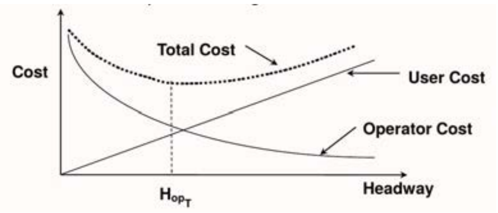
Macro Design Models for a Single Route

Outline

- Introduction to analysis approach
- Bus frequency model
- Bus size model
- Stop/station spacing model

Introduction to Analysis Approach

- Basic approach is to establish an aggregate total cost function including:
    - operator cost as f (design parameters)
    - user cost as g (design parameters)
- Minimize total cost function to determine optimal design parameter (s.t. constraints)
- Variants include:
    - Maximize service quality s.t. budget constraint
    - Maximize consumer surplus s.t. budget constraint



Bus Frequency Model: Square Root Rule

- **Problem** define bus service frequency on a route as a function of ridership
- total cost = operator cost + user cost

$$Z = c \bullet \frac{t}{h} + b \bullet r \bullet \frac{h}{2}$$

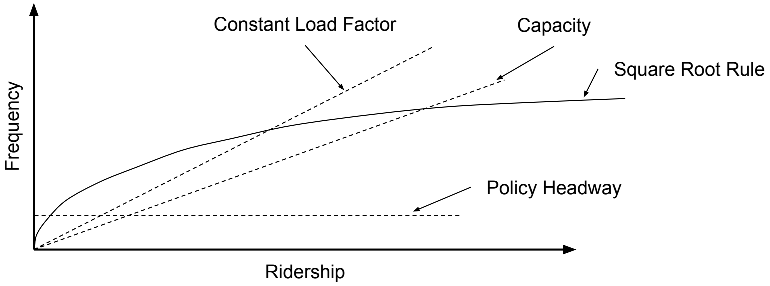
where Z = total (operator + user) cost per unit time
c = operating cost per unit time
t = round trip time
h = headway – the decision variable to be determined
b = value of unit passenger waiting time
r = ridership per unit time

Minimizing Z w.r.t. h yields:

$$h = \sqrt{\frac{2ct}{br}} \ or \ \sqrt{2\left(\frac{c}{b}\right)\left(\frac{t}{r}\right)}$$

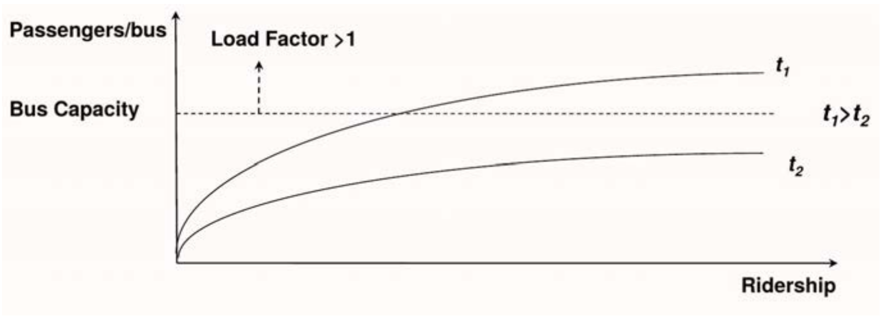
Bus Frequency Model: Square Root Rule

- This is the Square Root Rule with the following implications:
    - High frequency is appropriate where (cost of wait time/cost of operations time) is high
    - Frequency is proportional to the square root of ridership per unit time for routes of similar length



Bus Frequency Model: Square Root Rule

- Load factor is proportional to the square root of the product of ridership and route length



Bus Frequency Model: Square Root Rule

- Critical Assumptions
  - bus capacity is never binding
  - wait time savings are the only benefits of higher frequency
  - ridership f (frequency)
  - simple wait time model
  - budget constraint is not binding
- Possible Remedies
  - introduce bus capacity constraint
  - modify objective function
  - introduce r = f(h) and re-define objective function
  - modify objective function
  - introduce budget constraint

Bus Frequency Example

If

c = \$90/bus hour

b = \$10/passenger hour

t = 90 mins

r = 1,000 passengers/hour

Then

$h_{OPT} \approx 11$ mins

Bus Size Model

- **Problem** define optimal bus size for a route
- Assumptions
  - Desired load factor is constant
  - Labor cost/bus hour is independent of bus size
  - Bus dwell time costs per passenger are independent of bus size
- Using the same notation as before, plus
  - w = labor cost per bus hour
  - p = passenger flow past peak load point
  - k = desired bus load (the decision variable)

Then $Z = w \bullet \frac{t}{h} + b \bullet r \bullet \frac{h}{2}$

Now $h = \frac{k}{p}$ by assumption above

$\therefore \quad Z = \frac{wtp}{k} + \frac{brk}{2p}$

Minimizing Z w.r.t. k gives: $k_{OPT} = \sqrt{\frac{2p^2wt}{rb}}$

Bus Size Model

- Result is another square root model, implying that optimal bus size increases with:
  - round trip time
  - ratio of labor cost to passenger wait time cost
  - peak passenger flow
  - concentration of passenger flows
- Previous example extended with
  - p = 500 pass/hour
  - w = $40/bus hour
  - all other parameters as before
- Then k_{OPT} = 55 passengers

Stop/Station Spacing Model

- **Problem** determine optimal stop or station spacing
  - Trade-off is between
    - walk access time (increases with station spacing)
    - in-vehicle time (decreases as station spacing increases)
    - operating cost (decreases as station spacing increases)
  - Z = total cost per unit distance along route and per headway
  - T_{st} = time lost by vehicle making a stop
  - c = vehicle operating cost per unit time
  - s = station/stop spacing - the decision variable to be determined
  - N = number of passengers on board vehicle
  - v = value of passenger in-vehicle time
  - D = demand density in passenger per unit route length per headway
  - v_{acc} = value of passenger access time
  - w = walk speed
  - cs = station/stop cost per headway

Stop/Station Spacing Model

$$Z = \frac{T_{st}}{s}(c + N \bullet v) + \frac{c_s}{s} + \frac{s}{4} \bullet D \bullet \frac{v_{acc}}{w}$$

Minimizing Z w.r.t. s gives:

$$s_{OPT} = \left[\frac{4w}{Dv_{acc}}\left[c_s + T_{st}(c_v + Nv)\right]\right]^{1/2}$$

- Yet another square root relationship, implying that
  - station/stop spacing increases with
    - walk speed *w*
    - station/stop cost c_s
    - time lost per stop T_{st}
    - vehicle operating cost c_v
    - number of passengers on board vehicle *N*
    - value of in-vehicle time *v*
  - and decreases with
    - demand density *D*
    - value of access time v_{acc}

Bus Stop Spacing

U.S. Practice

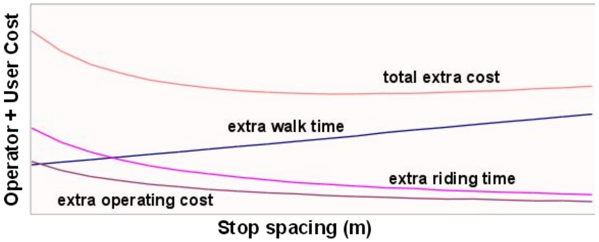
- 200 m between stops (8 per mile)
- shelters are rare
- little or no schedule information

European Practice

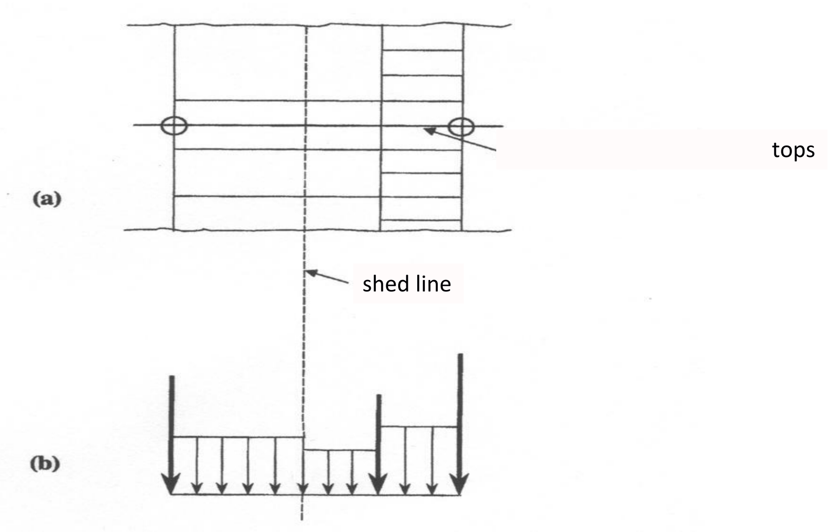
- 320 m between stops (5 per mile)
- named & sheltered
- up to date schedule information
- scheduled time for every stop

Stop Spacing Tradeoffs

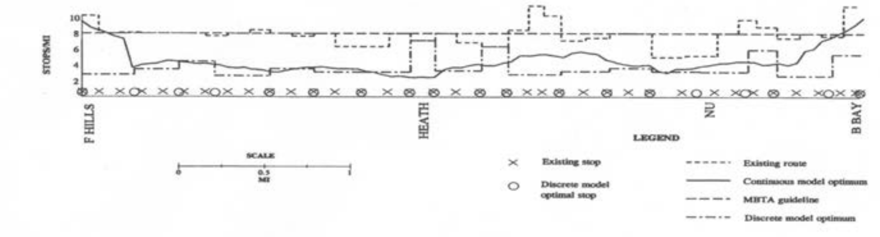
- Walking time
- Riding time
- Operating cost
- Ride quality



Walk Access: Block-Level Modeling



Example: MBTA Route 39

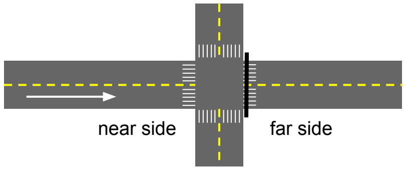


AM Peak Inbound results

- Average walking time up 40 s
- Average riding time down 110 s
- Running time down 4.2 min
- Save 1, maybe 2 buses

Furth, P.G. and A. B. Rahbee, "Optimal Bus Stop Spacing Using Dynamic Programming and Geographic Modeling." Transportation Research Record 1731, pp. 15-22, 2000.

Bus Stop Locations and Policies



- Far-side (vs. near-side)
  - less queue interference
  - easier pull-in
  - fewer pedestrian conflicts
  - snowbank problem demands priority in maintenance
- Curb extensions
  - benefit transit, pedestrians, and traffic (0.9 min/mi speed increase)
- Pull-out priority
  - it's the law in some states
- Reducing dwell time
  - vehicle design
  - fare collection
  - fare policy

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