12.005 Lecture Notes 13

Measurement of Displacement Gradient Tensor

Techniques of measurement of displacement gradient tensor:

- Leveling
- Triangulation
- Trilateration
- Very Long Baseline Interferometry (VLBI)
- Global Positioning System (GPS)

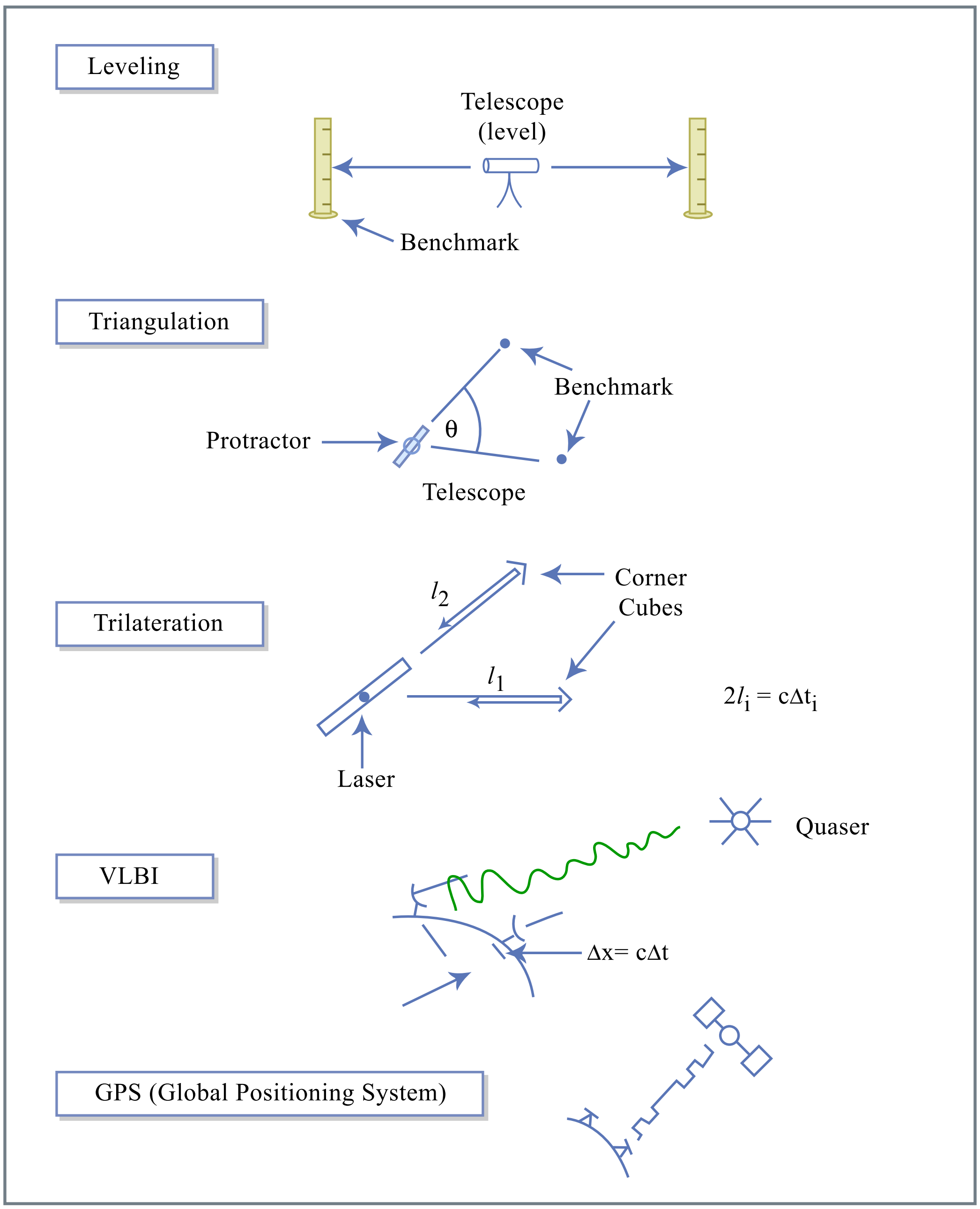


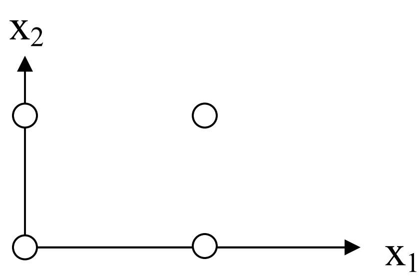
Figure 13.1

Figure by MIT OCW.

Sensitivity:

technique	angle	distance	height	orientation
leveling			Yes	
triangulation	Yes			
trilateration	Yes	Yes		
VLBI	Yes	Yes	Yes	Yes
GPS	Yes	Yes	Yes	Yes

Consider plane strain $(\varepsilon_{33} \equiv 0)$ and a four-benchmark network



Consider the following strain tensors:

Case 1. $\begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Case 2. $\begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & -\varepsilon & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\text{Case 3.} \begin{bmatrix} 0 & \varepsilon & 0 \\ \varepsilon & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Case 4.} \begin{bmatrix} 2\varepsilon & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Can they be observed using triangulation?

Can they be observed using trilateration?

Which angles would change?

Which line lengths would change?

What would the change in line length be?

$\underset{\sim}{\varepsilon}$ is a tensor, so $\underset{\sim}{\varepsilon}' = \underset{\sim}{\alpha}\underset{\sim}{\varepsilon}\underset{\sim}{\alpha}^T$, just as for stress

$$\alpha = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

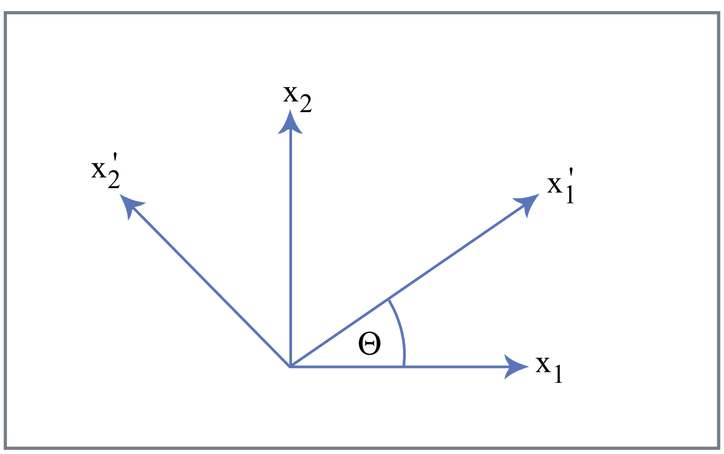


Figure 13.2

Figure by MIT OCW.

Evaluating this for plane strain is straightforward:

$$\varepsilon_{11}' = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} + \frac{\varepsilon_{11}-\varepsilon_{22}}{2}\cos 2\theta + \varepsilon_{12}\sin 2\theta$$
$$\varepsilon_{22}' = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} - \frac{\varepsilon_{11}-\varepsilon_{22}}{2}\cos 2\theta - \varepsilon_{12}\sin 2\theta$$
$$\varepsilon_{12}' = -\left(\frac{\varepsilon_{11}-\varepsilon_{22}}{2}\right)\sin 2\theta + \varepsilon_{12}\cos 2\theta$$

Note: Shear strains are tough to measure.

These relationships are useful in relating longitudinal strains $\frac{\delta l}{l}$ to the total strain tensor.

$\frac{\delta l}{l}$ can be easily measured using trilateration (10's km scale), strain gauge (10's mm scale).

For example, "delta rosetle"

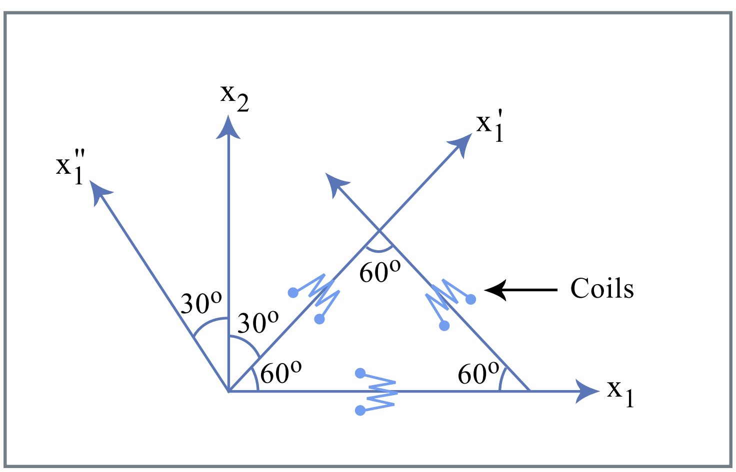


Figure 13.3

Figure by MIT OCW.

Equilateral triangle of transducers records elongations in x_1, x_1', and x_1''directions.

These can be related to $\underset{\sim}{\varepsilon}$.

Summary of infinitesimal strain:

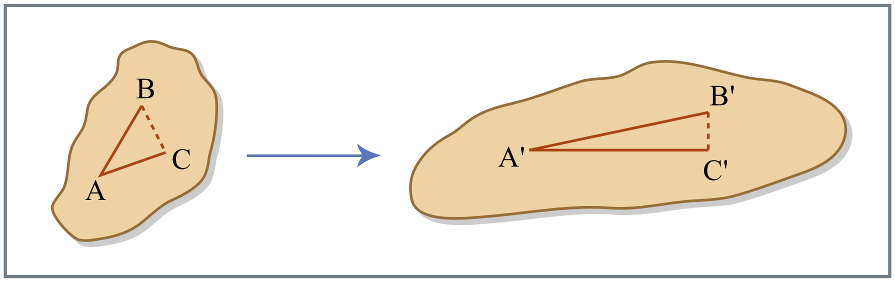


Figure 13.4

Figure by MIT OCW.

The above figure shows translation and rotation.

Both length change $\frac{\delta l}{l}$ and angle change $\delta\theta$ depend on orientations.

Solid lines $\frac{\delta l}{l} > 0$

Dashed lines $\frac{\delta l}{l} < 0$

$\angle BAC$ decreases

$\angle ABC$ increases

Strain tensor:

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix}$$

diagonal $\Rightarrow \frac{\delta l}{l}$ for lines along axes

off diagonal $\sim \frac{\delta\theta}{2}$ for deformation of axes

The strain tensor depends on the orientation of the coordinate system.

Geologic Strain indicators – pebbles elliptical