_{kl}$, show that $\tau_{ij} = \lambda\delta_{ij}\Delta + 2\mu\varepsilon_{ij}$.

(b) if $\rho\ddot{\vec{u}} = (\lambda+\mu)\nabla(\nabla\cdot\vec{u}) + \mu\nabla^2\vec{u}$, decompose into a wave equation for a propagating volume change $\nabla\cdot\vec{u}$ and a rotation $\nabla\times\vec{u}$, by using the following vector identities:

$$\nabla^2\vec{u} = \nabla(\nabla\cdot\vec{u}) - \nabla\times\nabla\times\vec{u}, \quad \nabla\cdot\nabla\times\vec{a} = 0 \quad \text{and} \quad \nabla\times\nabla\phi = 0,$$

where $\vec{a}$ is a general vector field and ϕ a general scalar field.

(3) The radii of the Earth, Moon, and Sun are 6,371 km, 1,738 km, and 695100 km, respectively. From Figures 1.1, 1.5 and 1.6, make a rough estimate of how long it takes a P-wave to traverse the diameter of each body.

(4) Assume that the S velocity perturbation plotted at 200km depth in Figure 1.7 extend throughout the uppermost 300 km of the mantle. Estimate how many seconds earlier a vertically upgoing *S*-wave will arrive at a seismic station in the middle of Canada, compared to a station in the eastern Pacific. Ignore any topographic or crustal thickness differences between the sites; consider only the integrated travel time difference though the upper mantle.

(5) Assuming that the P velocity in the ocean is 1.5 km/s, estimate the minimum and maximum water depths shown in Figure 1.8. If the crustal P velocity is 5 km/s, what is the depth to the top of the magma chamber from the sea floor?