12.005 Lecture Notes 21

Plates (continued)

Flexural equation: $D\dfrac{d^4w}{dx^4}+P\dfrac{d^2w}{dx^2}+\Delta\rho g w=q(x)$

where $D=\dfrac{Eh^3}{12(1-\nu^2)}$.

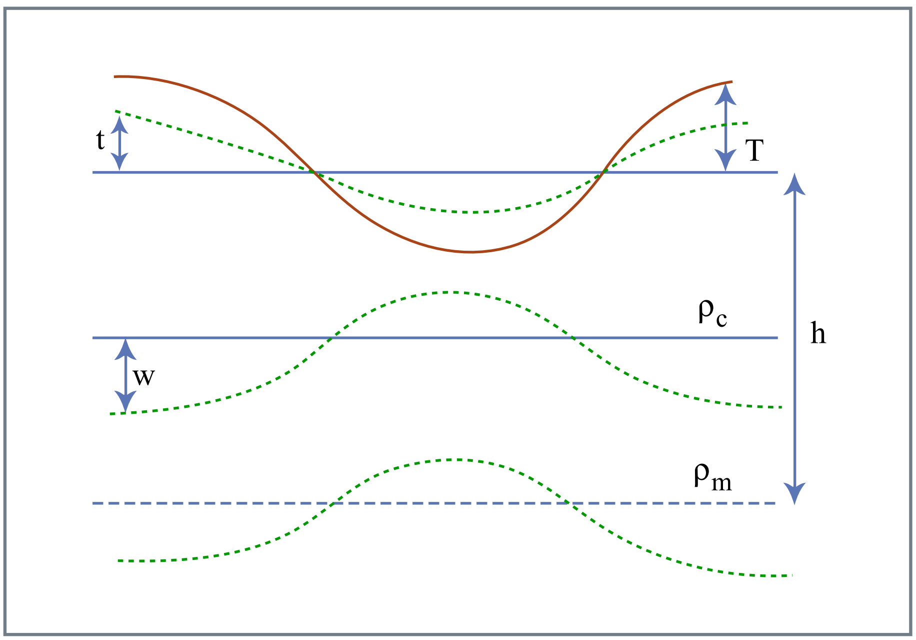

Figure 21.1

Figure by MIT OCW.

$T=T_0\cos kx=T_0\cos\dfrac{2\pi x}{\lambda}$

“Harmonic” load

$t=t_0\cos kx\,,\quad w=w_0\cos kx\,,\quad t_0=T_0-w_0$

$D\dfrac{d^4w}{dx^4}+\rho_m g w=t_0\rho_c g\cos kx$ when $P=0$

$$k^4 D w_0 \cos kx + \rho_m g w_0 \cos kx = t_0 \rho_c g \cos kx$$

$$w_0 = \frac{t_0}{\dfrac{\rho_m}{\rho_c} - 1 + \dfrac{Dk^4}{\rho_c g}}$$

Call $2\pi\left(\dfrac{D}{\rho_c g}\right)^{1/4} \equiv \lambda_l$ flexural wavelength

For λ ? λ_l, $(\lambda_l k)^4$ = 1, w_0 ; $\dfrac{t_0}{\dfrac{\rho_m}{\rho_c} - 1}$; isostacy

For λ = λ_l, w_0 ; $0 \Rightarrow$ uncompensated

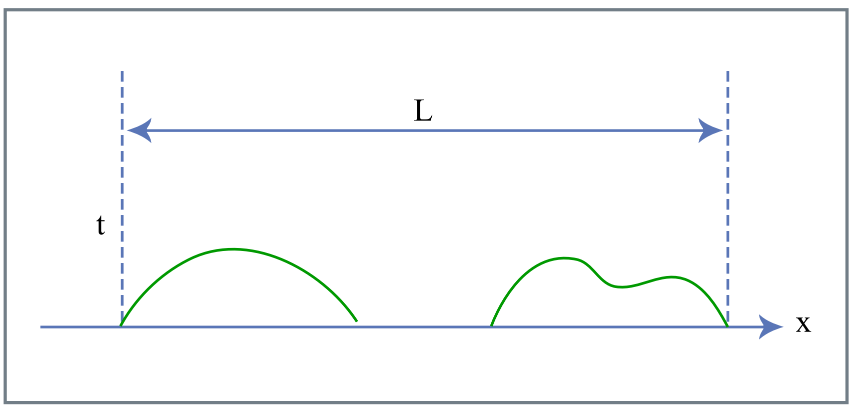

Figure 21.2

Figure by MIT OCW.

$$t(x) = \sum_{n=0}^{\infty}\left[t_n^c \cos\frac{2\pi n x}{L} + t_n^s \sin\frac{2\pi n x}{L}\right]$$

Find t_n^c, t_n^s Assume D, calculate w_n^s, w_n^c

Synthesize $w(x)$, compare to observations.

Plate subject to an end load

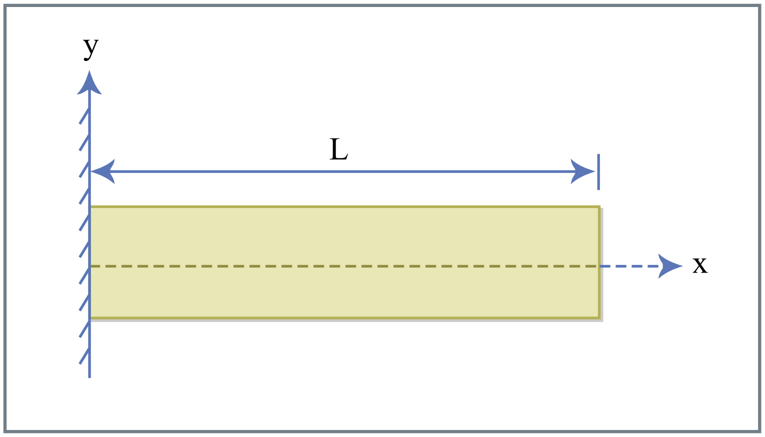

Figure 21.3

Figure by MIT OCW.

Shear force: $\frac{dV}{dx} = -q$

Since $q = 0$, $V =$ const. $= V_a$

Bending moment:

$$\frac{dM}{dx} = V + P\frac{dw}{dx}$$

Since $P = 0$, $\frac{dM}{dx} = V \Rightarrow M = V_a x + \text{const} \Rightarrow 0 \text{ at } x = L$

$$M = V_a(x - L)$$

Displacement:

$$\frac{d^4w}{dx^4} = 0 \quad \Rightarrow \quad \frac{d^3w}{dx^3} = \text{const}$$

But $M = -D\dfrac{d^2w}{dx^2} = 0$, $\quad \dfrac{dM}{dx} = -D\dfrac{d^3w}{dx^3} = V_a$

$$\frac{d^3w}{dx^3} = -\frac{V_a}{D}$$

$$\frac{d^2w}{dx^2} = -\frac{V_a}{D}(x-L)$$

Subject to w, $\dfrac{dw}{dx} = 0$ at $x = 0$

$$w = \frac{V_a x^2}{2D}(L - \frac{x}{3}) \quad \Rightarrow \quad \text{cubic displacement}$$