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ANALYSIS OF STRUCTURAL MEMBER SYSTEMS

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Preface

With the development over the past decade of computer-based analysis methods, the teaching of structural analysis subjects has been revolutionized. The traditional division between structural analysis and structural mechanics became no longer necessary, and instead of teaching a preponderance of solution details it is now possible to focus on the underlying theory.

What has been done here is to integrate analysis and mechanics in a systematic presentation which includes the mechanics of a member, the matrix formulation of the equations for a system of members, and solution techniques. The three fundamental steps in formulating a problem in solid mechanics—enforcing equilibrium, relating deformations and displacements, and relating forces and deformations—form the basis of the development, and the central theme is to establish the equations for each step and then discuss how the complete set of equations is solved. In this way, a reader obtains a more unified view of a problem, sees more clearly where the various simplifying assumptions are introduced, and is better prepared to extend the theory.

The chapters of Part I contain the relevant topics for an essential background in linear algebra, differential geometry, and matrix transformations. Collecting this material in the first part of the book is convenient for the continuity of the mathematics presentation as well as for the continuity in the following development.

Part II treats the analysis of an ideal truss. The governing equations for small strain but arbitrary displacement are established and then cast into matrix form. Next, we deduce the principles of virtual displacements and virtual forces by manipulating the governing equations, introduce a criterion for evaluating the stability of an equilibrium position, and interpret the governing equations as stationary requirements for certain variational principles. These concepts are essential for an appreciation of the solution schemes described in the following two chapters.

Part III is concerned with the behavior of an isolated member. For completeness, first are presented the governing equations for a deformable elastic solid allowing for arbitrary displacements, the continuous form of the principles of virtual displacements and virtual forces, and the stability criterion. Unrestrained torsion-flexure of a prismatic member is examined in detail and then an approximate engineering theory is developed. We move on to restrained torsion-flexure of a prismatic member, discussing various approaches for including warping restraint and illustrating its influence for thin-walled

open and closed sections. The concluding chapters treat the behavior of planar and arbitrary curved members.

How one assembles and solves the governing equations for a member system is discussed in Part IV. First, the direct stiffness method is outlined; then a general formulation of the governing equations is described. Geometrically nonlinear behavior is considered in the last chapter, which discusses member force-displacement relations, including torsional-flexural coupling, solution schemes, and linearized stability analysis.

The objective has been a text suitable for the teaching of modern structural member system analysis, and what is offered is an outgrowth of lecture notes developed in recent years at the Massachusetts Institute of Technology. To the many students who have provided the occasion of that development, I am deeply appreciative. Particular thanks go to Mrs. Jane Malinofsky for her patience in typing the manuscript, and to Professor Charles Miller for his encouragement.

JEROME J. CONNOR

Cambridge, Mass.
January, 1976

Contents

I—MATHEMATICAL PRELIMINARIES

1 Introduction to Matrix Algebra 3

1–1 Definition of a Matrix 3
1–2 Equality, Addition, and Subtraction of Matrices 5
1–3 Matrix Multiplication 5
1–4 Transpose of a Matrix 8
1–5 Special Square Matrices 10
1–6 Operations on Partitioned Matrices 12
1–7 Definition and Properties of a Determinant 16
1–8 Cofactor Expansion Formula 19
1–9 Cramer's Rule 21
1–10 Adjoint and Inverse Matrices 22
1–11 Elementary Operations on a Matrix 24
1–12 Rank of a Matrix 27
1–13 Solvability of Linear Algebraic Equations 30

2 Characteristic-Value Problems and Quadratic Forms 46

2–1 Introduction 46
2–2 Second-Order Characteristic-Value Problem 48
2–3 Similarity and Orthogonal Transformations 52
2–4 The *n*th-Order Symmetrical Characteristic-Value Problem 55
2–5 Quadratic Forms 57

3 Relative Extrema for a Function 66

3–1 Relative Extrema for a Function of One Variable 66
3–2 Relative Extrema for a Function of *n* Independent Variables 71
3–3 Lagrange Multipliers 75

4 Differential Geometry of a Member Element 81

4–1 Parametric Representation of a Space Curve 81
4–2 Arc Length 82

4–3 Unit Tangent Vector 85
4–4 Principal Normal and Binormal Vectors 86
4–5 Curvature, Torsion, and the Frenet Equations 88
4–6 Summary of the Geometrical Relations for a Space Curve 91
4–7 Local Reference Frame for a Member Element 92
4–8 Curvilinear Coordinates for a Member Element 94

5 Matrix Transformations for a Member Element 100

5–1 Rotation Transformation 100
5–2 Three-Dimensional Force Transformations 103
5–3 Three-Dimensional Displacement Transformations 109

II—ANALYSIS OF AN IDEAL TRUSS

6 Governing Equations for an Ideal Truss 115

6–1 General 115
6–2 Elongation—Joint Displacement Relation for a Bar 116
6–3 General Elongation—Joint Displacement Relation 120
6–4 Force-Elongation Relation for a Bar 125
6–5 General Bar Force—Joint Displacement Relation 130
6–6 Joint Force-Equilibrium Equations 130
6–7 Introduction of Displacement Restraints; Governing Equations 132
6–8 Arbitrary Restraint Direction 134
6–9 Initial Instability 137

7 Variational Principles for an Ideal Truss 152

7–1 General 152
7–2 Principle of Virtual Displacements 153
7–3 Principle of Virtual Forces 159
7–4 Strain Energy; Principle of Stationary Potential Energy 162
7–5 Complementary Energy; Principle of Stationary Complementary Energy 165
7–6 Stability Criteria 169

8 Displacement Method—Ideal Truss 178

8–1 General 178
8–2 Operation on the Partitioned Equations 178
8–3 The Direct Stiffness Method 180

8–4 Incremental Formulation; Classical Stability Criterion 191
8–5 Linearized Stability Analysis 200

9 Force Method—Ideal Truss 210

9–1 General 210
9–2 Governing Equations—Algebraic Approach 211
9–3 Governing Equations—Variational Approach 216
9–4 Comparison of the Force and Mesh Methods 217

III—ANALYSIS OF A MEMBER ELEMENT

10 Governing Equations for a Deformable Solid 229

10–1 General 229
10–2 Summation Convention; Cartesian Tensors 230
10–3 Analysis of Deformation; Cartesian Strains 232
10–4 Analysis of Stress 240
10–5 Elastic Stress-Strain Relations 248
10–6 Principle of Virtual Displacements; Principle of Stationary Potential Energy; Classical Stability Criteria 253
10–7 Principle of Virtual Forces; Principle of Stationary Complementary Energy 257

11 St. Venant Theory of Torsion-Flexure of Prismatic Members 271

11–1 Introduction and Notation 271
11–2 The Pure-Torsion Problem 273
11–3 Approximate Solution of the Torsion Problem for Thin-Walled Open Cross Sections 281
11–4 Approximate Solution of the Torsion Problem for Thin-Walled Closed Cross Sections 286
11–5 Torsion-Flexure with Unrestrained Warping 293
11–6 Exact Flexural Shear Stress Distribution for a Rectangular Cross Section 303
11–7 Engineering Theory of Flexural Shear Stress Distribution in Thin-Walled Cross Sections 306

12 Engineering Theory of Prismatic Members 330

12–1 Introduction 330
12–2 Force-Equilibrium Equations 331

12–3 Force-Displacement Relations; Principle of Virtual Forces 333
12–4 Summary of the Governing Equations 339
12–5 Displacement Method of Solution—Prismatic Member 340
12–6 Force Method of Solution 349

13 Restrained Torsion-Flexure of a Prismatic Member 371

13–1 Introduction 371
13–2 Displacement Expansions; Equilibrium Equations 372
13–3 Force-Displacement Relations—Displacement Model 375
13–4 Solution for Restrained Torsion—Displacement Model 379
13–5 Force-Displacement Relations—Mixed Formulation 383
13–6 Solution for Restrained Torsion—Mixed Formulation 389
13–7 Application to Thin-Walled Open Cross Sections 395
13–8 Application to Thin-Walled Closed Cross Sections 405
13–9 Governing Equations—Geometrically Nonlinear Restrained Torsion 414

14 Planar Deformation of a Planar Member 425

14–1 Introduction; Geometrical Relations 425
14–2 Force-Equilibrium Equations 427
14–3 Force-Displacement Relations; Principle of Virtual Forces 429
14–4 Force-Displacement Relations—Displacement Expansion Approach; Principle of Virtual Displacements 435
14–5 Cartesian Formulation 445
14–6 Displacement Method of Solution—Circular Member 449
14–7 Force Method of Solution 458
14–8 Numerical Integration Procedures 473

15 Engineering Theory of an Arbitrary Member 485

15–1 Introduction; Geometrical Relations 485
15–2 Force-Equilibrium Equations 488
15–3 Force-Displacement Relations—Negligible Warping Restraint; Principle of Virtual Forces 490
15–4 Displacement Method—Circular Planar Member 493
15–5 Force Method—Examples 499
15–6 Restrained Warping Formulation 507
15–7 Member Force-Displacement Relations—Complete End Restraint 511
15–8 Generation of Member Matrices 517

15–9 Member Matrices—Prismatic Member 520
15–10 Member Matrices—Thin Planar Circular Member 524
15–11 Flexibility Matrix—Circular Helix 531
15–12 Member Force-Displacement Relations—Partial End Restraint 535

IV—ANALYSIS OF A MEMBER SYSTEM

16 Direct Stiffness Method—Linear System 545

16–1 Introduction 545
16–2 Member Force-Displacement Relations 546
16–3 System Equilibrium Equations 547
16–4 Introduction of Joint Displacement Restraints 548

17 General Formulation—Linear System 554

17–1 Introduction 554
17–2 Member Equations 555
17–3 System Force-Displacement Relations 557
17–4 System Equilibrium Equations 559
17–5 Introduction of Joint Displacement Restraints; Governing Equations 560
17–6 Network Formulation 562
17–7 Displacement Method 565
17–8 Force Method 567
17–9 Variational Principles 570
17–10 Introduction of Member Deformation Constraints 573

18 Analysis of Geometrically Nonlinear Systems 585

18–1 Introduction 585
18–2 Member Equations—Planar Deformation 585
18–3 Member Equations—Arbitrary Deformation 591
18–4 Solution Techniques; Stability Analysis 597

Index 605