

### **Lab 3: Hillslope Profile Modeling (1-D) [due: Nov. 8]**

Matlab code will be available on Langtang on the MIT server.

In this lab you will learn to work with Matlab and use a simple numerical model to explore geological and climatic controls on hillslope forms. You will use a Matlab script (“hillslope\_profile3.m”) for modeling hillslope evolution under imposed sets of processes and boundary conditions (these are the geological and climatic conditions). The hillslope\_profile3.m script is written for you as a “jump start” into numerical modeling and Matlab programming. The script is as simple and straight forward as possible (nothing too fancy), so you can and should work your way through it carefully and make sure you understand every step -- working in teams on this is encouraged.

Once you are comfortable with the code, there are a series of exercises given below to guide you through an analysis of the controls on hillslope evolution under a variety of conditions. All analyses will assume transport-limited conditions. The code can handle soil production-limited transport as well (works well for creep or if initial conditions are not too far from steady-state).

Some of you may wish to modifying the code for specific calculations/outputs, but this is not necessary to complete the exercise. You will probably want to rename hillslope\_profile3.m if you develop your own extensions to the code.

For each exercise you are asked to turn in:

- (a) print-outs of model runs that demonstrate your findings, including a complete list of model parameters used;
- (b) a brief explanation of (1) why you chose the parameter values you did and (2) your interpretation of the results; and
- (c) brief answers to any direct questions asked below (read carefully – *answer all sentences that end with a question mark*). Be complete but do not go overboard - this is a lab exercise, you are not asked for a lengthy report: limit yourself to a paragraph or two maximum for each exercise.

#### **Exercise 1. Achieving steady state - Model Confirmation and Response Time (13pts).**

Initial condition: a triangular hill (straight sided); slope = .25; assume a channel at the toe of the slope incising at 1 mm/yr.

Case 1: a 40 meter-long hillslope

Case 2: a 20 meter-long hillslope

Case 3: a 10 meter-long hillslope

Problem 1: For cases 1-3, quantitatively verify the analytical solution for a soil-creep dominated landscape (i.e do you get a parabola with the correct curvature – either derive this value from your solution or plot the analytical solution against the numerical). Check how the solution depends on cell size by using  $dx = 2m, 1m, 0.5m$ . How long does it take to get to steady state (a good way to test for convergence to steady state is to plot the erosion rate at the divide normalized by the steady state erosion rate as a function of time – this is done in one of the figures; the best way is to re-write the code to iterate until the normalized erosion rate is within a specified tolerance of 1.0 [e.g.,  $1.0 \pm .01$ ] and output the final timestep in years)? Test whether  $T_{eq} \sim L^2/K$  using your 10, 20, and 40 meter cases and a range of K values. How sensitive is  $T_{eq}$

## **Surface Processes and Landform Evolution (12.163/12.463)**

Fall, 04 -- K. Whipple

to cell size? How much lowering of the divide is required before the hillslope achieves a steady-state form (report as % initial relief)?

Problem 2: For cases 1-2 only, quantitatively verify the analytical solution for a sheetwash dominated landscape (in this simple code, we ignore  $X_c$  – we simply allow some erosion at the first node by setting the first node a finite distance away from the divide ( $x_0 = 0.2 \cdot dx$ ). What happens if you do not do this and do not explicitly treat rainsplash erosion at the divide? Using  $x_0 = 0.2 \cdot dx$ , how long does it take to get to steady state? How much lowering of the divide is required?

Briefly comment on the difference between these two hillslope profiles. What sets response time (time to steady state)? Comment on the probable sensitivity of the solution to the bogus way I “faked” erosion at the ridge crest in the rainsplash/sheetwash case.

### **Exercise 2. Real World Rainsplash Erosion Equation (7pts).**

This problem does not require the Matlab code.

Question 1. Rainsplash is generally modeled as a slope-dependent (diffusive) process with  $q_s = kS$ . In lecture we talked about all the factors that affect the value of  $k$ . What are these factors?

Many of these factors have been evaluated in experimental plots under controlled, artificial rain storms. Based on work in Kenya (sandy substrate .5-2 mm only) Tom Dunne and coworkers developed the following equation to describe rainsplash erosion:

$$q_s = 110.0 D^{1.55} I \left( e^{-0.718 D^{-1.28} C} \right) \left( 1 - \left[ e^{-2.30 S} \right] \right)$$
$$D = 0.1882 I^{0.182}$$

where  $q_s$  is sediment flux during the storm [ $\text{cm}^2/\text{hr}$ ],  $D$  is median drop size [ $\text{cm}$ ];  $C$  is cover density [ $\text{cm}^2/\text{cm}^2$ ];  $I$  is the rainfall intensity [ $\text{cm}/\text{hr}$ ]; and  $S$  is the slope [ $\text{m}/\text{m}$ ]. Note that in this type of empirical relationship the units matter! Effectively the coefficients have units (like the Manning Equation).

Problem1: Write a function (or use matlab or excel or another spreadsheet) to calculate  $q_s$  as a function of hillslope gradient (use the following ranges:  $S$ : 0.01 – 0.2;  $C$ : 0.1-0.5;  $I$ : 0.3 – 1.0

- What approximate form of the generic transport law  $q_s = kx^m S^n$  fits the predictions (i.e. do a regression using a power-law regression model)?
- How does this result compare with dePloey’s empirical observation that  $q_s \sim s^{0.7-0.9}$ ?
- What other factors besides storm intensity and cover density must we consider before the “ $k$ ” found in your regression above can be “converted” into an effective  $k$  useful for long-term hillslope evolution modeling?
- For reasonable values of these other factors (feel free to discuss in groups) how does this “ $k$ ” compare with typical values found for soil creep ( $10^{-2}$ - $10^{-3}$   $\text{m}^2/\text{yr}$ )?

**Exercise 3. Using Hillslope Modeling to “Date” Features with Known Initial Shapes (15pts).**

This type of analysis has become common in neotectonics, paleo-seismicity, and (to a lesser extent) in Quaternary chronologic work. Consider a lateral moraine crest. These are reasonably modeled as beginning life (at the time of glacial retreat) as approximately triangular ridges, with slopes approximately at the angle of repose. Which slope (inner or outer) is most likely to have an angle of repose slope (33 degrees)? If we know the processes active (i.e.  $n$  and  $m$  values), and can constrain process rates (i.e.  $k$  values) through other means, we can model the evolution of these slope forms and derive an estimate of the age of the feature.

Consider the large moraine ridges (say 15m high) in the eastern Sierra Nevada (Permeable sandy till with granitic boulders and only 10-15% silt and clay; semi-arid climate, ca. 10” ppt/yr; some snow in winter, 10-30% veg cover of sage and sparse grasses; a moderate population of pocket gophers, rabbits and foxes). These moraines are deposited on relatively flat alluvial fan surfaces. The fans are still active today. What processes are likely active? Which is likely to dominate slope transport? What evidence would you look for in the field to confirm or disprove your intuition?

Problem 1. Using your favorite process or combination of processes and your best estimate of a reasonable rate constant (discussion again encouraged) model the slope evolution over a 10,000 year period. What measurable aspect of moraine morphology will be most sensitive to age (i.e. what should we look for in the field)? What boundary conditions did you chose? Why? How sensitive are your results to boundary conditions?

Problem 2. Use the same model to consider a certain realistic scenario. Over a 20,000 year period one might expect the active channel of the alluvial fan to migrate over and erode the toe of the moraine, effectively stripping away material. Suppose just once in this 20,000 year period the stream did just that. Suppose that for 200 years at the 10,000-yr mark it chewed away at the moraine toe (a constant lowering rate of 1cm/yr). What effect does this have on the final moraine profile? Can you tell that it happened or is the evidence erased? Would your estimate of moraine age be seriously thrown off? Would your impression of what type of process was operative on the moraine slope be changed if you only had the final profile shape to look at?

**Exercise 4. Scarp-Height Dependence of Effective Diffusion Coefficients? (15pts)**

Pierce and Colman (1986, GSA Bulletin, v. 97, p. 869-885) find that the effective K value fit to scarp profiles (assumed initially at angle of repose) of the same age vary as a function of both scarp height and “microclimate” (i.e. north or south facing).

Based on your experience with hillslope processes and modeling, what explanations might you be able to offer for this observation? Can some combinations of processes and boundary conditions yield an apparently “diffusive” evolution (rounding of crest and toe, while lowering overall slope) and yet yield slope-length dependent bulk erosion rates as well? If slope-length dependent processes are required, what are these processes and what evidence would you expect in the field? In what combination would you expect soil creep, sheetwash, and rapid mass wasting processes to contribute to sediment transport off these scarps? Would the relative importance of different processes vary with time as the scarp is eroded (Hint: consider the initial condition)? Would they vary with scarp height?

Explore the consequences of the Roering non-linear diffusion model ( $m=0$ ,  $n=1$ ,  $rflag=2$ ) acting on scarps ranging from 5 to 20-m high (above flat, non-eroding surfaces). What might such a non-linear diffusion model represent? Does non-linear diffusion provide a simple explanation for the Pierce and Colman data? One approach is to output  $S_{max}$  ( $= [dz/dx]_{centerline}$ ) from your final solution (after time T) and back-calculate  $K_{effective}$  from the analytical solution for the linear diffusion model and plot this as a function of hillslope length (the code currently writes these values to the screen at the end of each run – they are only meaningful for the Roering case, and for scarps with initial  $\tan\_alpha = 0.8$  situated in the middle of extensive flat surfaces) (with some editing of the code you could automate generating a plot of  $K_{effective}$  vs scarp height, but it is easy enough to simply run 5-10 cases and plot the results). How do you interpret this result? What difference does the boundary condition make (ie. Compare solutions for scarps on non-eroding flat platforms to that where all material is removed at the base)?