1.224J Carrier Systems
Problem Set 2, due 11/17/2003

<u>Problem 1:</u>
Consider the network shown below. Our objective is to minimize the cost of shipping a product from cities to delivery regions. Each city has an unlimited supply of the product. The demands at region A, B, and C are 80, 70, and 40 respectively. We can satisfy the demand at each region by shipping from any city or any combination of cities. In order to ship between 2 locations, a shipping lane must be opened. If opened, a shipping lane has a capacity of 100 units. The costs are shown in Table 1. First, we have the fixed cost of opening a lane, whih depends only upon the city from which the lane originates. For instance the shipping lane from 1 to A, 1 to B, 1 to C have the same fixed cost: $1000. Second, we have shipping cost (per unit) for shipping from each city to each region.

City i	Fixed Cost per lane from city (f_i)	Shipping cost per unit of demand (c_{ij})		
		A	**B**	**C**
1	1000	10	20	20
2	900	20	10	30
3	800	30	20	20
Demand (b_j)		**80**	**70**	**40**
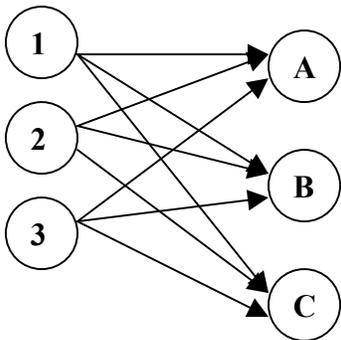
(a) Write a MIP formulation for this cost minimizing problem.
(b) Solve it in Excel
(c) Model and solve each of the modifications. (Turn in both the new optimal solution and the inequalities you used to capture the constraints.)
   • All demand for a given region must be satisfied by a single source
   • Each city can only have a maximum of 2 shipping lanes open.
   • If lane 1 to A is open, than lane 2 to A has to be open
   • If lanes 1 to B and 2 to B are open, then lane 3 to B has to be closed
   • If either lane 1 to C or 2 to C is open, lane 3 to C must be open

Problem 2 (1999 Exam Question 2)

Consider the following integer problem:

$$MIN \quad 6x_1 + 9x_2 + 4x_3 + x_4$$

s.t.

$$2x_1 + 4x_2 + 3x_3 + x_4 \geq 6$$
$$x_i \in \{0,1\}, \forall i$$

Consider the following lower bound algorithm:
- Step 0: Set x_i=0 for each i=1,2,3,4. Let the candidate list contain items 1, 2, 3 and 4.
- Step 1: Determine the ratio of each item's cost to its constraint "contribution"
- Step 2: Select the item in the candidate list with the smallest ratio. Call it i and remove i from the candidate list.
- Step 3: Given x_j values for each item j removed from the candidate list; set the value of x_i to the minimum of
  - x_i=1
  - x_i such that $2x_1 + 4x_2 + 3x_3 + x_4 = 6$
- Step 4: If $2x_1 + 4x_2 + 3x_3 + x_4 = 6$, STOP! Otherwise, return to Step 2.

Example:
The ratios are 6/2, 9/4, 4/3, and 1/1. Item 4 has the smallest ratio. We set x_4 as large as possible (=1). The next smallest ratio is for item 3. We set x_3=1 and the left hand side of the constraint sums to 4. The next item is item 2. If we set x_2=1/2, then the constraint is satisfied and we stop. The simple algorithm solves the LP relaxation and yields a solution (lower bound with cost of 9.5).

(a) Find the upper bound on the optimal integer solution and describe how you obtained it.

(b) Using the lower bound algorithm, fill-in the attached branch and bound tree to find the optimal integer solution. Clearly mark the branches you prune and indicate the reason for pruning.

9.5

| 0 | 1/2 | 1 | 1 |

$x_1=1$ $x_1=0$

$x_2=1$ $x_2=0$ $x_2=1$ $x_2=0$

$x_3=1$ $x_3=0$ $x_3=1$ $x_3=0$ $x_3=1$ $x_3=0$ $x_3=1$ $x_3=0$

$x_4=0$ $x_4=1$ $x_4=0$ $x_4=1$ $x_4=0$ $x_4=1$ $x_4=0$ $x_4=1$ $x_4=0$ $x_4=1$ $x_4=0$ $x_4=1$ $x_4=0$ $x_4=1$
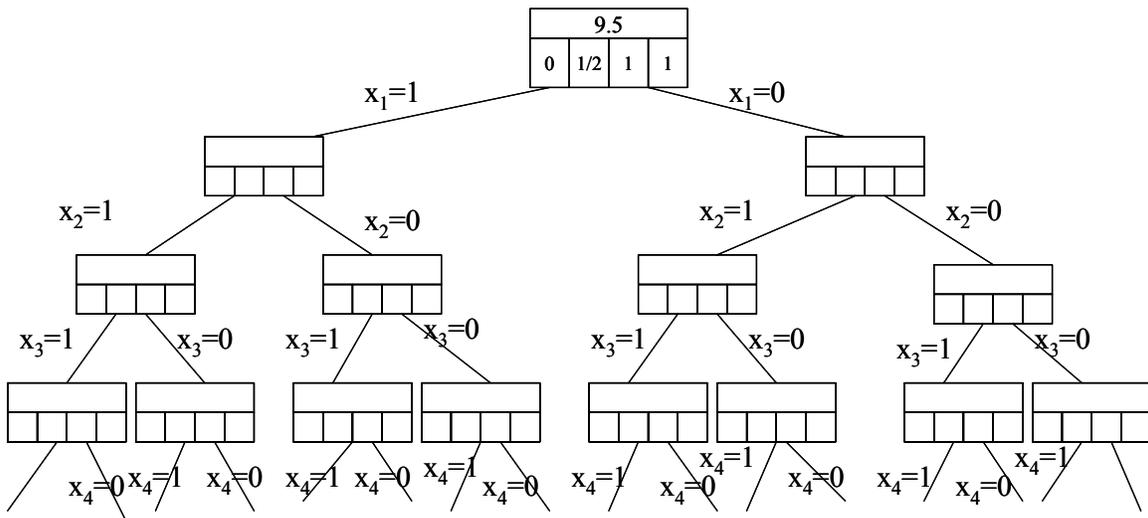
(c) Next, we add the following constraint to the formulation.

$10x_1 + 8x_2 + 4x_3 + 7x_4 <= 17$

When you solve the LP relaxation and print it out, the printer fails and you have the following incomplete output. Fill in the missing entries and explain your reasoning.

Variable	Value	Red. Cost
X2	0.5	-----

Constraint	Value	Shadow price
Constr1	-----	-2.25
Constr2	15	----

(d) Based on this output, what is the most you would be willing to pay to reduce the first constraint from 6 to 5?