Essentials of Geophysics 12.201/501
Problem Set 2

Suggested reading: Chapter 5 in (the old edition of) Turcotte and Schubert.

Problem One

- Start by writing down a general formula for the variation of the gravitational acceleration with radius for a spherical body like the Earth, in which the density is a function of radius only.
- Now, derive and plot the expression for $g(r)$ assuming $d\rho/dr = 0$.
- Next, consider a two-step variation of density for a planetary body much like the Earth in the sense that it has a core. Derive a general expression for $g(r)$ where $\rho = \rho_c$ for $0 \leq r \leq b$ and $\rho = \rho_m$ for $b \leq r \leq a$. Plug in densities and radii adequate for the Earth and plot the variation of g with r. You must obtain a curve similar to the one pictured in Fowler, Fig. 4.31, p. 111.
- In a planet, there is no variation of gravitational acceleration with increasing depth. What does this tell you about the distribution of density within the planet? Express your answer in terms of mean density and total radius.

Problem Two
Consider a spherical sheet mass of radius r_s. The mass distribution on the sheet is described by $\sigma(l)$, l being the spherical harmonic. Likewise, the gravity potential on the spherical shell is $U(l)$. (See Figure 1)

- Derive an expression for the potential outside and inside of the shell, $U_out(r, l)$ and $U_{in}(r, l)$ in terms of the gravitational potential on the mass sheet $U(l)$ and r_s.
- Find the gravity expressions, $g_{out}(r, l)$ and $g_{in}(r, l)$.

- (Graduate students only, extra credit for undergrads) Use Gauss's law (see class notes) to show that at the shell ($r = r_s$):

$$g_{out}(l) - g_{in}(l) = 4\pi G\sigma(l) \tag{1}$$

- Rewrite the expressions $U_{out}(r,l)$, $U_{in}(r,l)$, $g_{out}(r,l)$, and $g_{in}(r,l)$ (as obtained in parts (a) and (b)) as functions of $\sigma(l)$ using equation **??**.

Problem Three

- Consider a buried sphere of radius R with a uniform density anomaly $\Delta\rho$. Derive an expression for the surface gravity anomaly Δg, measured positively downward. Plot the gravity profile for varying x on the surface.

- Two identical spheres are buried at the same depth. Their densities are twice that of the surrounding material. Plot the surface anomalies to show what happens as the spheres are brought closer together. Plot the anomalies that result when both spheres are moved vertically.

- Discuss your results in terms of the *non-uniqueness* of the data. What can determined from surface measurements of gravity anomalies?

Problem Four

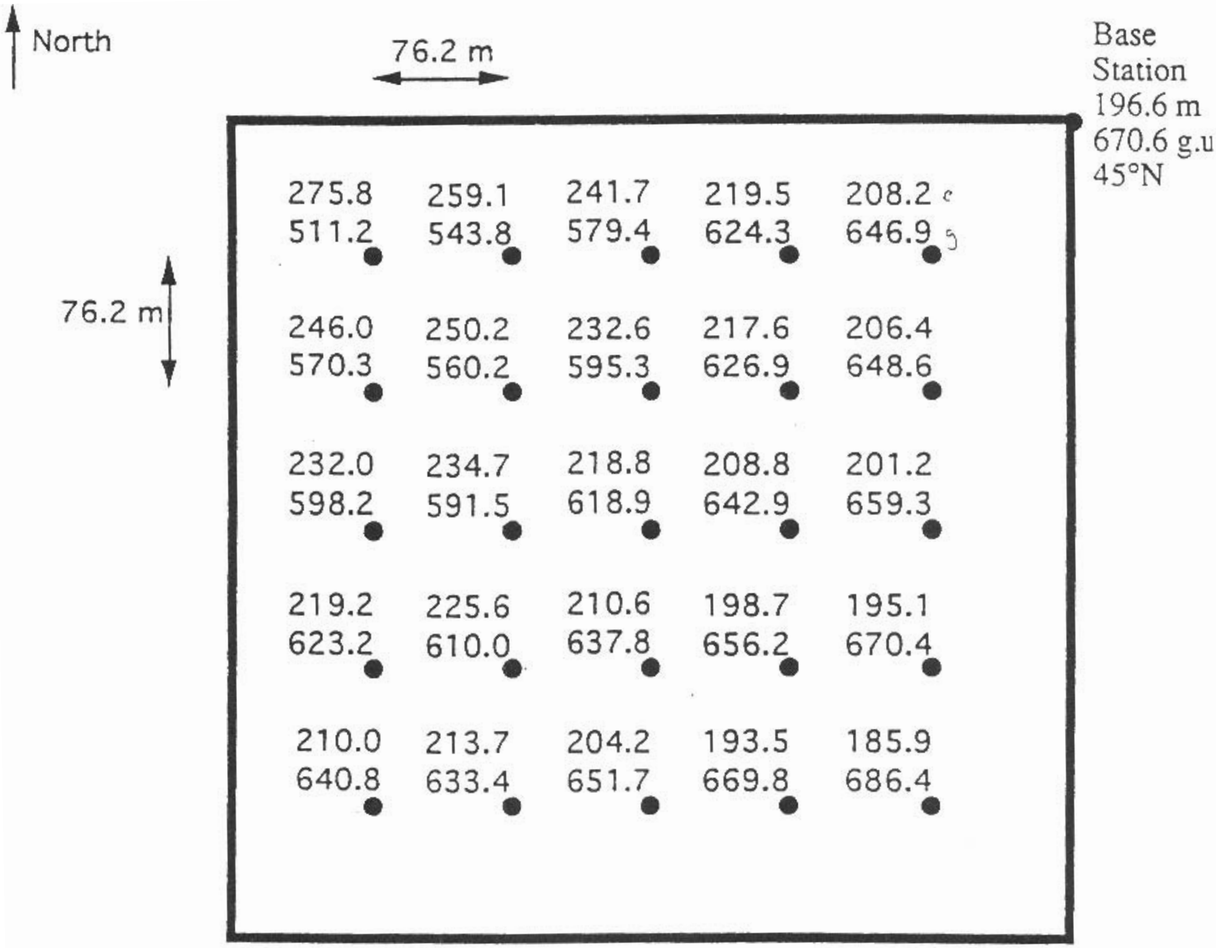
- Derive an expression for the gravity anomaly (measured

  positively downward) of an infinitely long horizontal cylinder of radius R with anomalous density $\Delta\rho$ buried at depth b beneath the surface, in terms of x, the horizontal distance from the surface measurement to the point on the surf ace directly over the cylinder axis.

- You are attempting to find the location and depth of a tunnel beneath a hillside. The tunnel has an approximately cylindrical cross-section. The map in Figure 2 shows station elevation in meters and raw gravity in gravity units (10 g.u. $=$ 1 milligal, 1 milligal$= 10^{-5}$ ms^{-2}. Do all the necessary reductions to the data in order to find the location of the tunnel. (You may ignore terrain corrections, Eotvos corrections, and tidal corrections.)

- Plot up a cross-section of the gravity anomaly from the tunnel along a line perpendicular to its strike. Using the formula for the gravity anomaly from a buried cylinder, what can you conclude about the depth and radius of the tunnel? Consider both air and water in the tunnel.

- Helpful hints. The Matlab functions , `mesh`, `meshgrid`, `plot`, `plot3`, `fmins`, and the concept of the `function`-environment (similar to the `subroutine` in FORTRAN) will come in handy. You will want to minimize the root mean square error between your measurements and the predicted gravity anomaly profile based on the cylinder model.



Terrain density = 2650 kg/m^3

First number is elevation in meters,
second is gravity in gravity units.