12.005 Lecture Notes 26

Growth and Decay of Boundary Undulations

Growth: Rayleigh - Taylor Instability

- salt domes
- diapirs
- continental delamination

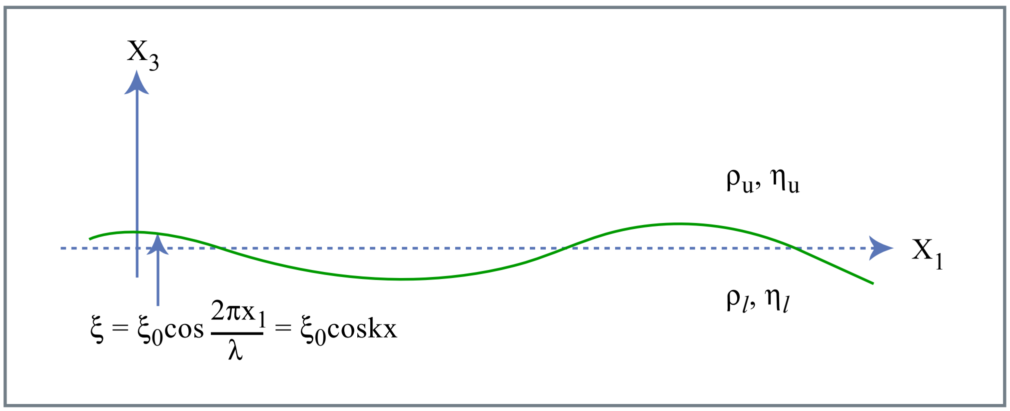

Figure 26.1

Figure by MIT OCW.

General problem: topography on an interface

$$\xi = \xi_0 \cos kx_1 \qquad k = \frac{2\pi}{\lambda}$$

(1) If $\rho_u < \rho_l$ topography decays as $\xi_0 e^{-t/\tau}$.

(2) If $\rho_u > \rho_l$ topography grows.

Initially $\xi = \xi_0 e^{t/\tau}$.

Eventually many wavelengths interact, problem is no longer simple.

Characteristic time τ depends on $\Delta\rho, \eta_u, \eta_l$, thickness of layers, …

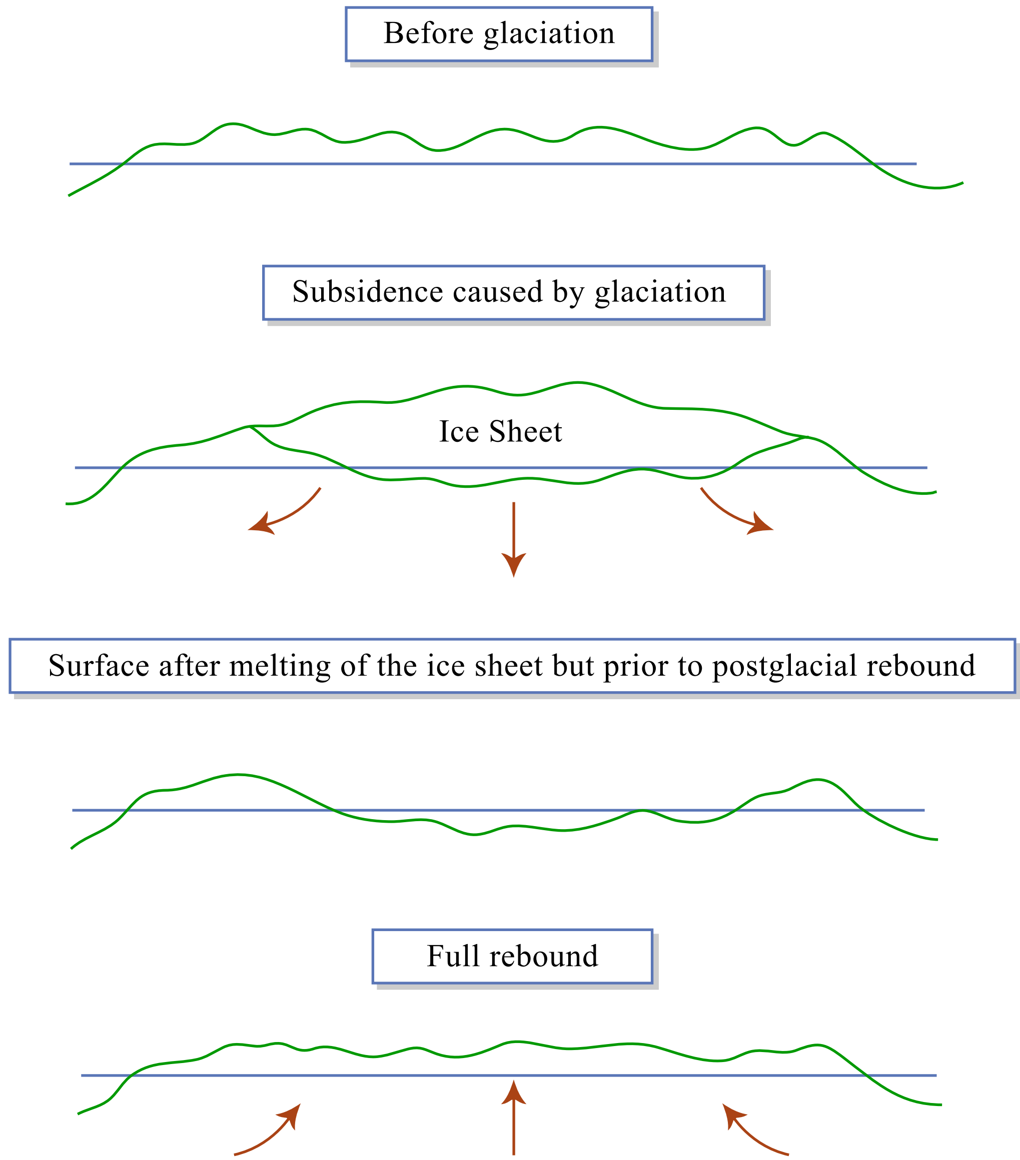

Figure 26.2. Subsidence due to glaciation and the subsequent postglacial rebound.

Figure by MIT OCW.

- Weight of ice causes viscous flow in the mantle.
- After melting of ice, the surface rebounds – "postglacial rebound".
- Different regions have different behaviors (e.g., Boston is now sinking).

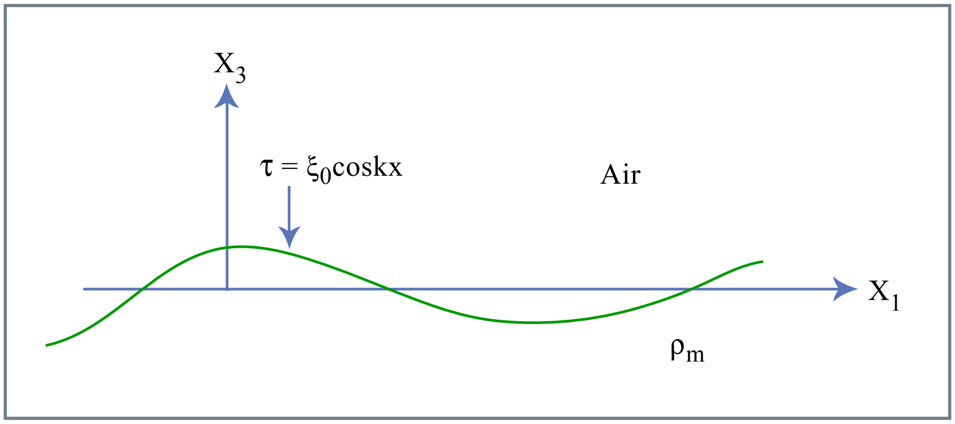

Figure 26.3
Figure by MIT OCW.

Problem: how to reconcile physical boundary conditions with mathematical description?

Decay: Postglacial rebound (1/2 space, uniform η)

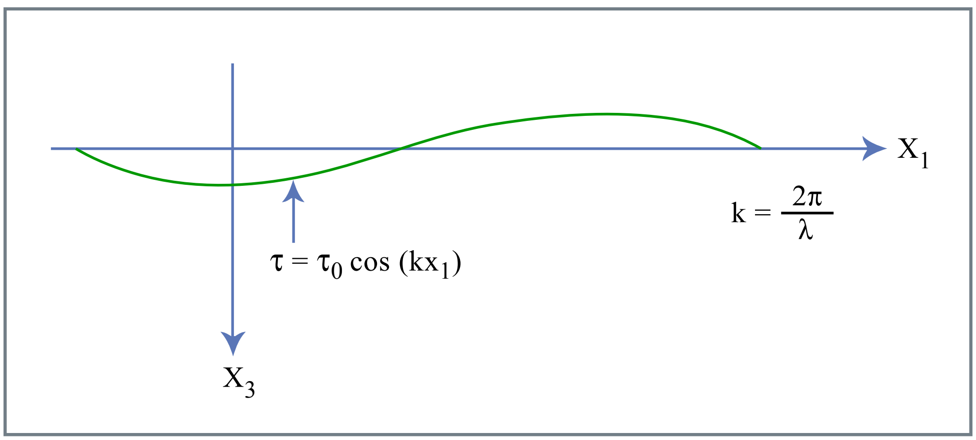

Figure 26.4
Figure by MIT OCW.

- Assume uniform η
- Subtract out lithostatic pressure $P = p - \rho g x_3$
- Assume ρg uniform
- Use stream function Ψ

$$v_1 = -\frac{\partial \Psi}{\partial x_3} \qquad v_3 = \frac{\partial \Psi}{\partial x_1}$$

$$\Rightarrow \nabla^4 \Psi = 0$$

Solution: $\Psi = \left[(A + Bkx_3)\exp(-kx_3) + (C + Dkx_3)\exp(kx_3) \right] \cdot \sin kx_1$

Boundary conditions:

at $x_3 = 0$ (mathematical, not physical)

$$\sigma_{33} = \rho g \zeta$$

$$\sigma_{13} = 0 = \eta \left(\frac{\partial v_1}{\partial x_3} + \frac{\partial v_3}{\partial x_1} \right)$$

at $x_3 \to \infty$, must be bounded

$$\Rightarrow C = D = 0$$

In order that $\sigma_{13} = 0$ at $x_3 = 0$,

$$-\frac{\partial^2 \Psi}{\partial {x_3}^2} + \frac{\partial^2 \Psi}{\partial {x_1}^2} = 0$$

$$\Rightarrow B = A$$

or $\Psi = A(1 + kx_3)\exp(-kx_3) \cdot \sin kx_1$

Then

$$v_1 = Ak^2 x_3 \exp(-kx_3) \cdot \sin kx_1$$

$$v_3 = Ak(1 + kx_3)\exp(-kx_3) \cdot \cos kx_1$$

at $x_3 = 0 \quad v_3 = \dot{\zeta} = Ak\cos(kx_1)$

Now

$$\sigma_{33} = -p + 2\eta \dot{\varepsilon}_{33}$$

$$\dot{\varepsilon}_{33} = 0 \quad \text{at } x_3 = 0$$

To get p, use $-\frac{\partial p}{\partial x_i} + \eta \frac{\partial^2 v_i}{\partial x_j \partial x_j} + \rho x_1 = 0$

for $i = 1$

$$\Rightarrow \; -\frac{\partial p}{\partial x_1} + \eta\left(\frac{\partial^2 v_1}{\partial {x_1}^2} + \frac{\partial^2 v_1}{\partial {x_3}^2}\right) = 0$$

Substitute for v_1 and integrating $\Rightarrow p\big|_{x_3 = 0} = 2\eta k^2 A \cos kx_1$

But $p = -\rho g \zeta \Rightarrow A = -\dfrac{\rho g \zeta_0}{2k^2 \eta}$

Or $\dot{\zeta}_0 = -\dfrac{\rho g \zeta_0}{2k\eta} = -\dfrac{\rho g \lambda \zeta_0}{4\pi\eta}$

Or $\zeta_0 = \zeta_0\big|_{t=0} \exp(-\dfrac{\rho g t \square}{2k\eta}) = \zeta_0\big|_{t=0} \exp(-\dfrac{t}{\tau})$

where $\tau = \dfrac{2k\eta}{\rho g \square} = \dfrac{4\pi\eta}{\rho g \lambda}$

Solving for η: $\eta = \dfrac{\rho g \lambda \tau}{4\pi}$

For curves shown,

$$\left.\begin{array}{l} \tau : \ 5000 \text{ yr} \\ \lambda : \ 3000 \text{ km} \end{array}\right\} \Rightarrow \eta : \ 10^{21} \text{ Pa}$$

Note: stream function $\sim \exp(-kx_3) = \exp(-\dfrac{2\pi x_3}{\lambda})$

Falls off to $\sim 1/e$ at $x_3 : \dfrac{\lambda}{2\pi}$

Senses to fairly great depth

$\Rightarrow$ postglacial rebound doesn’t reveal the details of mantle viscosity structure, but only the gross structure.