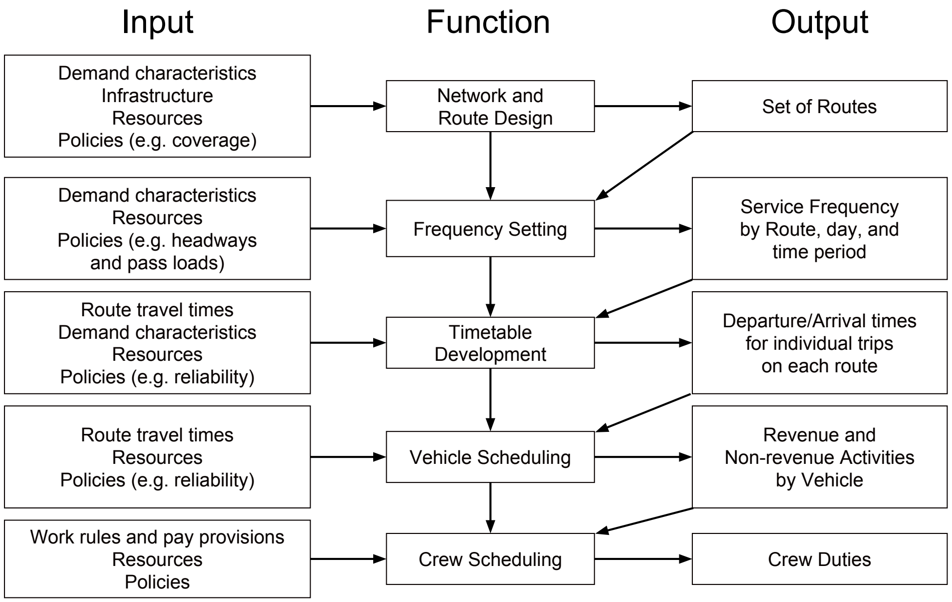
Frequency Determination

Outline

- Service Planning Hierarchy
- Introduction to Scheduling
- Setting Running Times and Cycle Times
- Frequency Determination

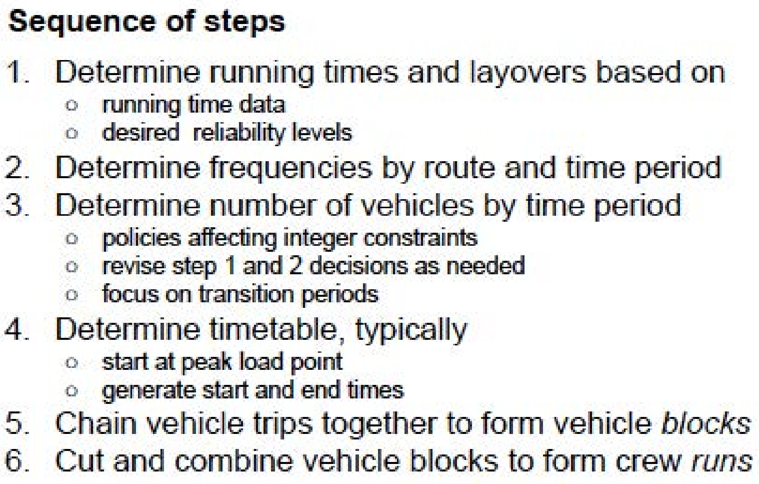
Service Planning Hierarchy



Service Planning Hierarchy

Planning Step	**Frequency of Decisions**	**Principal Consideration**	**Principal Analysis Type**
Network Design	Infrequent	Service	Judgment & Manual
Frequency Setting			
Timetable Development			
Vehicle Scheduling			
Crew Scheduling	Frequent	Cost	Computer-Based

Introduction to Scheduling



Sequence of steps

1. Determine running times and layovers based on
   - running time data
   - desired reliability levels
2. Determine frequencies by route and time period
3. Determine number of vehicles by time period
   - policies affecting integer constraints
   - revise step 1 and 2 decisions as needed
   - focus on transition periods
4. Determine timetable, typically
   - start at peak load point
   - generate start and end times
5. Chain vehicle trips together to form vehicle *blocks*
6. Cut and combine vehicle blocks to form crew *runs*

Common Issues

- Integrality constraints
    - If book times are 26 minutes each way, recovery time is 5 minutes at each terminus, and desired frequency is 10 per hour:

$$n_V = \left\lceil \frac{2 \cdot (26+5)}{6} \right\rceil = \lceil 10.\bar{3} \rceil = 11$$

    - Trade-off between shortening cycle time by 2 minutes to save 1 vehicle, or not?
    - In a similar case, but if desired frequency is 1 per hour, choice is to:
        - shorten cycle time by 2 minutes, or
        - interline with another route having cycle time of 58 minutes or less
- Marginal cost of additional trips
    - A single trip for a vehicle/crew in peak period is typically uneconomic
        - eliminating the single trip and saving the vehicle/crew costs
        - adding additional trips to make a minimum sized “piece of work”
    - Where and when you add extra trips will affect costs.
- Hard constraints
    - Contract terms include hard/soft constraints which determine feasibility

Setting Running Times and Cycle Times

- Key Input Data
    - Actual running times
    - Current operations practices, e.g., time points
- Typical Steps
    - Define time points
    - Define time periods
    - For each time period
        - set scheduled running time for full route and for each time point
        - set recovery time at end of trip
- Example of Current Practice
    - Use median running time for scheduled time
    - Set half-cycle time (scheduled time + minimum recovery time for 1-way trip) to 95th percentile of cumulative running time distribution

Analysis of AVL Data Using Hastus ATP

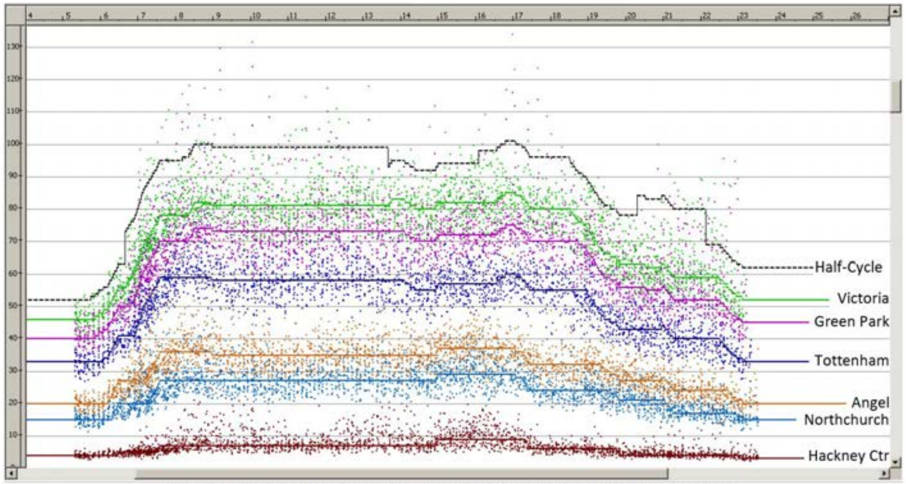


Figure 2: Route 38 Southbound - Suggested Running and Half-Cycle Times

Prepared by Kevin Muhs (3/15/2011) for TfL using GIRO HASTUS ATP software.

Simple Rules and Current Practice

- Frequencies typically based on
    - policy headways – vary by time of day and route type
    - maximum loads – vary by time of day and route type
    - These represent constraints rather than decision algorithms.
- Maintain constant maximum load factor over periods
    - at a level below official maximum load factor
    - may vary by time period
- Maintain constant average occupancy level over periods
    - subject to capacity constraint
    - may also be subject to a maximum time for loads above a specified level

Importance of Frequency Determination

- Major short-range planning decision
  - Affects service quality through wait time and crowding
  - Affects transit path selection (assignment) in complex networks
- Two different contexts
  - Developed country city
    - ridership sensitive to service quality
    - sparse network, little transit path choice
    - maximum acceptable crowding levels specified
    - defined level of subsidy available
  - Developing country city
    - ridership constrained by capacity
    - crowding levels very high
    - dense network, significant transit path choice

Developed Country Frequency Determination Problem

- Decision variables
  - headway on each route for each time period
- Objective function
  - maximize consumer surplus + social ridership benefit
    - (b × wait time savings) + (a × ridership)
- Constraints
  - total subsidy is exhausted
  - total fleet size is not exceeded
  - headway meets policy maximums and loading maximums

Furth, P.G. and N.H.M. Wilson, “Setting Frequencies on Bus Routes: Theory and Practice,” Transportation Research Record 818, 1981, pp 1-7

Maximize Social Surplus (multiple routes problem)

Context

- Given a fixed fleet size and subsidy,
- Determine optimal allocation of this fleet to the various routes (thus setting the frequencies on the routes)

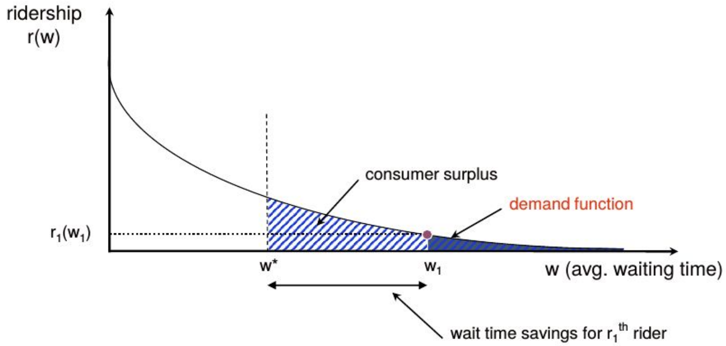
Formulation

- Maximize social surplus across all routes
- Subject to
  - subsidy not exceeded
  - fleet size not exceeded
  - level of service is acceptable (meets service delivery policy)

Maximize Social Surplus

Social Surplus

1. Consumer Surplus
   - Recall that waiting time is a function of headway



Maximize Social Surplus

- For a given headway *h**, *w** = *f*(*h**)
- Consumer surplus is

$$CS = b\int_{w^*}^{\infty} r(w)\mathrm{d}w$$

where

b = monetary value of waiting time

CS = savings in wait time cost that accrues to riders who would have been prepared to ride at higher waiting times

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Maximize Social Surplus

2. Social Benefits (of transit)
    - mobility for non-auto owners
    - reduced congestion
    - reduced pollution
    - reduced energy consumption
    - positive land use effects

- All of these benefits are highly associated with ridership
    - Social benefit for a route = a · r(w)
    - where a = monetary value of social benefit associated with an additional rider less the fare

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Maximize Social Surplus

- Since *w* = *f*(*h*), we can derive *r*(*h*) from *r*(*w*), i.e.
- *r*(*h*) = *r*(*f*(*h*))

Total social surplus to maximize:

$$CS + SB = \sum_{\text{routes } i}\left[b\int_{h_i^*}^{\infty} r(h)\mathrm{d}h + ar(h_i^*)\right]$$

where h_i^* is the headway on route whose optimal value is to be determined (decision variable)

Maximize Social Surplus: Constraints

Subsidy

$$\sum_{routes}[\text{operating cost - fare revenue}] = \text{subsidy limit}$$

$$\sum_{routes\ (i)}[c(h_i^*) - F \cdot r(h_i^*)] = S_o$$

fare

Fleet Size

$$\sum_{routes\ (i)} \frac{\text{round - trip time}}{h_i^*} \le \text{Fleet size, M}$$

Level of Service

$h_i^* < h_0$ headway standard

$g(h_i^*) < l_0$ load standard

vehicle load

Maximize Social Surplus

Critical Assumptions and Limitations

- independence across routes
  - In model, ridership on a route depends only on the headway of that route.
  - In reality, ridership also depends on headways on competing routes and complementary routes (transfers).
- network design is not considered

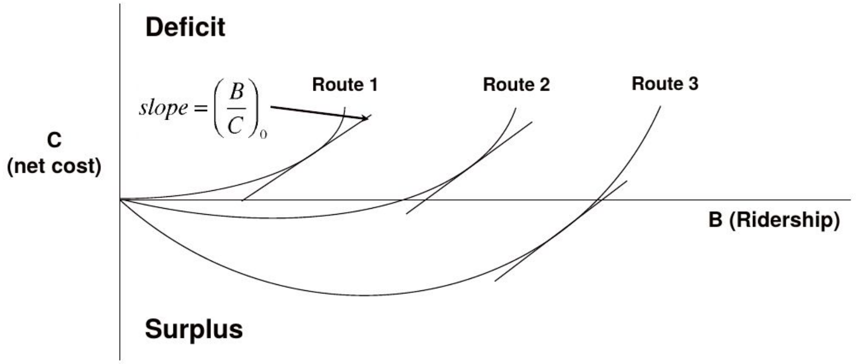
Advantages

- ridership = *f*(frequency)
- captures trade-offs across routes
- introduces system wide budget constraint

Efficiency in Subsidy Allocation

This is a resource allocation problem.

For optimality, allocate enough resources to each route so that Marginal Benefit/Cost Ratio is same on each route.



Furth & Wilson (1981) Findings

- Square root rule is valid where constraints are not binding
- Problem can be solved using lagrangian relaxation and single variable search techniques (not very complex)
- Existing scheduling practice over allocates service to peak and to long, high ridership routes
- Minimizing wait time assuming fixed demand gives similar solutions to more complex objective and variable demand
- Best allocation of resources is quite robust with respect to objectives and parameters assumed

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Developing Country City Frequency Determination Problem

Objectives

- minimize crowding levels
- minimize waiting times

Constraints

- loading feasibility (vehicle capacity)
- passenger assignment
- total fleet size

Passenger Assignment Heuristic Approach

1. Classify flow into:
   a. captive flow (CF) - any OD pairs with only one feasible path
   b. variable flow (VF) - OD pairs with more than one feasible path
2. Assign VF in proportion to frequency share on acceptable routes, consistent with random bus arrival process

$$\frac{D_i}{\sum_{j \in J} D_j} = \frac{F_i}{\sum_{j \in J} F_j}$$

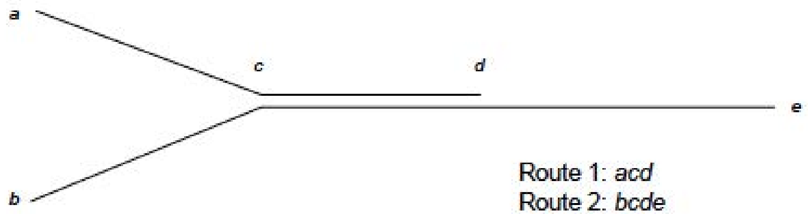
where

D_i = demand assigned to route i for specific OD pair
F_i = frequency offered on route i
J = set of acceptable routes

Models

- Normative (Ideal) Model
  - assign passenger flows to routes with minimum round trip vehicle time among all acceptable paths
  - compute frequency and fleet size required on this assignment basis
- Descriptive (Realistic) Model
  - assign passengers to alternative acceptable paths in proportion to frequency share in an iterative process
- The difference in the total fleet sizes from the normative and descriptive models indicates the extent of inefficiency resulting from the overlapping route structure.

Simple Example of Overlapping Routes



Route 1: *acd*
Route 2: *bcde*

- OD pair *cd* is VF, all other pairs are CF
- Ideally, *cd* flow would be assigned to route 1, which is shorter, but in reality these passengers will take route 1 or 2, whichever arrives first.
- Some *ce* passengers may be forced to board route 1 buses, then make a transfer at *d* to route 2

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