Woods Hole Oceanographic Institution and
Massachusetts Institute of Technology
Subject 12.097

HOMEWORK 2

Assigned 17 January 2006
Due 23 January 2006

1. **Buoyancy.** Consider a *1 meter* long cylindrical buoy with a radius of *15 cm*. For the buoy to be neutrally buoyant at a depth of *2000 m* what must its mass be? Neutral buoyancy is when the buoyant force on an object is equal to the weight of the object. The buoyancy force (F_B) is the weight of water displaced by the object ($FB = \rho_{SW} \cdot V \cdot g$). The density of seawater is *1025 kg/m³*. By changing the volume of the buoy, its buoyancy can be changed. If the buoy is lengthened by *0.5 cm*, what is the buoyancy force on the buoy? Assuming that a drag force of F_d = *1.5729* is exerted on the moving buoy, how long will it take to reach the surface from *2000 m* depth? (Remember that for $F = m \cdot a$, F equals the sum of the forces acting on the object).

    The density of fresh water is *1000 kg/m³*. What would be the buoyancy force on this buoy in fresh water? What would happen to the buoy? Assuming the same mass and radius, how long must the buoy be to be neutrally buoyant in fresh water?

2. **Deployment planning & data acquisition.** An *in situ* chemical sensor has a storage capacity of *1 million bits*. The sensor must have a resolution of *0.1 ppm* over a *0-100 ppm* range. This means each measurement must be *10 bits* long (10 bits in binary goes up to 1024 in base 10). Assume each measurement must also have an *32 bit* time stamp. To deploy this sensor for a year, how often can measurements be recorded? If a calibration routine is performed once a day which generates *256 bits* of data, how often can measurements be made over the course of the year? If you could collect data once an hour, how long could you deploy the instrument (you still need a calibration procedure every day)?

3. **Raman spectroscopy** is a type of vibrational spectroscopy in which you interrogate a sample with a monochromatic light source (a laser) and look at the backscattered radiation which is shifted in wavelength. The shift is due to the vibrational modes of the molecule and can be used to identify the molecule. The intensity of the backscattered radiation (I_R) is dependent on the laser intensity (I_L), the concentration (C), the Raman scattering efficiency of the molecule (σ), and instrumental factors (η): $I_R = I_L \cdot \sigma \cdot \eta \cdot C$. We take the intensity to be the area under the Raman peak (A). If you have a mixture of two molecules, their peak areas and concentrations are related by:

$$\frac{C_1}{C_2} = \frac{A_1}{A_2} \frac{\sigma_2}{\sigma_1} \frac{\eta_2}{\eta_1}$$

    The signal-to-noise ration necessary for detection of a peak is *S/N* = 3. Assume the RMS noise is 50, and the peak area is 1500 counts for a concentration of 50 mol/cm³. What is the lowest concentration that can be detected? Consider a two-part mixture, where the peak areas are the same, but $C_2 = 5 \cdot C_1$. If σ_1 is 2.5, what is σ_2? (You can assume that the instrument parameters do not change - i.e., $\eta_1 = \eta_2$)