12.215 Modern Navigation

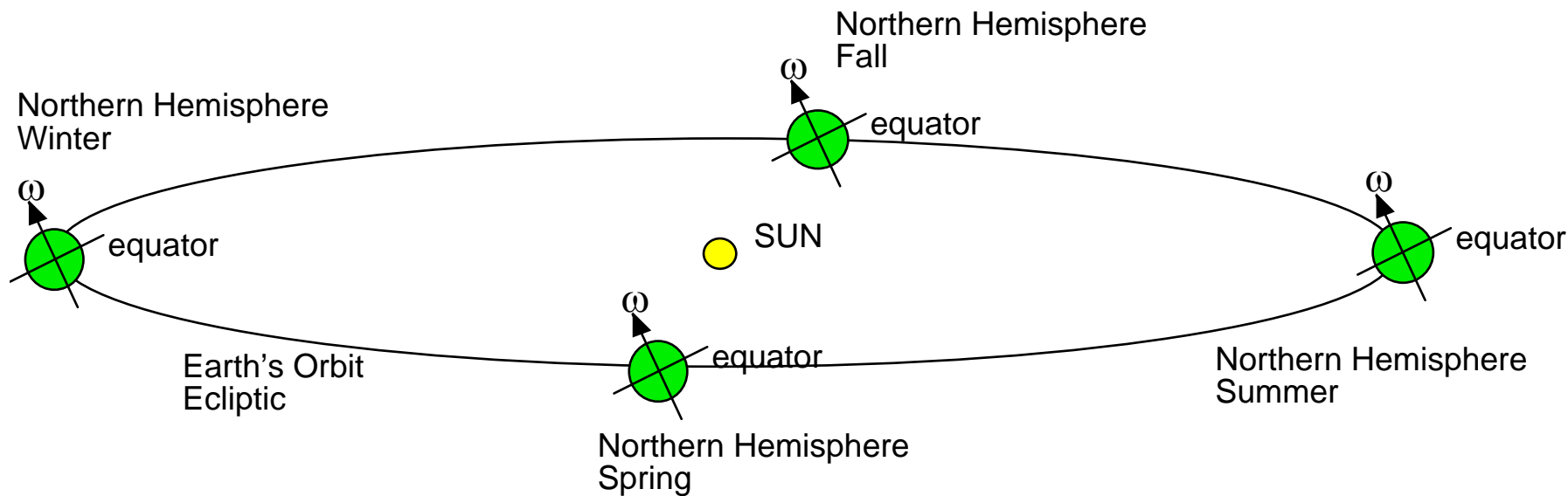**Thomas Herring**

Review of Monday's Class

- Spherical Trigonometry
  - Review plane trigonometry
  - Concepts in Spherical Trigonometry
    - Distance measures
    - Azimuths and bearings
  - Basic formulas:
    - Cosine rule
    - Sine rule
  - Basic applications

Today's Class

- Motion of the Earth and Sun
  - Geometry of Earth/Sun system
  - Astronomical coordinates
  - Motion of the Earth around the sun
  - Equation of Time
- Astronomical positioning
  - Latitude and Longitude determination using astronomical bodies
- Error contributions to latitude and longitude measurements.

Geometry of Earth Sun System

- Figure below shows the basic geometry

Northern Hemisphere
Fall

ω
equator

Northern Hemisphere
Winter

ω
equator

SUN

Earth's Orbit
Ecliptic

ω
equator

Northern Hemisphere
Spring

ω
equator

Northern Hemisphere
Summer

Geometry of Earth Sun

- The Earth's equator plane is inclined at ~23.5^o to the orbit plane (called the ecliptic)

- It takes ~365.25 solar days for one orbit (hence a leap-year every 4 years, and the odd rule about leap years at century boundaries because the value is not exactly 365.25 solar days.

- A solar-day is the length of time (on average) for the sun to move from noon to noon. Because the earth moves in its orbit by a little during the day, the length of time for stars to come to the same place in the sky is a little bit shorter.

- A sidereal-day is the length of time for stars to come back to the same point in the sky

- There are ~366.25 sidereal days in a year (the extra day is basically one rotation due to the orbit in one year).

Time systems

- There are a number of time systems encountered in astronomy and navigation.
- Time keep by our watches is related to Universal Time Coordinated (UTC). Used to be called Greenwich Mean Time (GMT)
  - UTC is based on atomic time standards (Cesium clocks) and is an average over Cesium clocks operated all around the world. The US clocks are operated at the US Naval Observatory in Washington DC
  - The International Earth Rotation Service (IERS) (and formally the Bureau International de Le Heure (BIH)) coordinates these activities and publishes corrections to the time systems operated in each country

Time systems

- The time defined by atomic clocks runs at a constant rate. Unfortunately, we tend to perceive time by the rotation of the Earth which is not uniform. There is a slowing of the rate of rotation of Earth (about 1 second every 18 months) and there are fluctuations due mainly to changes in atmospheric winds and processes in the fluid core.
- Time defined by the rotation of the Earth is called UT1 and is solar day system.
- UTC has discontinuities, call leap-seconds, that are added to keep it aligned with UT1. (When the atomic second was adopted in the mid-1950s the rates were the same and so leap-seconds were not needed. They were introduced in the mid-1960s after the Earth rotation rate had slowed enough that the difference between UT1 and UTC had reached several seconds.

Time systems

- The difference between UT1 and UTC has be measured and the IERS coordinates these measurements and published differences between UT1 and UTC. (They also decide when leap seconds need to be added).

- Sidereal time is derived from UT1 and measures times in sidereal seconds. If we ran our watches on sidereal time, the stars would also be in the same place in the sky at the same time. (With solar time, the stars rise 4 minutes earlier each night).

Solar time

- Solar time is based on the mean solar day, but the time that Sun reaches its highest point each day (around noon) varies through out the year. The difference between noon at Greenwich and when the sun is at its highest point (or highest elevation angle) is call the Equation of Time.

- There are two components to the equation:

  - The Earth's orbit is eccentric (e=0.0167) and so moves at different speeds through the orbit, causes an annual variation.

  - The equator is included to the orbit plane (obliquity of the ecliptic) by ~23.5^o and this causes a semi-annual variation.

  - Combination of the two effects cause changes in the time of noon at Greenwich by -14 to +16 minutes (see URL:

  http://www.nmm.ac.uk/
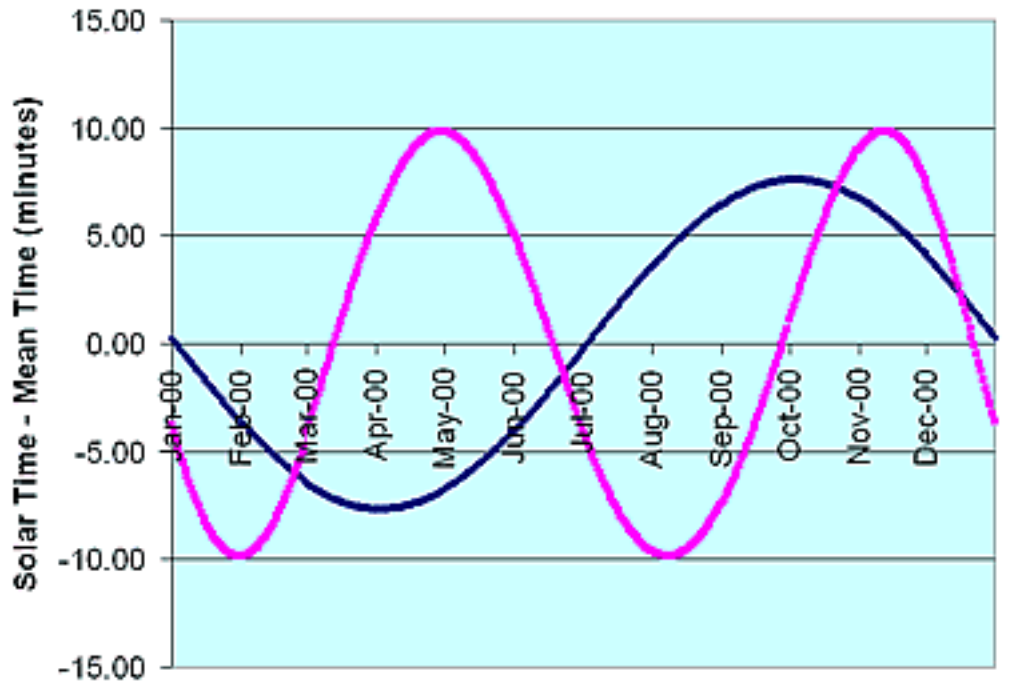
Equation of Time

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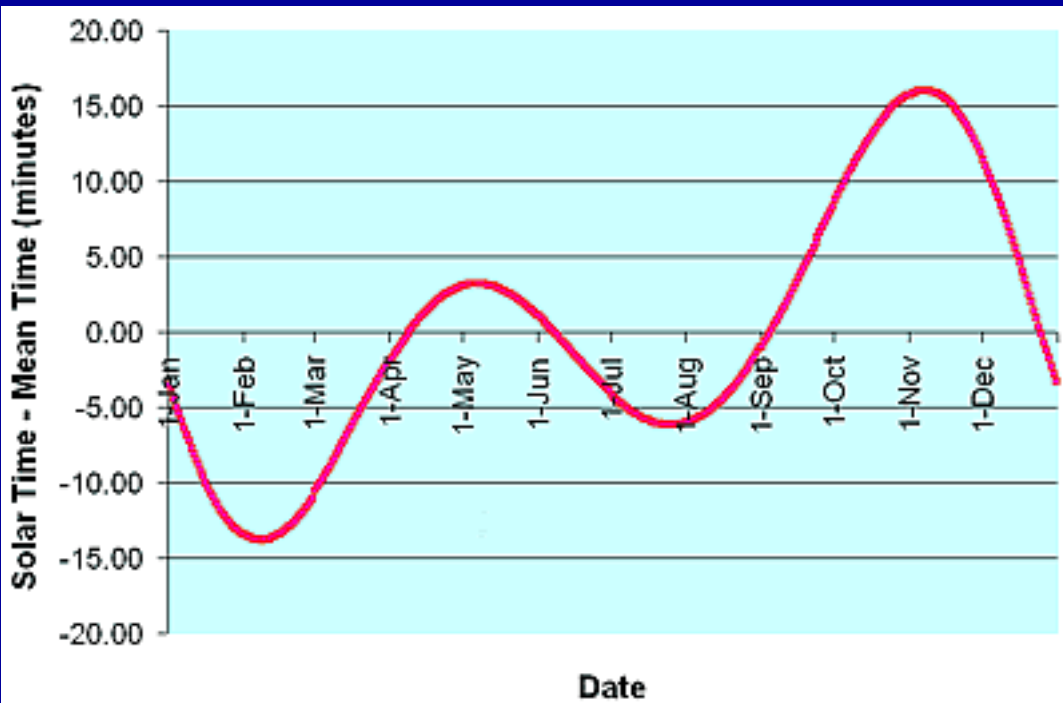
For Longitude determination using the sun, this effect must be accounted for (15 minutes of time~225 nautical miles)

Astronomical position determination

- To determine position using astronomical measurements (or a sextant) requires relating positions of celestial objects to Earth coordinates.

- Celestial coordinates:
  - Declination: Measured from equator and it astronomical coordinate equivalent to latitude
  - Right Ascension: Angle measured along the equator (similar to longitude) but origin is the intersection of the equator and ecliptic planes (called the first point of Aries).

- Celestial coordinates are specified in a non-rotating or slowly rotating frame. The diurnal rotation of the Earth is not in the coordinates.

Celestial coordinates

- Since the celestial coordinates are given in a system attached to the equator of the Earth, they would change slowly with time due to precession (26,000 year motion of the rotation axis, about an axis perpendicular to the orbit plane), and nutation (nodding of the rotation axis in space due to gravitational torque on the equatorial bulge.

- Because of these motions of the rotation axis (and hence in the equator) in space, celestial coordinates are generated in a number of systems.
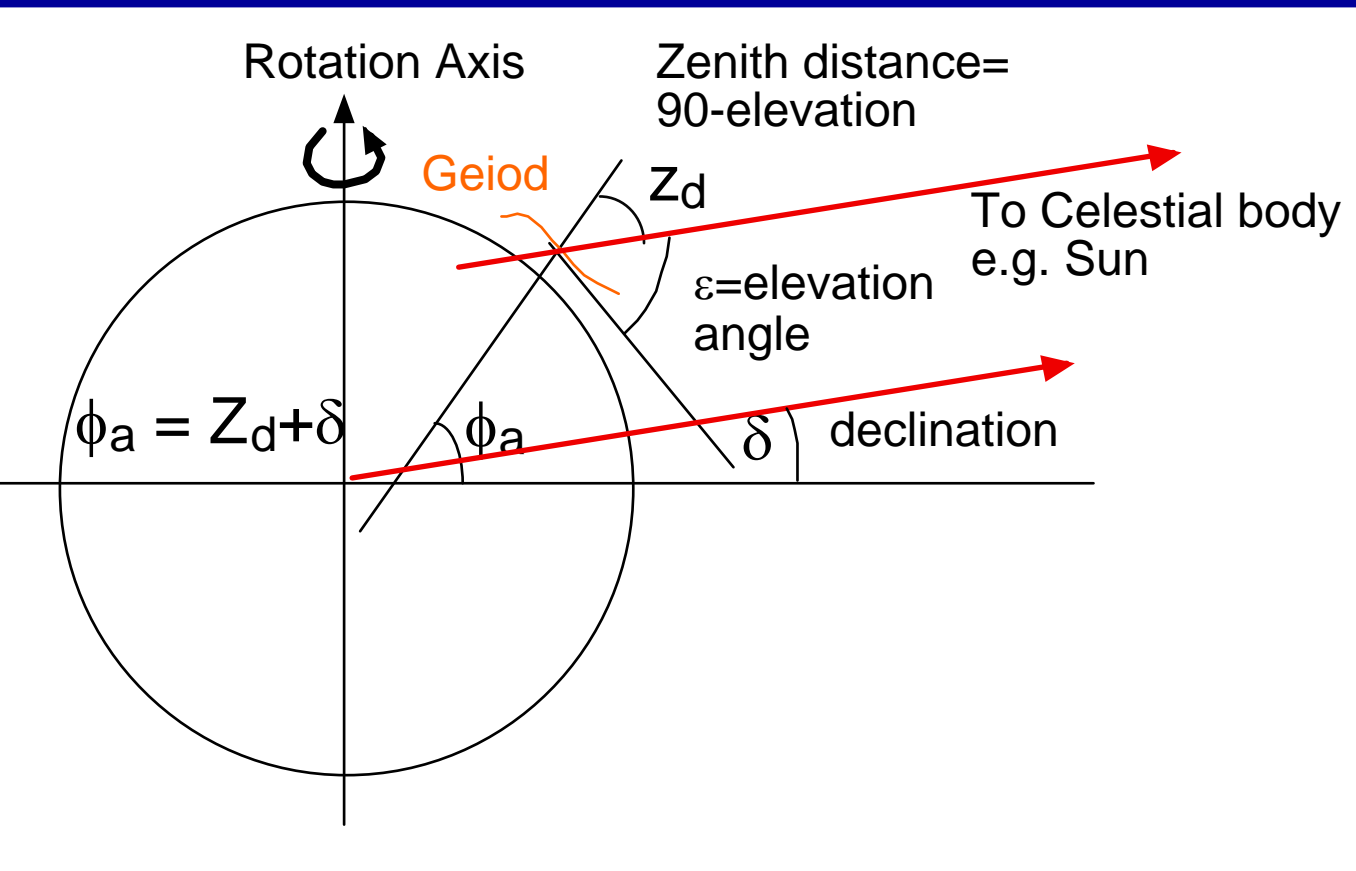
Celestial coordinates

- Fundamental coordinates of stars are given in a system which corresponds to the equator and ecliptic orientations at a specific time, call coordinates of epoch. Current system is call J2000 and is the position at Jan 1.5, 2000.

- The other common system is the coordinates of date corresponding to the equator and ecliptic orientations at the day of interest.

- There is a mathematical relationship through the application of a series of rotation matrices, that allow the systems to be related.

- For navigation, positions of date are used and these can be found in almanacs (or can be computed). We discuss almanacs in the next lecture.

Celestial positioning

- The easiest method for determining latitude and longitude is to make measurements of the elevation angle to the Sun at its highest point and to note the time at which this event occurs. This method requires access to accurate time which was the major advance made in determining longitude by the Harrison clocks. The book Longitude details these developments

- To determine latitude, the declination of the sun needs to be known on the day (obtained from almanacs) and the elevation measured (usually over a period of time so that highest point reached can be determined).

Latitude determination

Rotation Axis

Zenith distance= 90-elevation

Geiod

Z_d

To Celestial body e.g. Sun

ε=elevation angle

$\phi_a = Z_d + \delta$

ϕ_a

δ declination

When the sun is at the highest elevation, it is in the plane of the meridian and

ϕ_a=90-e+δ

Longitude determination

- The Greenwich observatory publishes tables of the time that the sun will cross the meridian in Greenwich (equation of time) and the difference between the time of the meridian crossing of the Sun at your location and the time in Greenwich, is the longitude of your location in time units. (Multiple by 15 to get degrees).

- Your time has to be converted to UTC which means that the time zone needs to be known (Boston for example is 5 hours from Greenwich except when daylight savings time is in effect and then it is 4 hours).

Errors in positioning

- Several types of errors can effect latitude and longitude determination
- For latitude:
  - Atmospheric bending of the light from the Sun can cause errors of several minutes of arc (several nautical miles). There are approximate equations that allow this to be corrected.
  - There are other errors associated with sextant measurements which we will discuss when we use the instrument
- For longitude:
  - Judging when the sun is at its highest point is difficult because the elevation angle changes slowly at the time
  - Error in knowledge of local time is a large error especially id no external calibration is possible (e.g., when clocks were first used).

Summary of Today's class

- Motion of the Earth and Sun
  - Geometry of Earth/Sun system
  - Astronomical coordinates
  - Motion of the Earth around the sun
  - Equation of Time
- Astronomical positioning
  - Latitude and Longitude determination using astronomical bodies
- Error contributions to latitude and longitude measurements. Later we will see how when we can make these measurements with a sextant.