1.225J (ESD 205) Transportation Flow Systems

Lecture 3

Modeling Road Traffic Flow on a Link

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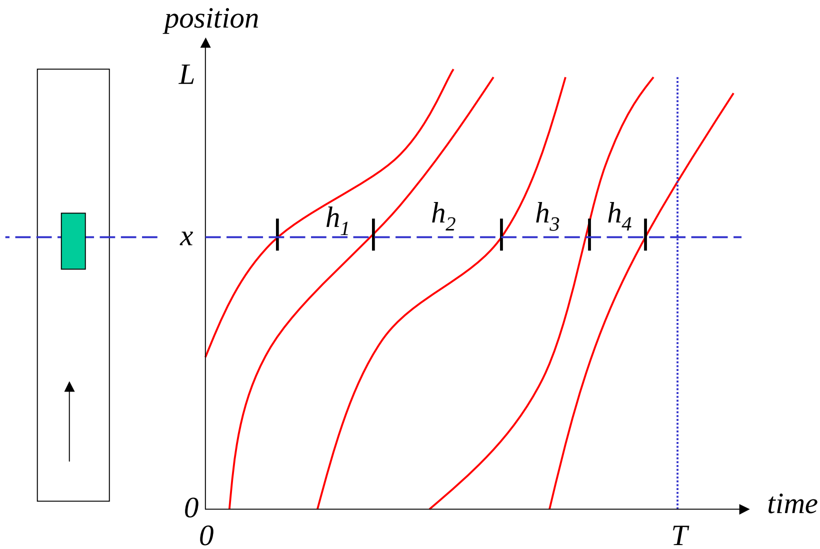
Lecture 3 Outline

- Time-Space Diagrams and Traffic Flow Variables
- Introduction to Link Performance Models
- Macroscopic Models and Fundamental Diagram
- Volume-Delay Function
- (Microscopic Models: Car-following Models
- Relationship between Macroscopic Models and Car-following Models)
- Summary

Time-Space Diagram: Analysis at a Fixed Position



Flows and Headways

- *m(x)*: number of vehicles that passed in front of an observer at position *x* during time interval *[0,T]*. (ex. *m(x)=5*)
- Flow rate: $q(x) = \frac{m(x)}{T}$
- Headway $h_j(x)$: time separation between arrival time of vehicles *i* and *i+1*
- Average headway: $\bar{h}(x) = \frac{\sum_{j=1}^{m(x)} h_j(x)}{m(x)}$
- What is the relationship between *q(x)* and $\bar{h}(x)$?

Flow Rate vs. Average Headway

- If T is large, $T \approx \sum_{j=1}^{m(x)} h_j(x)$

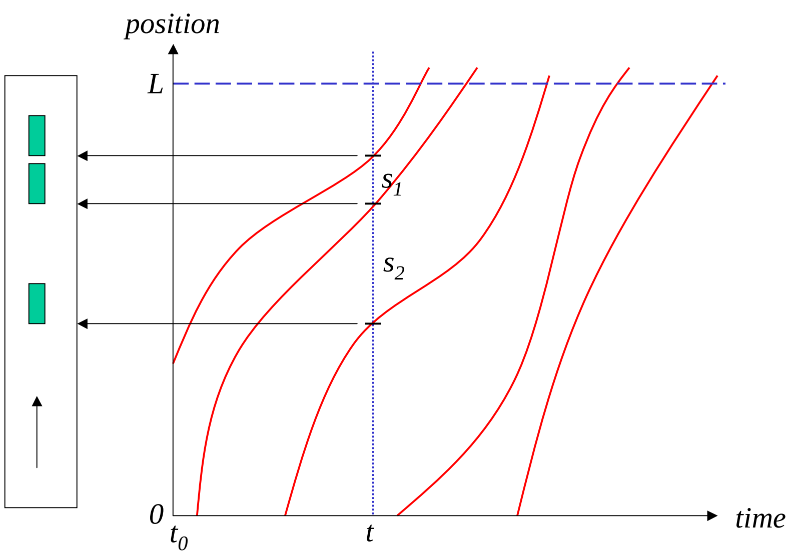
- Then, $\frac{1}{q(x)} = \frac{T}{m(x)} \approx \frac{\sum_{j=1}^{m(x)} h_j(x)}{m(x)} = \bar{h}(x)$

  $\Rightarrow q(x) \approx \frac{1}{\bar{h}(x)}$ This is intuitively correct.

- *q(t)* is also called **volume** in traffic flow system circles (i.e. 1.225)

- *q(t)* is also called **frequency** in scheduled systems circles (i.e. 1.224)

Time-Space Diagram: Analysis at Fixed Time



Density and Average Spacing

- $n(t)$: number of vehicles in a road stretch of length L at time t
- Density: $k(t) = \frac{n(t)}{L}$
- $s_i(t)$: spacing between vehicle i and vehicle $i+1$
- $L \approx \sum_{i=1}^{n(t)} s_i(t)$
- $\frac{1}{k(t)} = \frac{L}{n(t)} \approx \frac{\sum_{i=1}^{n(t)} s_i(t)}{n(t)} = \bar{s}(t)$
- $k(t) \approx \frac{1}{\bar{s}(t)}$ (Is this intuitive?)

Performance Models of Traffic on a Road Link

- Link: a representation of a highway stretch, road from one intersection to the next, etc.
- Example of measures of performance:
  - Travel time
  - Monetary or environmental cost
  - Safety
- Main measure of performance: travel time

- 3 types of models:
  - Macroscopic models: Fundamental diagram (valid in static (stationary) conditions only. Long roads and long time periods)
  - Microscopic models: Car-following models (no lane changes)
  - Volume-delay functions

Macroscopic Flow Variables

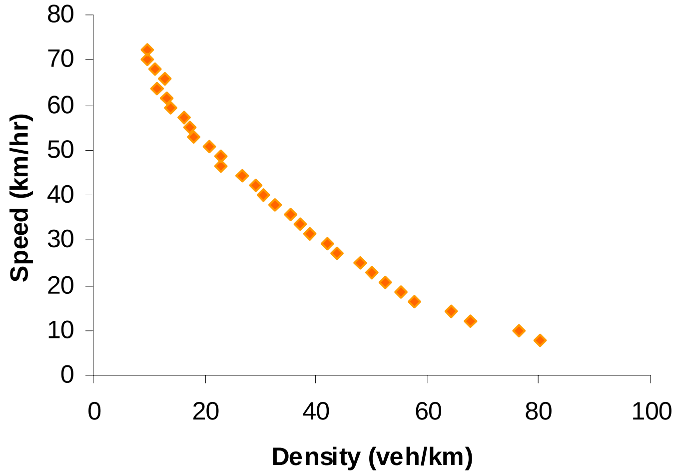
- Three macroscopic flow variables of a link:
  - Average density *k* (also denoted by ρ)
  - Average flow *q*
  - Average speed *u* (also denoted *v*)

- Relationships between variables:
  - $q = uk$
  - (k,q) curve: **Fundamental diagram**
  - Fundamental diagram is a property of the road, the drivers and the environment (icy, sunny, raining)
- 3 variables + 2 equations ⇒ only one variable can be an independent variable (But one of the variables (k, u, q) can not be independent)

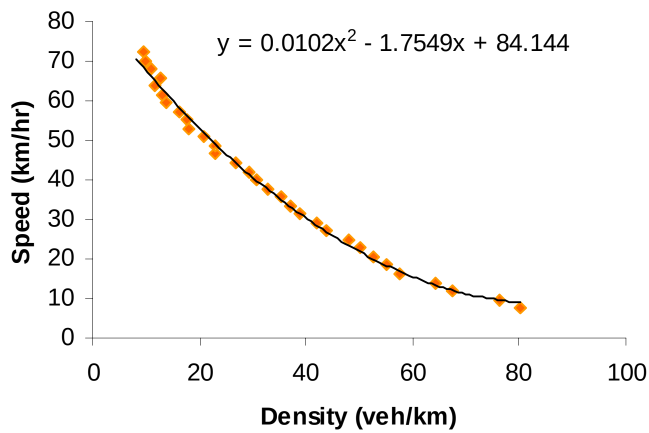
Data Collected from Holland Tunnel (Eddie, 63)

Speed (km/hr)	Average Spacing (m)	Concentration (veh/km)	Number of Vehicles
7.56	12.3	80.1	22
9.72	12.9	76.5	58
11.88	14.6	67.6	98
14.04	15.3	64.3	125
16.2	17.1	57.6	196
18.36	17.8	55.2	293
20.52	18.8	52.6	436
22.68	19.7	50	656
24.84	20.5	48	865
27	22.5	43.8	1062
29.16	23.4	42	1267
31.32	25.4	38.8	1328
33.48	26.6	37	1273
35.64	27.7	35.5	1169
37.8	30	32.8	1096
39.96	32.2	30.6	1248
42.12	33.7	29.3	1280
44.28	33.8	26.8	1162
46.44	43.2	22.8	1087
48.6	43	22.9	1252
50.76	47.4	20.8	1178
52.92	54.5	18.1	1218
55.08	56.2	17.5	1187
57.24	60.5	16.3	1135
59.4	71.5	13.8	837
61.56	75.1	13.1	569
63.72	84.7	11.6	478
65.88	77.3	12.7	291
68.04	88.4	11.1	231
70.2	100.4	9.8	169
72.36	102.7	9.6	55
74.52	120.5	8.1	56

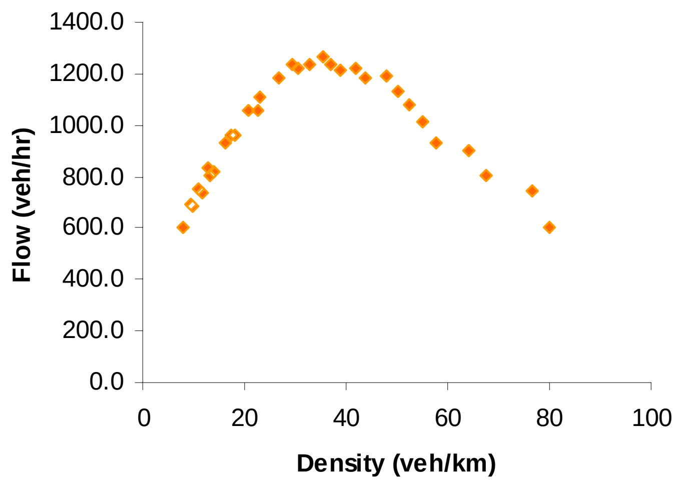
(Density, Speed) Diagram for the Field Data



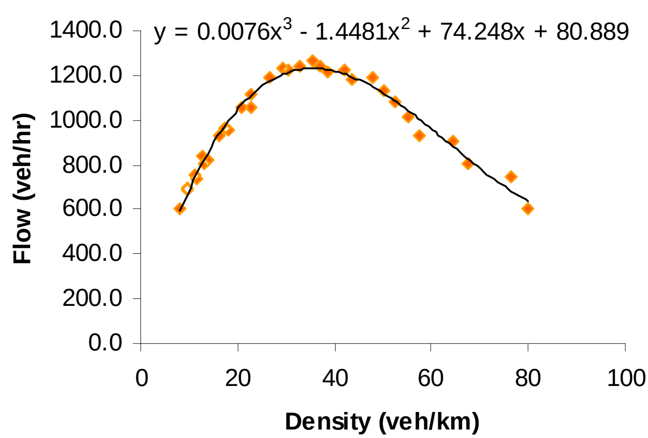
(Density, Speed) Diagram with a Fitted Curve



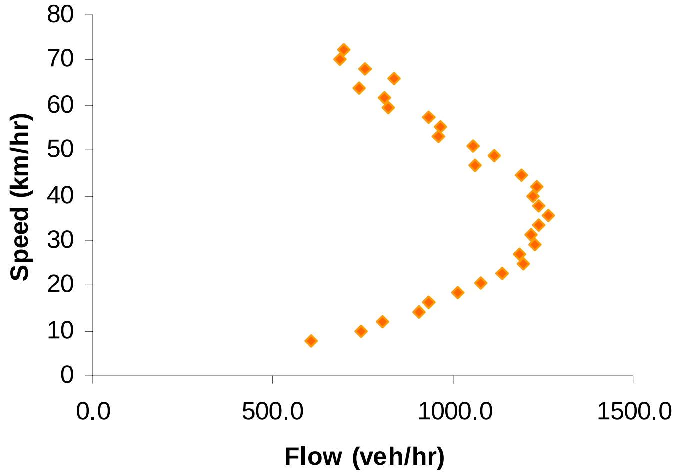
(Density, Flow) Diagram from the Field Data



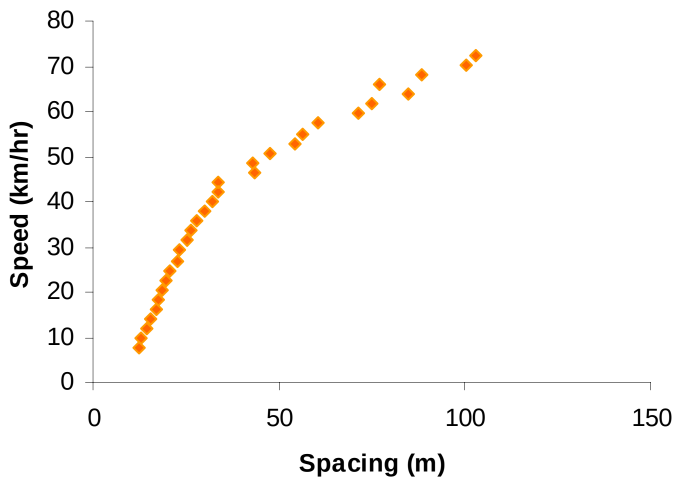
(Density, Flow) Diagram with a Fitted Curve



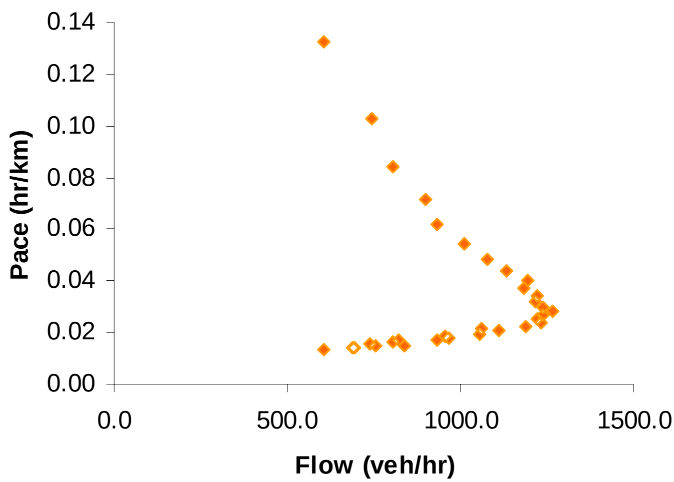
(Flow, Speed) Diagram from the Field Data



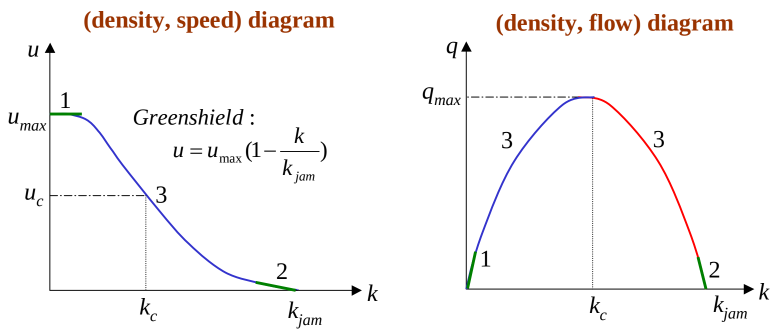
(Spacing, Speed) Diagram from the Field Data



(Flow, Pace) Diagram from the Field Data



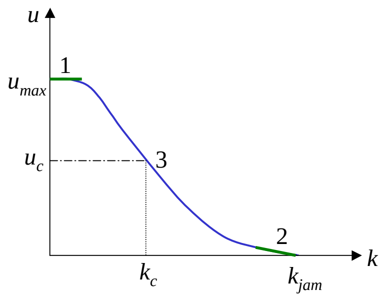
Relationships between Flow Variables



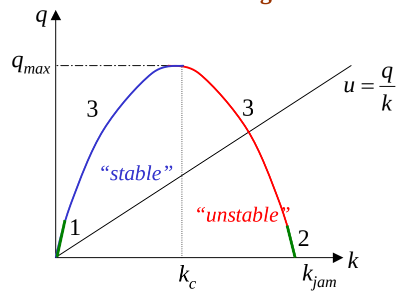
- k_{jam}: **jam density** (the highway stretch is like a parking lot!)
- k_{jam}^{-1} = a car length
- $q = uk$
- $q_{max} = q(k_c)$ is the **maximum flow**, or **link capacity**
- $u_c = u(k_c) = \frac{q_{\max}}{k_c}$

Fundamental Diagram

(density, speed) diagram



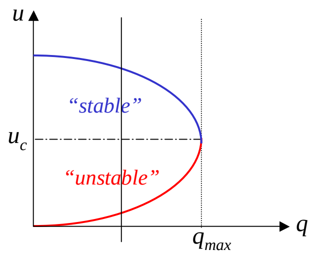
Fundamental diagram



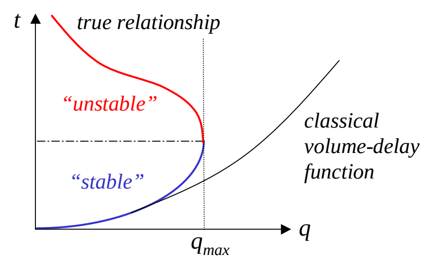
- $k \in [k_c, k_{jam}]$: arise when flow is slower down stream due to lane drops, a slow plowing-truck, etc
- k_c is **critical,** since it marks the start of an **"unstable"** flow area where additional input of cars decrease flow served by the highway
- (k, q) diagram is **fundamental** since it represents the three variable as compared to the other diagrams

Derived Diagrams

(flow, speed) diagram



(flow, travel time) diagram



- ❑ In general, q cannot be used as a variable (why?)
- ❑ In the road network planning area:
  - q is also called **volume**
  - travel time is also called **travel "delay"**
  - In the case of volume-delay functions, q is used as a variable

Examples of Classical Volume-Delay Functions

- Notation:
  - q is the link flow
  - $t(q)$ is the link travel time
  - c is the **practical capacity**
  - α and β are calibration parameters

- Davidson's function:
  - $t(q) = t(0)[1 + \alpha \frac{q}{c - q}]$

- US Bureau of Public Roads
  - $t(q) = t(0)[1 + \alpha (\frac{q}{c})^{\beta}]$

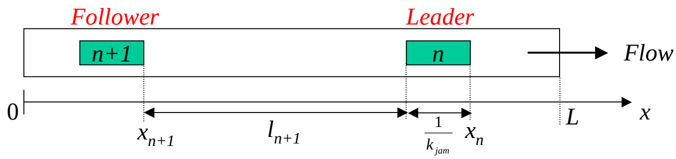
Observations on Classical Volume-Delay Functions

- Examples where the classical model may be acceptable:
  - Delay at a signalized link
  - $q < q_{max}$ (mild congestion)

- What makes the classical model interesting?
  - It is a function (There is only one value for a given q)
  - Typical functions used are increasing with q, and
  - their derivatives are also increasing ("it holds water" $\Leftrightarrow$ it is convex)
  - The above are analytical properties that have been adopted to study the properties of, and design solution algorithms for, network traffic assignment models (Lectures 4-6)
  - $\Rightarrow$ An example of tradeoffs made between realism and computational tractability

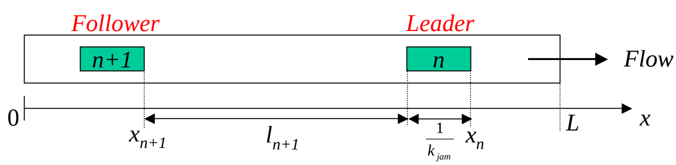
Link Travel Time Models: Car-Following Models

- Notation:



- $x_n(t) - x_{n+1}(t) = \text{spacing (space headway)} = l_{n+1}(t) + \frac{1}{k_{jam}}$
- speed of vehicle n : $\frac{dx_n(t)}{dt} = \dot{x}_n(t)$
- acceleration (decceleration) of vehicle n : $\frac{d\dot{x}_n(t)}{dt} = \frac{d^2 x_n(t)}{dt} = \ddot{x}_n(t)$
- $\dot{x}_n(t) - \dot{x}_{n+1}(t) = \dot{l}_{n+1}(t)$
- car-following regime: $l_{n+1}(t)$ is below a certain threshold

Link Travel Time Models: Car-Following Models



- Simple car-following model:

$$\ddot{x}_{n+1}(t+T) = a\dot{l}_{n+1}(t) = a(\dot{x}_n(t) - \dot{x}_{n+1}(t))$$

$$T : \textit{reaction time} \quad (T \approx 1.5 \sec)$$

$$a : \textit{sensitivity factor} \quad (a \approx 0.37 s^{-1})$$

- Questions about this simple car-following model:
  - Is it realistic?
  - Does it have a relationship with macroscopic models?

From Microscopic Models To Macroscopic Models

- Simple car-following model: $\ddot{x}_{n+1}(t) = a(\dot{x}_n(t) - \dot{x}_{n+1}(t)) \;\; (T = 0)$
- Fundamental diagram: $q = q_{\max}(1 - \frac{k}{k_{jam}})$
- Proof of "equivalency"

$$\ddot{x}_{n+1}(y) = a(\dot{x}_n(y) - \dot{x}_{n+1}(y))$$

$$\ddot{x}_{n+1}(y)dy = a(\dot{x}_n(y) - \dot{x}_{n+1}(y))dy = a\dot{l}_{n+1}(t)dy$$

$$\int_0^t \ddot{x}_{n+1}(y)dy = \int_0^t a\,\dot{l}_{n+1}(t)dy$$

$$u_{n+1}(t) - u_{n+1}(0) = a(l_{n+1}(t) - l_{n+1}(0))$$

$$u_{n+1}(t) = a\,l_{n+1}(t) + u_{n+1}(0) - al_{n+1}(0)$$

If $l_{n+1}(t) = 0$, then $u_{n+1}(t) = 0 \Rightarrow u_{n+1}(0) - al_{n+1}(0) = 0$

From Microscopic Model to Macroscopic Model

$$u_{n+1}(t) = al_{n+1}(t) = a(\frac{1}{k_{n+1}(t)} - \frac{1}{k_{jam}})$$

$$\Rightarrow u = a(\frac{1}{k} - \frac{1}{k_{jam}})$$

$$\Rightarrow q = uk = a(\frac{1}{k} - \frac{1}{k_{jam}})k = a(1 - \frac{k}{k_{jam}})$$

If $k = 0$, then $q = a$

Since $q = a \geq a(1 - \frac{k}{k_{jam}})$, then $a = q_{\max}$

$$\Rightarrow q = q_{\max}(1 - \frac{k}{k_{jam}})$$

- Note: if $k \rightarrow 0$, then $u \rightarrow \infty$. Does this make sense?

Non-linear Car-following Models

- $$\ddot{x}_{n+1}(t+T) = a_0 \frac{\dot{x}_n(t) - \dot{x}_{n+1}(t)}{\left(x_n(t) - x_{n+1}(t)\right)^{1.5}}$$
  $$= a_0 \frac{\dot{l}_{n+1}(t)}{\left(l_{n+1}(t) + \frac{1}{k_{jam}}\right)^{1.5}}$$
- If $T = 0$, the corresponding fundamental diagram is:
  $$q = u_{\max} k \left[1 - \left(\frac{k}{k_{jam}}\right)^{0.5}\right]$$

Flow Models Derived from Car-Following Models

$$\ddot{x}_{n+1}(t+T) = a_0 \dot{x}_{n+1}^m(t+T) \frac{\dot{x}_n(t) - \dot{x}_{n+1}(t)}{\left(x_n(t) - x_{n+1}(t)\right)^l}$$

l	m	Flow vs. Density
0	0	$q = q_m \left(1 - \frac{k}{k_{jam}}\right)$
1	0	$q = u_c k \ln\left(\frac{k_{jam}}{k}\right)$
1.5	0	$q = u_{\max} k \left[1 - \left(\frac{k}{k_{jam}}\right)^{0.5}\right]$
2	0	$q = u_{\max} \left(1 - \frac{k}{k_{jam}}\right)$
2	1	$q = u_{\max} k \exp\left(1 - \frac{k}{k_{jam}}\right)$
3	1	$q = u_{\max} k \exp\left[-\frac{1}{2}\left(\frac{k}{k_{jam}}\right)^2\right]$

Lecture 3 Summary

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- Microscopic Models: Car-following Models
- Relationship between Macroscopic Models and Car-following Models