III-1 DRY SOIL : STRESSES

PART III DRY SOIL: No. 1 STRESSES (p 1/6)

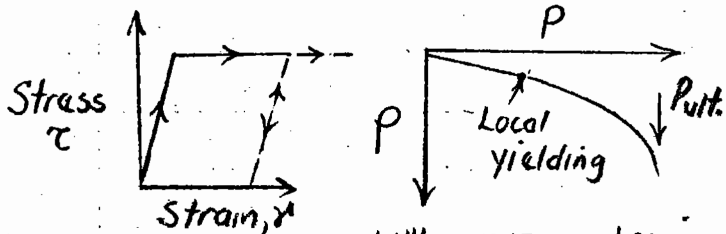
1. INTRODUCTION

1.1 Soil Stresses = force/unit area on plane thru soil: σ = normal stress, τ = shear stress

1.2. Part III deals w/ strength-deformation behavior of dry soil (sand), but principles wrt $\sigma' = \sigma - u$ apply to soils with H_2O.

1.3 Approaches to predict load-settlement for footing on soil:

(a) Constitutive eqn. for soil + F.E. code (Soil Model)
(Elasto-Plastic)

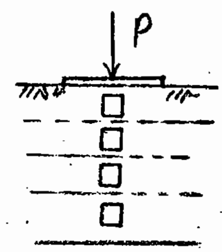


Whittle: MIT-E3 & SI

Remarks: A lot research on soil modeling with increasing use in practice for major projects (nuclear, offshore...)

(b) Lambe's "Stress Path" method

(1) Lab tests on soil samples from each layer
- Initial stresses
- Δ stresses
- Measure strains



(2) $\rho = \Sigma(H_i \cdot \varepsilon_i)$

Remarks: Problems w/ sample disturbance (esp. sands) + sometimes difficult estimate stresses

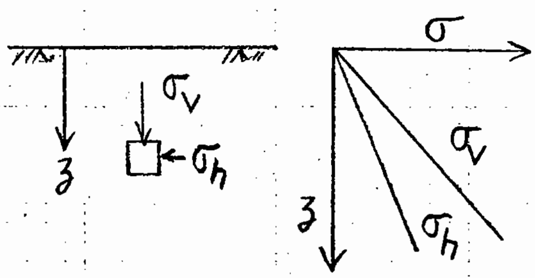
(c) Empirical prediction using results from in situ penetration tests, e.g. SPT N values for settlement of sands (clays use a or b usually)

1.4 Coverage: a) Initial stresses & Δ stresses due to construction; stress paths
b) Stress-strain-strength behavior of granular soils
c) Applications to lateral earth pressures, stability & settlement

2. GEOSTATIC STRESSES

2.1 Definition = stresses acting within horizontal soil deposit due to self weight of soil. Only for vertical 1-D load & unload, $\tau_v = \tau_h = 0$

2.2 Estimation ($\sigma \equiv \sigma'$ for dry soil)



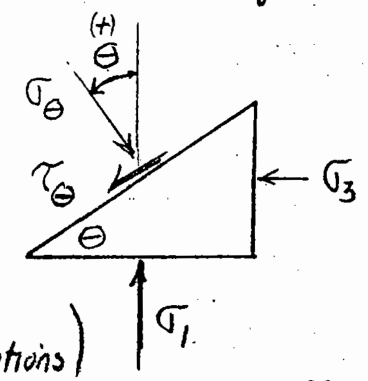
- $\sigma_v = \int_0^z \gamma' \cdot dz$; γ' = soil unit weight
- $\sigma_h = K_0 \sigma_v$
- K_0 = coef. of earth pressure at rest $= \sigma_h' / \sigma_v'$ for no lateral strain
- $K = \sigma_h' / \sigma_v'$ = lateral stress ratio

Note: $K_0 \approx 1/2$ for NC soil & incr. w/ incr. OCR

3. PRINCIPAL STRESSES & MOHR CIRCLE

3.1 Principal Stresses: For any state of stress at a point have 3 ⊥ planes with $\tau = 0$ called principal planes with $\sigma_1 \geq \sigma_2 \geq \sigma_3$

3.2 From Force Equilibrium (Plane ⊥ to σ_2 direction)



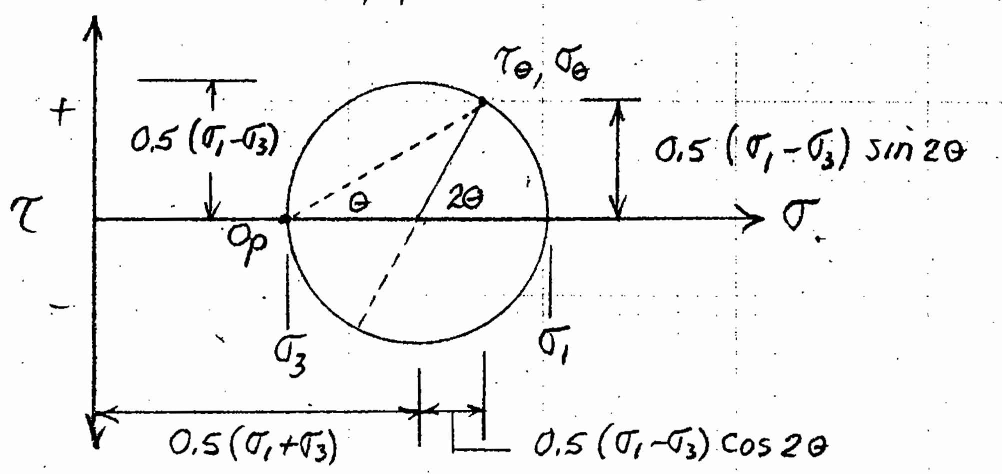
(See p2a for derivations)

θ = ∡ between planes of σ_1 & σ_θ (↺+)

τ_θ = positive when counter clockwise

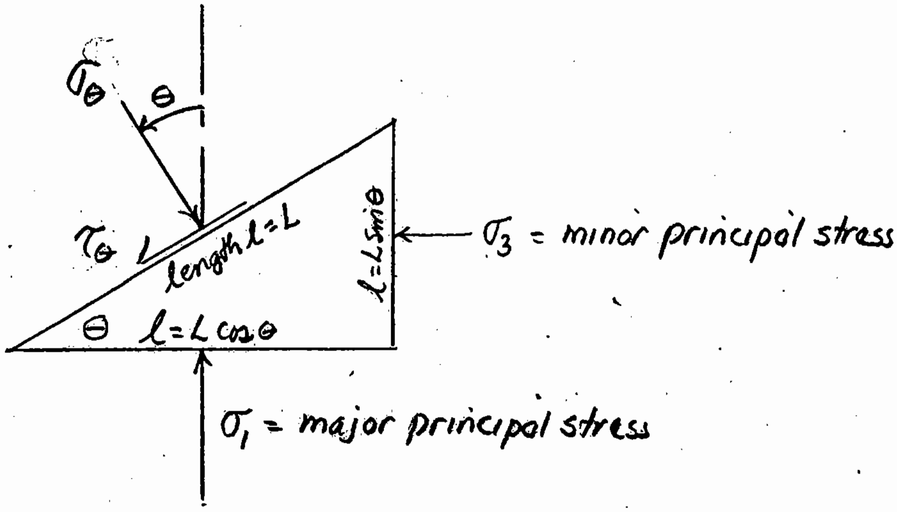
$$\sigma_\theta = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$$
$$= 0.5(\sigma_1+\sigma_3) + 0.5(\sigma_1-\sigma_3)\cos 2\theta$$
$$\tau_\theta = (\sigma_1-\sigma_3)\sin\theta\cos\theta = 0.5(\sigma_1-\sigma_3)\sin 2\theta$$

3.3 Mohr Circle (By plotting σ_θ & τ_θ at varying θ; stresses ⊥ to σ_2)



NOTES:
- Max. $\tau = 0.5(\sigma_1-\sigma_3)$
- Σ σ on 2 ⊥ planes = constant = $(\sigma_1+\sigma_3)$
- τ on 2 ⊥ planes of same magnitude but opposite sign
- Usually only deal with stresses ⊥ to σ_2 direction
- Minimum information needed to define state of stress =
  1) σ_θ & τ_θ on one plane of known orientation, θ
  2) PLUS....

Supplement for Section 3.2 : Derivation of Eqn. for Mohr Circle



$\underline{\Sigma V = 0}$ $\quad \sigma_1 \cdot L\cos\Theta = \sigma_\Theta \cdot L \cdot \cos\Theta + \tau_\Theta \cdot L \cdot \sin\Theta$

Eq. a $\quad \sigma_1 \cos^2\Theta = \sigma_\Theta \cos^2\Theta + \tau_\Theta \cos\Theta \sin\Theta \qquad \times \cos\Theta / L$

$\underline{\Sigma H = 0}$ $\quad \sigma_3 \cdot L\sin\Theta = \sigma_\Theta \cdot L \cdot \sin\Theta - \tau_\Theta \cdot L \cdot \cos\Theta$

Eq. b $\quad \sigma_3 \sin^2\Theta = \sigma_\Theta \sin^2\Theta - \tau_\Theta \cos\Theta \sin\Theta \qquad \times \sin\Theta / L$

Add Eq. a & b $\quad \underline{\sigma_1 \cos^2\Theta + \sigma_3 \sin^2\Theta} = \sigma_\Theta \overbrace{(\cos^2\Theta + \sin^2\Theta)}^{=1} = \underline{\sigma_\Theta} \qquad$ QED

$\qquad\qquad$ (1) $\qquad$ (2) $\qquad\qquad$ (3) $\qquad\qquad$ (4)

Substract Eq a & b $\quad \sigma_1 \cos^2\Theta - \sigma_3 \sin^2\Theta = \sigma_\Theta (\cos^2\Theta - \sin^2\Theta) + \tau_\Theta\, 2\cos\Theta \sin\Theta$

Term (3) $\quad \sigma_1 \cos^2\Theta (\cos^2\Theta - \sin^2\Theta) + \sigma_3 \sin^2\Theta (\cos^2\Theta - \sin^2\Theta)$ Substitute for σ_Θ

Term (1)+(2)+(3) $\quad \sigma_1 \cos^2\Theta (\underbrace{1 + \sin^2\Theta - \cos^2\Theta}_{2\sin^2\Theta}) - \sigma_3 \sin^2\Theta (\underbrace{1 + \cos^2\Theta - \sin^2\Theta}_{2\cos^2\Theta})$

Term (1)+(4) $\quad 2\sigma_1 \cos^2\Theta \sin^2\Theta - 2\sigma_3 \sin^2\Theta \cos^2\Theta = 2\tau_\Theta \cos\Theta \sin\Theta$

$$\sigma_1 - \sigma_3 = \tau_\Theta \frac{\cos\Theta \sin\Theta}{\cos^2\Theta \sin^2\Theta} \qquad \times \frac{1}{2\cos^2\Theta \sin^2\Theta}$$

$$\therefore\ \underline{\tau_\Theta = (\sigma_1 - \sigma_3)\cos\Theta \sin\Theta = 0.5(\sigma_1 - \sigma_3)\sin 2\Theta} \qquad \text{QED}$$

$\cos^2\Theta + \sin^2\Theta = 1 \quad ; \quad \cos\Theta \sin\Theta = 0.5 \sin 2\Theta$

PART III -1 STRESSES (p3/6)

3.4 Origin of Planes = Op

Point on Mohr circle such that any line drawn thru Op parallel to plane of interest will intersect circle at the stresses on that plane. (Better than using 2θ)

4. STRESS PATHS (Essential to "survival" in MIT soil mechanics subjects)

4.1 Important Restriction

- Should only use for states of stress wherein principal stresses act in vertical and horizontal directions

4.2 Purpose & Definition

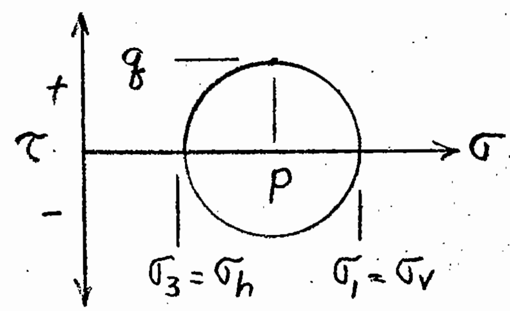
To simplify presentation of states of stress by using peak points (q & p) rather than Mohr circles + Op

$q = 0.5(\sigma_v - \sigma_h)$ & $p = 0.5(\sigma_v + \sigma_h)$

q = positive for $\sigma_v > \sigma_h$ ($K<1$)

= negative for $\sigma_v < \sigma_h$ ($K>1$)

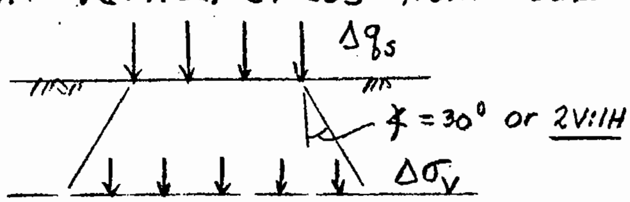
$\sigma_v = p + q$ and $\sigma_h = p - q$



4.3 Examples (See p 3a)

5 STRESS DISTRIBUTION: TYPICAL APPROACHES (Surface loads)

5.1 Vertical Stress from Codes



5.2 Finite Element with Layered Non-Linear $\sigma - \varepsilon$ Model

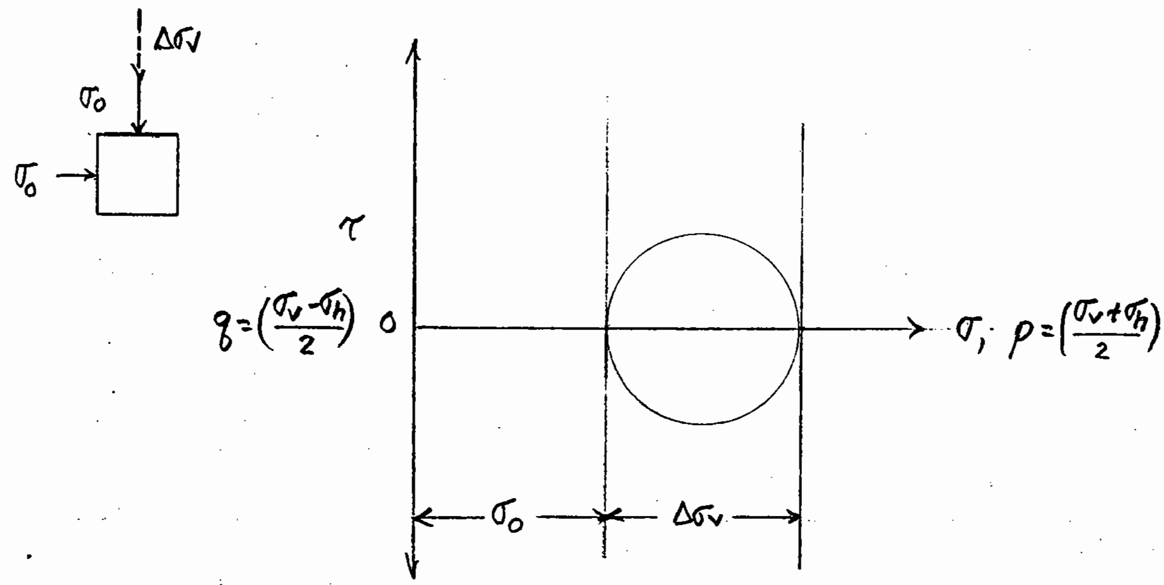
e.g. Hyperbolic

$$\sigma = \frac{\varepsilon}{a + b\varepsilon}$$

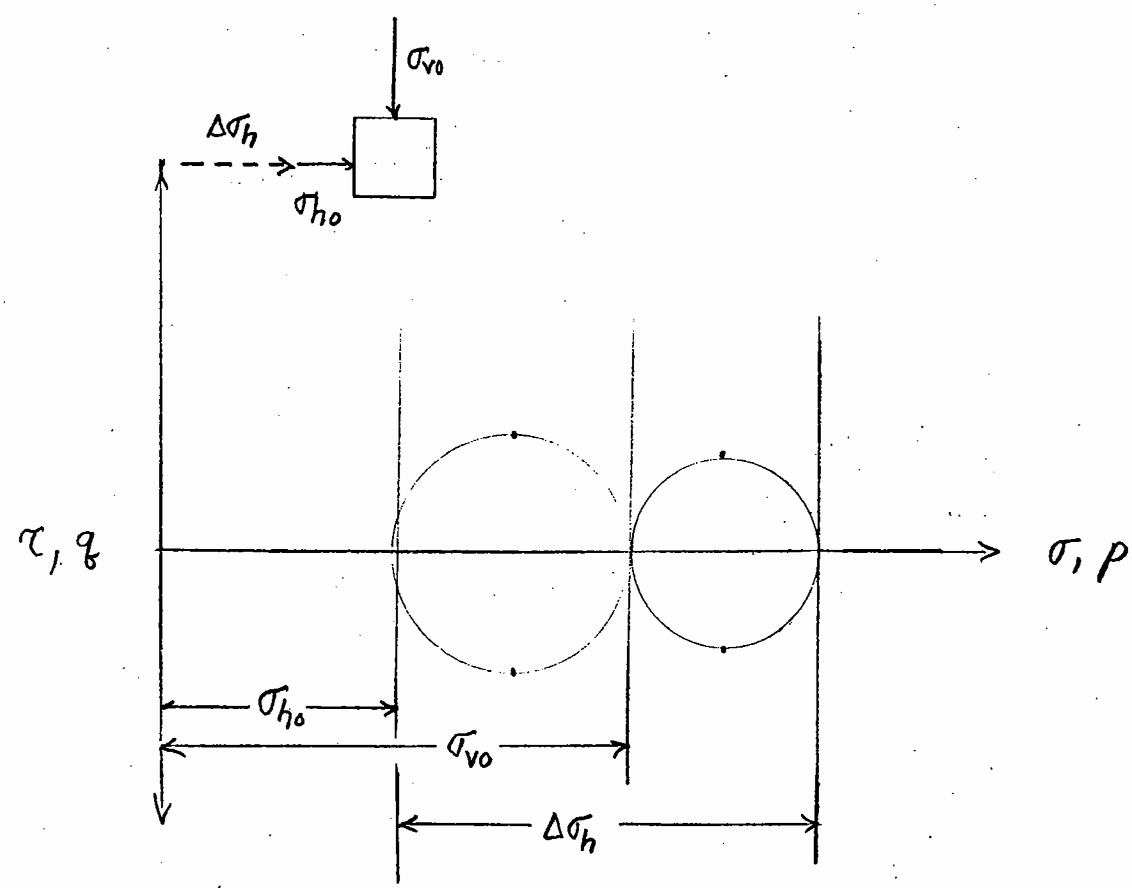
5.3 Theory of Elasticity - Most Common

4.3 Examples of Stress Paths

1) Isotropic (hydrostatic) compression to σ_0; then increase σ_v



2) 1-D (K_0) compression of NC sand to σ_{v0}; then increase σ_h



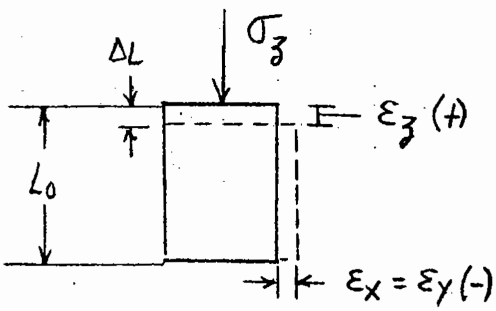
6. THEORY OF ELASTICITY

6.1 Conventional Assumptions

Theory	Soil
5) Homogeneous (same properties everywhere)	Often highly variable
4) Isotropic (properties independ. direction)	Cross anisotropic
1) Elastic (all strains recoverable)	Usually only true for very small strains
2) Linear (Hooke's Law : $\sigma \propto \varepsilon$)	Ditto
3) Small strains (Engr strain $\varepsilon = \frac{\Delta L}{L_0}$; Natural strain = $\frac{\Delta L}{L}$)	Usually O.K.

6.2 Evaluation of Elastic Constants (Linear & Isotropic)

- Constitutive relationship defined by two constant properties
- If run unconfined compression test (compressive ε is positive)



Young's modulus $E = \sigma_z / \varepsilon_z$

Poisson's ratio $\mu = -\varepsilon_x / \varepsilon_z$

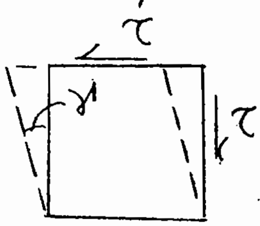
Notes: If $\varepsilon_{vol.} = \varepsilon_z + \varepsilon_x + \varepsilon_y = \varepsilon_z(1-2\mu) = 0 \rightarrow \mu = 0.5$

If $\varepsilon_x = \varepsilon_y = 0 \rightarrow \mu = 0$

If $\frac{1}{3}(\Delta\sigma_z + \Delta\sigma_x + \Delta\sigma_y) = \Delta\sigma_{oct} = 0$, then theory predicts that $\varepsilon_{vol.} = 0$

6.3 Real Soil Behavior

- E & μ vary with stress level & stress path
- μ' can be > 0.5 or < 0 ($\mu = 0.5$ saturated undrained shear; $\mu' \approx 1/3$ for drained shear clays)
- Results pure shear where $+\Delta\sigma_1 = -\Delta\sigma_3$ & $\Delta\sigma_2 = 0$



Shear modulus $G = \frac{\tau}{\gamma} = \frac{E}{2(1+\mu)}$

Altho. $\Delta\sigma_{oct} = 0$, $\varepsilon_{vol.} \neq 0$

- Cross-Anisotropic (1-D = K_0 loading history) $\rightarrow$ 5 constants
  E_v , E_h , G_{hv} , μ_{hv} & μ_{hh}

7. ELASTIC STRESS DISTRIBUTION

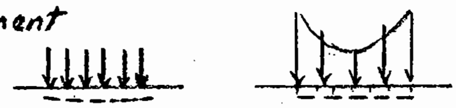
7.1 Best Reference

- Poulos & Davis (1974) "Elastic Solutions for Soil & Rock Mechanics", J. Wiley & Sons, 411p Also NAVFAC DM 7.1 (1982)

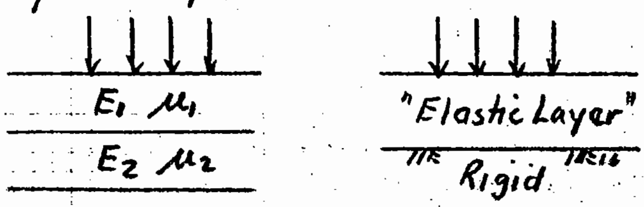
7.2 Typical Variables

(1) Boussinesq = Elastic Half Space

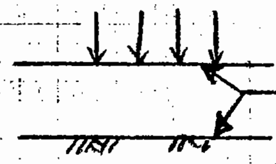
- Point vs distributed
- Shape of area : circle, square, rectangle, strip, ...
- Uniform stress vs uniform displacement



(2) Layered System (Burmister)

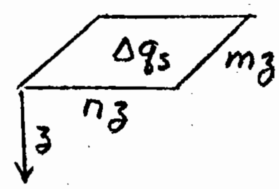


(3) Boundaries



Frictionless vs Full Adhesion (more realistic)

7.3 Boussinesq Charts for $\Delta\sigma_v$ (Westergaard less realistic since $\varepsilon_h = 0$)



(1) $\Delta\sigma_v = \Delta q_s \cdot f(m,n)$ is independent of μ

(2) Section 8.3 (know well, esp. p104)

(3) Values of $f(m,n)$

- When m & $n < 0.3 \rightarrow$ approx. Point Load
- When m or $n > 5 \rightarrow$ approx ∞ Load (in that direction)

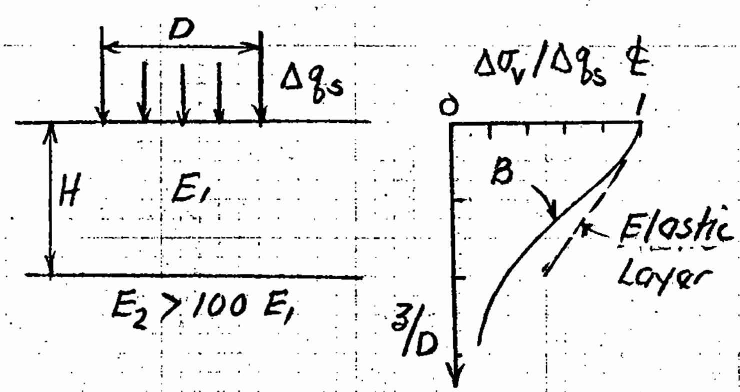
For m & $n > 5$ $\begin{cases} \text{• Under corner, } f = \\ \text{• Under edge, } f = \end{cases}$

7.4 Pressure Bulb (Refers to $\Delta\sigma_v / \Delta q_s$)

- Zone under loaded area where stress increase is significant, say $\Delta\sigma_v > 10\text{–}15\%\ \Delta q_s$ See Figs. 8.4 & 8.5
- Extends down to $z = 2B$ for circle/square; $z = 4B$ for strip

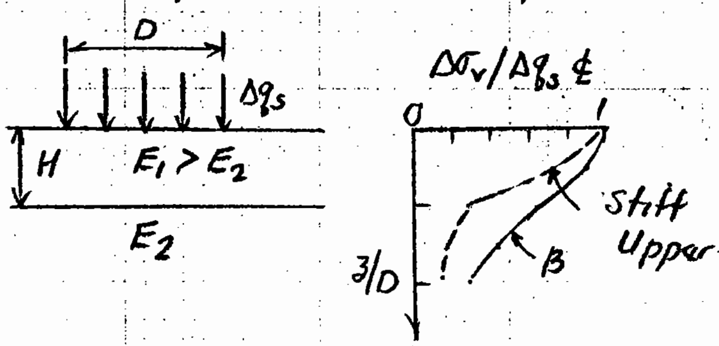
7.5 Effects of Layered System

(1) "Rigid base" → "Elastic layer" (Soft over stiff)



- B = Boussinesq
- Rigid base increases $\Delta\sigma_v$ within overlying soil which is less effective in distributing the load

(2) Stiff layer over Soft layer → Decreased $\Delta\sigma_v$ in both layers



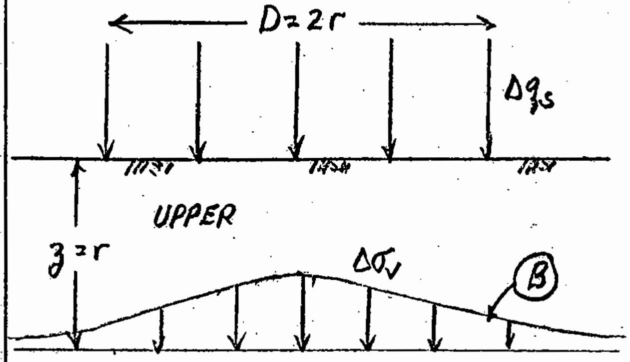
- For $D/H = 2$, at $z = H$:
  $E_1/E_2 = 10 \rightarrow \times 0.45$
  $= 100 \rightarrow \times 0.1$
- Rigid pavement design
- See Fig. 8, Sheet A

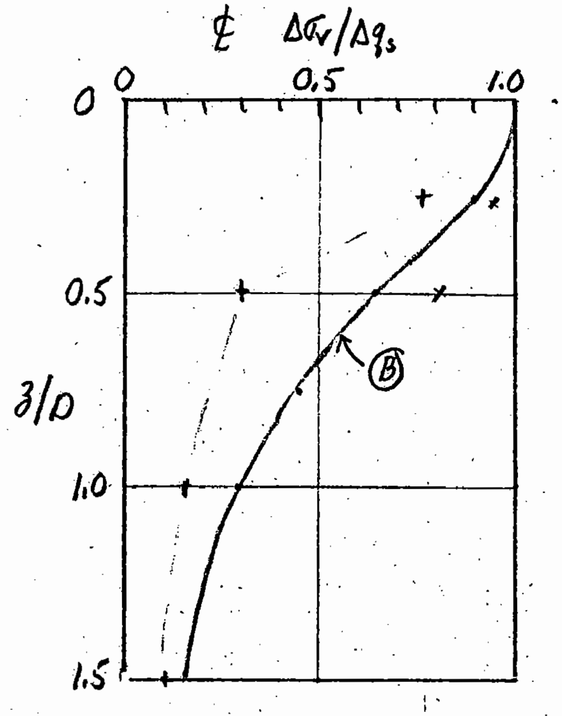
7.6 Effect of Poisson's Ratio

(1) None on $\Delta\sigma_v$ (Except see Fig. 9, Sheet A)

(2) Very significant on $\Delta\sigma_h$ for non-strip loadings (Class discussion)

(a) LAYERING





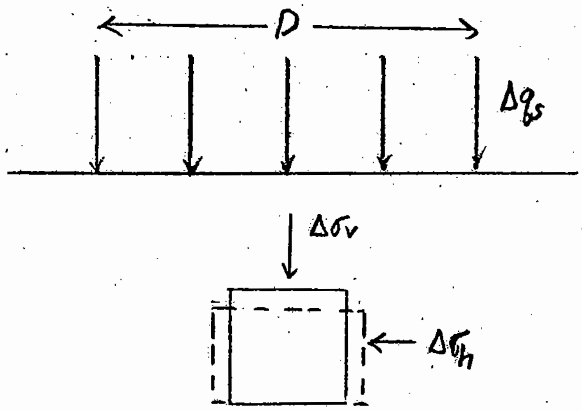
(P) from Fig. 8, Sheet A

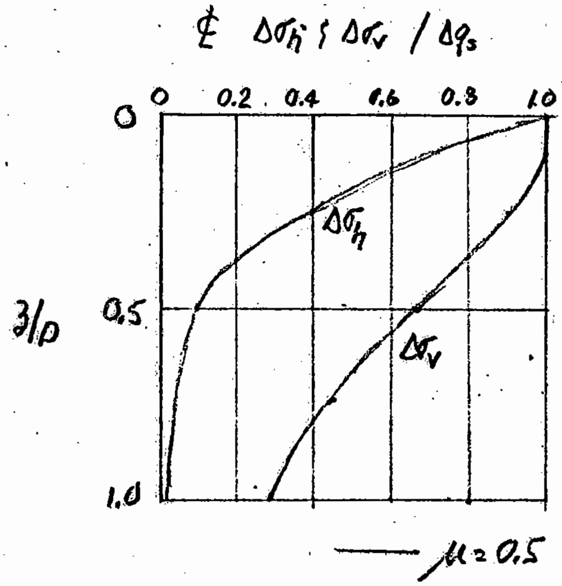
(B) Boussinesq = homogeneous elastic half space

(P) Stiff upper layer (Pavement)

(R) Soft upper layer (Clay over Rock)

(b) POISSON'S RATIO





- Undrained loading, $\mu = 0.5$
- Drained loading, $\mu' \approx 1/3$