# Airline Operations
Lecture #3

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Summary Lecture #2

- Achieving good passenger service reliability at an acceptable operating costs
- Disrupted passengers suffer long delays on average (320 minutes) versus non disrupted passengers (14 minutes)
- Connecting itineraries have a much higher risk of being disrupted than local itineraries (2.7x)
- Late disruptions are often difficult to recover the same day, much higher flight delay and cancellations at the end of the day
- Delays accumulate along the day, resulting in relatively high percentage of overnight passengers among disrupted (20%), but still small percentage (0.7% of passengers)

Average flight delay per hour, August 2000
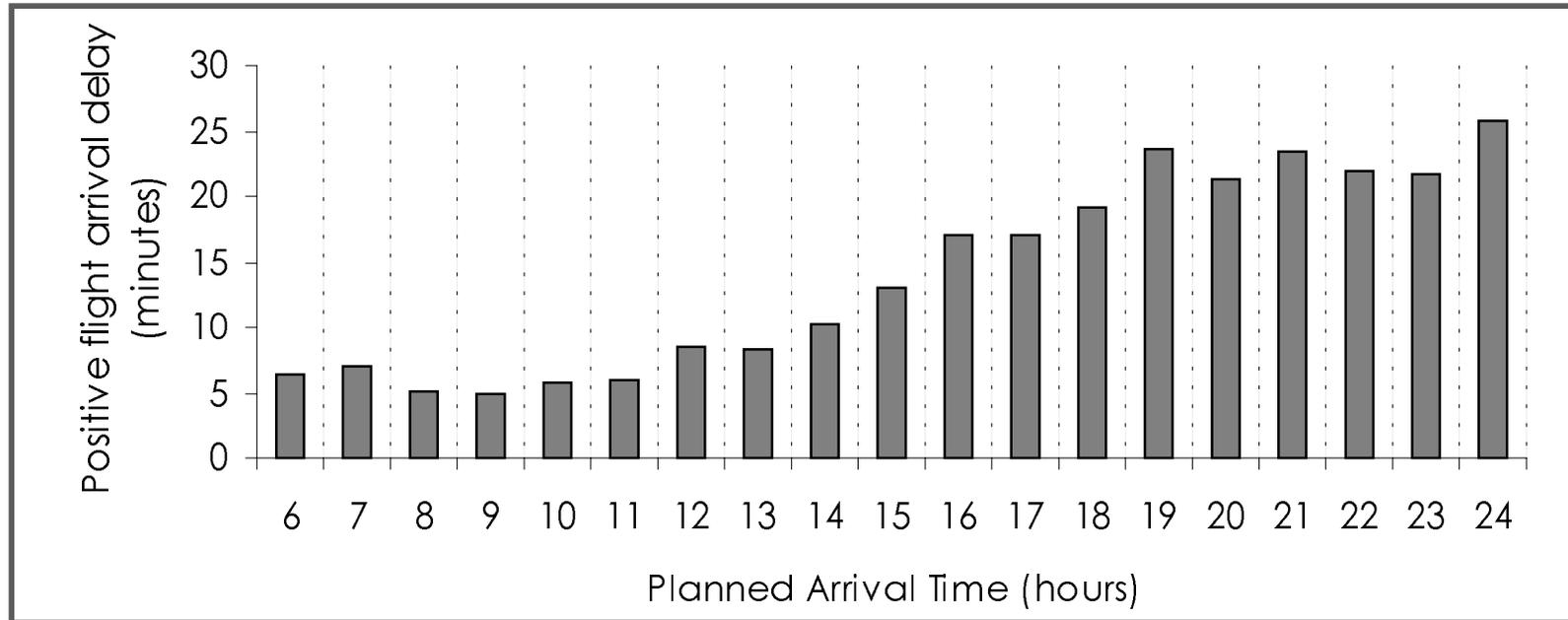
Our approach

- **Wisely postpone artificially departures to maintain bank integrity and prevent passengers from missing connections**
- **Wisely canceled flights if necessary to prevent delays to propagate and the negative effects on passengers**
- **We want our solutions to be feasible for aircraft (maintenance) and crews (schedule)**
- **Guarantee solution feasibility:**
  - ➢ **Artificially postponing flight departures does not disrupt more crews:**
    - • **Maintain flight sequence feasibility (duty)**
    - • **Does not include flight copies that violate crew regulation (Maximum Duty Elapsed Time)**
- **Do we guarantee maintenance routing feasibility?**

Summary Lecture #2 (Cont.)

- **Minimize Sum of Disrupted Passengers (M1)**
  - ➤ **Works well (20CPU) for day with severe flight schedule disruptions. Why?**
    - **Because number of variables relatively small (O(F + I) and number of constraints O(F + I))**
    - **And binary variables**
  - ➤ **Downside: do not consider disrupted passenger and non disrupted passenger delays: May decide to postpone a flight by 30 minutes with 100 passenger on board to recover only 1 disrupted passenger who could have been recovered effectively**
- **Minimizing Sum of Passenger Delays (M2)**
  - ➤ **Problem becomes much bigger if all the recovery itineraries are included**
  - ➤ **Hard to solve using B&B**
  - ➤ **(M1/M2) equivalent to (FAM/ODFAM): capacity constraints tend to lead to fraction solutions of LP relaxation**

Minimizing Sum of Disrupted Passengers

$$\text{Minimize}\left(\sum_{p\in P} n_p \times \rho_p\right)$$

$$\text{st}: \quad \sum_{t\in T_f} x_f^t + z_f = 1$$

$$\sum_{(f,t)\in In(j)} x_f^t + y_f^{t-} = \sum_{(f,t)\in Out(j)} x_f^t + y_f^{t+}$$

$$\sum x_f^{\bullet} + y_f^{\bullet} = \text{Res}(a, ft, \bullet)$$

$$\rho_p \geq z_f$$

$$x_f^t + \sum_{g\in C(u)\,|\,d(g)<a(f)} x_g^u - \rho_p \leq 1$$

$$\rho_p \in [0;1]; \ x_{f,a}^t \in \{0,1\}; \ y_f^t \geq 0$$

➢ **Objective: Minimize sum of disrupted passengers**

➢ **Flight coverage constraints**

➢ **Aircraft balance for each sub fleet type**

➢ **Initial and end of the day aircraft resource constraints**

➢ **Passenger cancellation constraints**

➢ **Missed connected passengers constraints**

➢ **Only flight copy variables, x, have to be binary**

Minimizing passenger delay

- **Need to consider all potential copies of recovery itineraries for each passenger**

- **Large scale problem: 500,000 integer variables; 12 hours CPU using B&B deep first search methodology**

$$\text{Min} \quad \sum_{p \in P} \sum_{i \in I_p} b_p^i q_p^i$$

$$\sum_{t \in T_f} x_f^t + z_f = 1 \quad \forall f \in F$$

$$\sum_{(f,t) \in In(j)} x_f^t + y_f^{t-} = \sum_{(f,t) \in Out(j)} x_f^t + y_f^{t+}$$

$$\sum x_f^0 + y_f^{0+} = j_\bullet$$

$$\sum_{i \in I_p} q_p^i = n_p$$

$$\sum_{p \in P} \sum_{i \in I_p} \delta_{fi}^t q_p^i \leq C_f \times x_f^t$$

$$q_p^i \geq 0; x_f^t \in \{0,1\}; y_f^t \geq 0$$

Summary Lecture #2 (Cont.)

➢ **Approximate models to minimize sum of passenger delay**
- From Model #1, estimate delay if itinerary is disrupted.
- From Model #2, limit the number of itinerary copy to include only good ones.

➢ **Objective function: minimizing estimated passenger dissatisfaction**
- Fine grained down to Passenger Name Record
- Assign a cost (expected future revenue loss of delay d for PNR p) based on:
  - Fare class
  - Disruption history
  - Loyalty (FFP)
- Same objective can be used in sorting passengers for recovery priority

Lecture #3 Outline

- **Airline schedule recovery framework**

- **Aircraft routing feasibility**

- **Disrupted passenger re-routing under seat uncertainty:**

  - ➢ **Heuristics**

  - ➢ **Optimal**

  - ➢ **Optimal with bumping control**

Airline system state:
Aircraft: position, maintenance, operational
Crews: position, disruption status, duty time, flight time, etc.
Passengers: position, destination, PAT, disruption status

Crew operations recovery,
Repair pairings

Operations forecasts

Flight copy generation algorithm

optimizer

Flight departure times, X* and flight cancellations Z*

Aircraft routing based on (X*,Z*)

∃ Feasible route R?

No

Yes

Prevent infeasible aircraft route swaps
Modify flight departure solution
Obtain feasible aircraft route R'
and associated optimal solution (X'*,Z'*)

Recovery priority policies

Optimal disrupted passenger re-routing
Considering seat availability uncertainty

Resource Dependability: Ripple effects

PC

Cockpit Crew rest at SMF

PC
LAX-SMF

SMF-LAX

PC
CC
A

CC
DFW-SNA

SNA-SJC

CC: deadheading
SNA-RNO

SNA-SEA

DFW-LAX

A
LAX-ONT

ONT-LAX

A

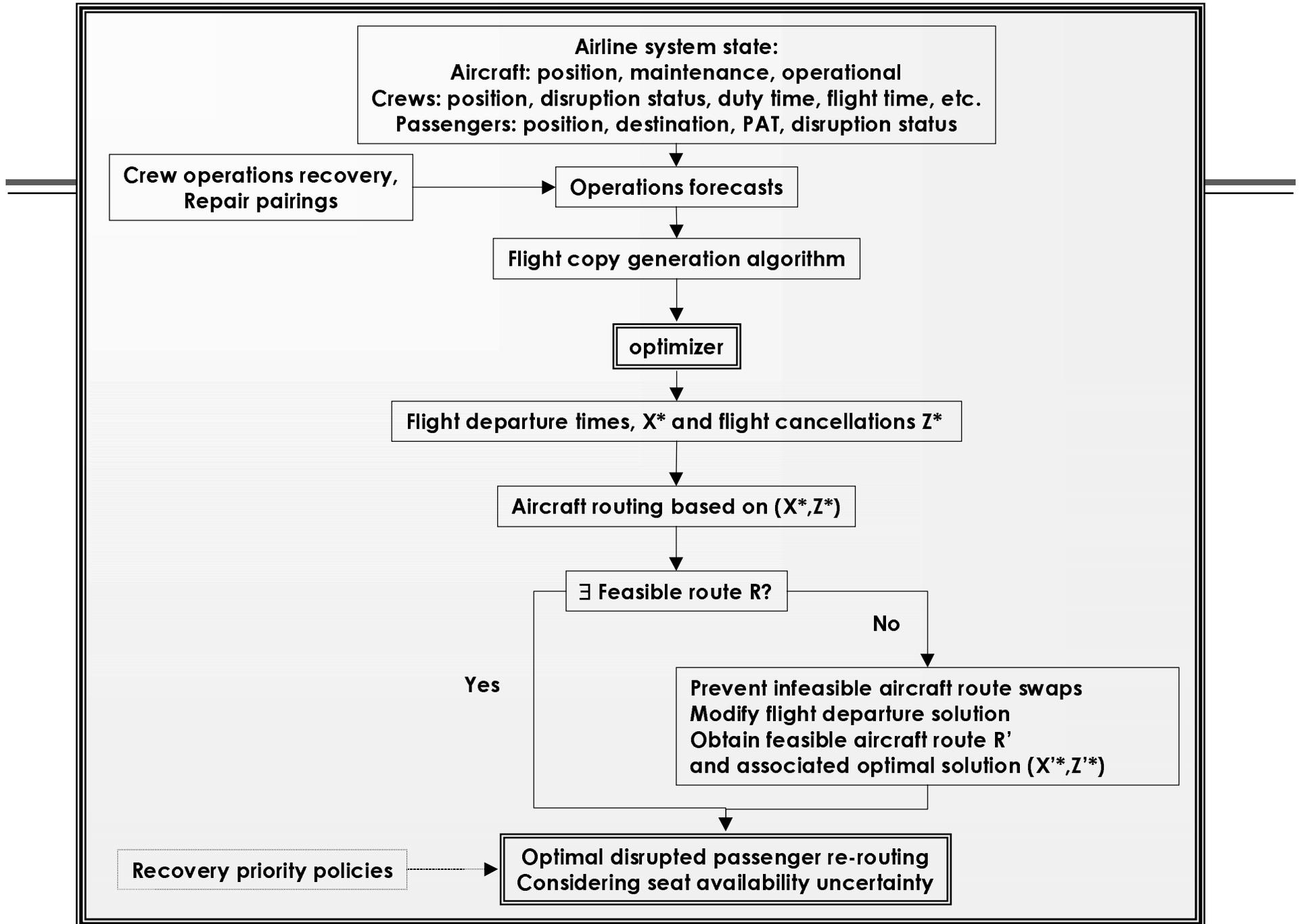Aircraft
maintenance at
ONT
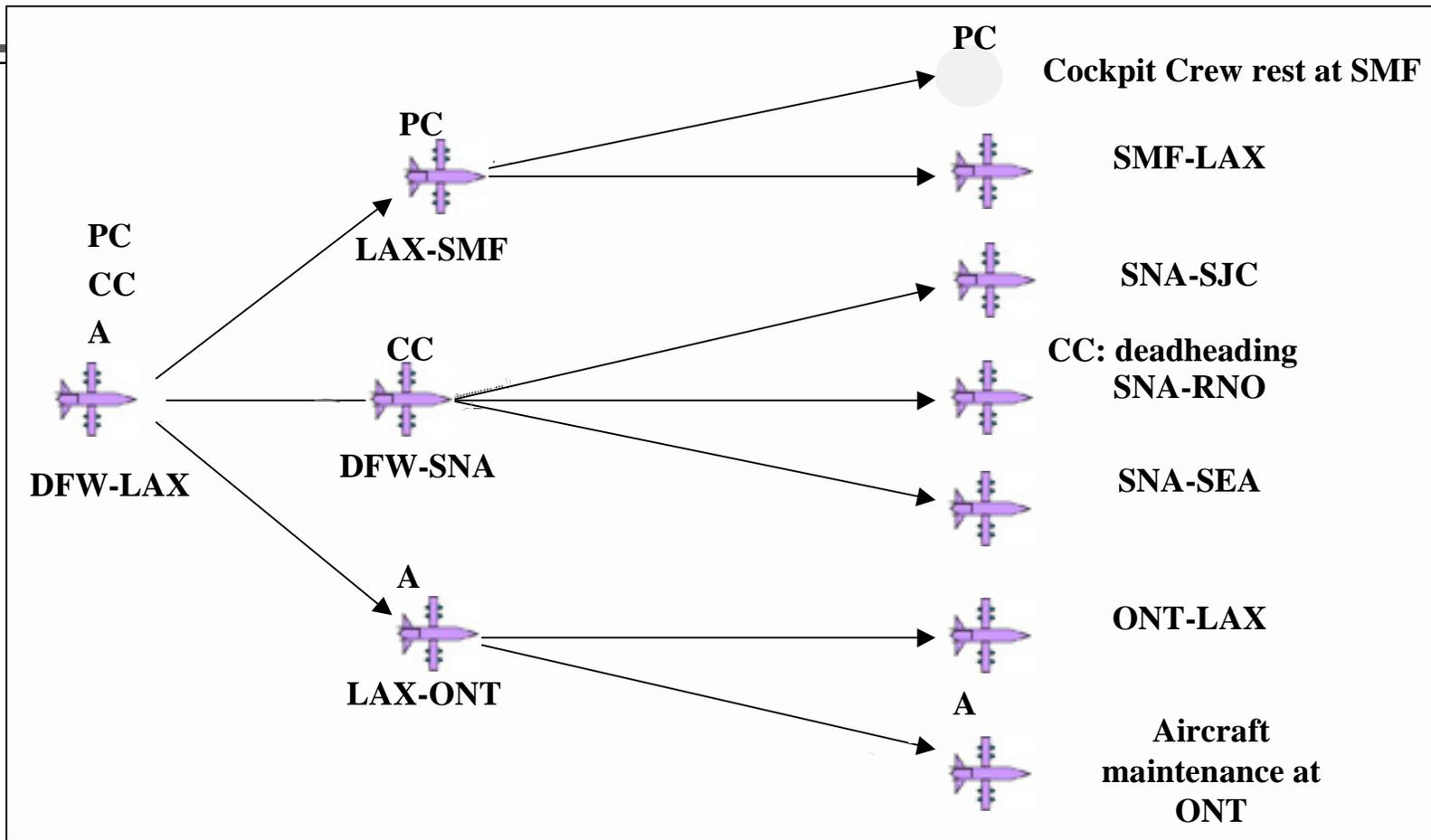
PC: Pilot Crew; CC: Cabin Crew; A: Aircraft

*Source: Sabre, 1998*

Flight copy generations

- **We have developed a technique to minimize the number of flight copies**
- **Four types of flight copies are generated:**
  - ➤ **Aircraft ready times**
  - ➤ **Copies to prevent passengers from missing connections**
  - ➤ **Consequence of type 2, aircraft postponement propagation**
  - ➤ **Schedule (for cancellations)**

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Considering seat availability uncertainty

Routing recovery

- Define maintenance critical aircraft, aircraft that have to be at a maintenance station before the end of the day

- <u>Routing feasibility</u>: if all the maintenance critical aircraft are at a maintenance station at the end of the day

- Identify all the swapped aircraft routes of maintenance critical aircraft: Set MCS.

- For each swap s, can we select a non critical aircraft with a route going to a maintenance station?
  - ➤ If yes, withdraw s from MCS, assign aircraft to new routes
  - ➤ Otherwise do the algorithm:

Neighborhood search algorithm
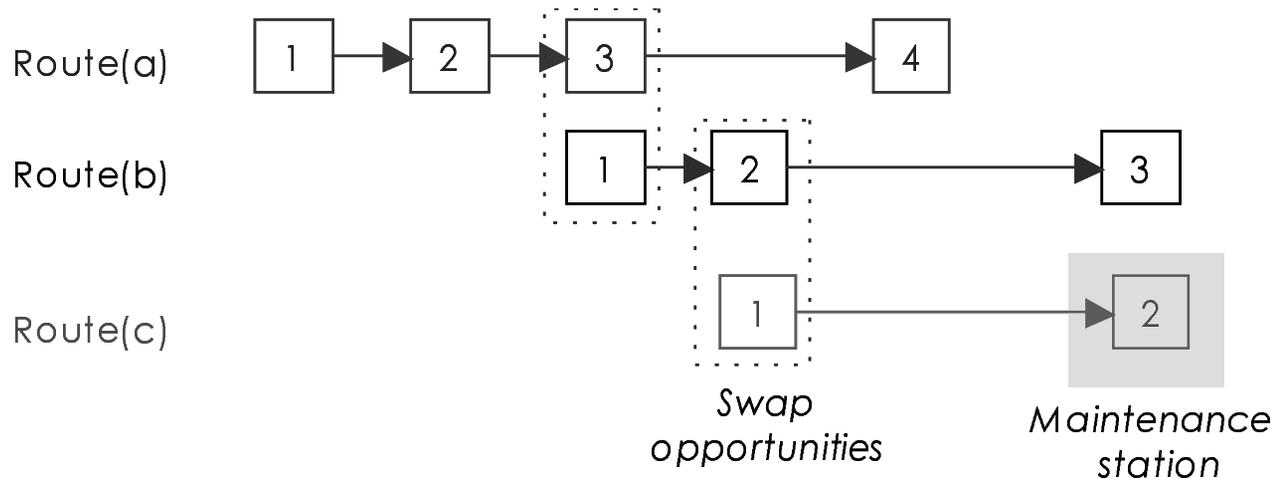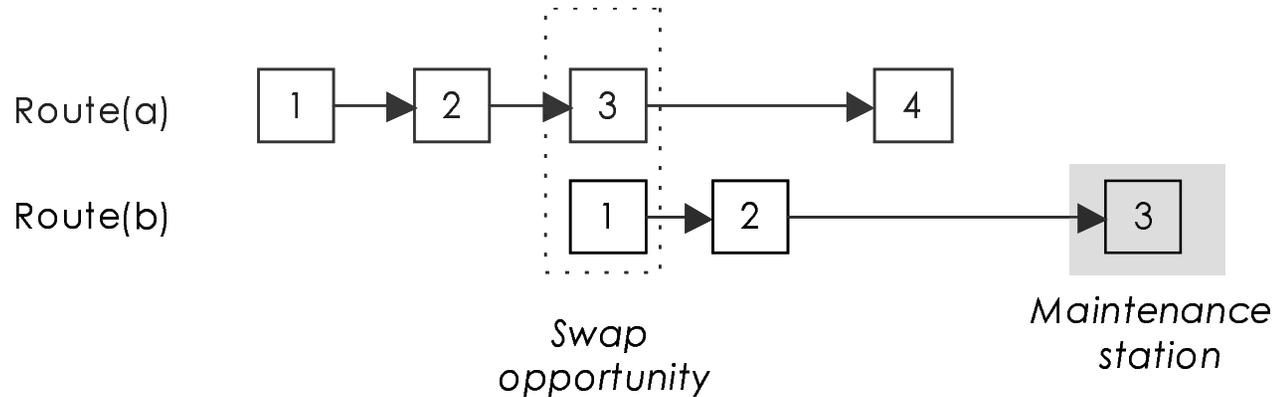
Neighborhood search algorithm to recover from routing infeasibility

For each infeasible swap s in MCS do:

➤ **STEP 1:** For each window of readiness WR(n), search for an aircraft with a route that goes terminates at a maintenance station before the end of the day of operations and have a readiness windows that intersect with WR(n) (including the infeasible routes). If one route is found, swap the two aircraft routes and move to the next infeasible swap. Otherwise, no simple route swap is found for all readiness windows and go to STEP 2.

➤ **STEP 2:** Generate a feasible route that goes from WR(n) to a maintenance station and all the sub-routes belong to non critical route.

➤ **STEP 3:** If no route swap found, include swaps in set of infeasible swaps

Forbid flight copies leading to routing infeasibility and run the optimization model again.

Algorithm's complexity

- STEP 1: runs in O(A*F)
- STEP 2: runs in O(A*F^n) with n number of route swap opportunities

- Can we find routing feasible solutions effectively using this approach?

Routing disrupted passenger under seat uncertainty

- Build the list of disrupted passengers according to a priority rule (First Disrupted First Recovered, fare class, loyalty (FFP))

- High fare tickets are often fully refundable. No shows (NS rate ≈20%).

- Number of seats available on flight f is uncertain

- Passenger centric approach: for each passenger in recovery list what is the recovery itinerary with the lowest expected arrival delay

Example

Flight	#	PDT	FDT	PAT	FAT	Feasibility probability
A-H	1	3:00PM	3:05PM	4:00PM	4:03PM	100%
A-B	2	3:30PM	3:35PM	5:30PM	5:31PM	70%
H-B	3	4:30PM	4:32PM	5:30PM	5:34PM	70%
H-B	4	5:00PM	5:01PM	6:00PM	6:05PM	70%
H-B	5	6:00PM	6:04PM	7:00PM	7:03PM	70%

Recovery itinerary	Flight string	Arrival delay (minutes)	Feasibility probability
A-B	2	271	70%
A-H-B	1-3	273	70%
A-H-B	1-4	305	70%
A-B	1-5	363	70%

Fast heuristic model

- **Heuristics chooses itinerary #1**
- **Probability of staying overnight is 30% whereas it is 2.7% for itinerary 2 and itinerary 2 arrives only 2 minutes after itinerary 1.**
- **Sub optimal model but very fast (O(log(I))**

Optimal routing algorithm

- **State = {Airport location, Forecasted Flight Time Departure}**

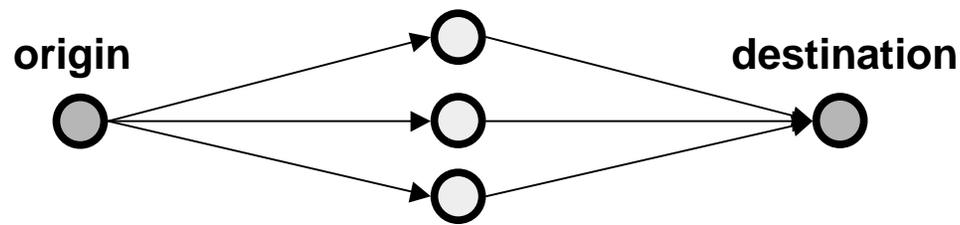| State S(i)	Loc(S(i))	Time(S(i))	Flight(S(i))
1	A	3:05PM	f(1)
2	A	3:35PM	f(2)
3	H	4:32PM	f(3)
4	H	5:01PM	f(4)
5	H	6:04PM	f(5)
6	B	3:35PM	T
7	B	4:32PM	T
8	B	5:01PM	T
9	B	6:04PM	T

Optimal routing algorithm (Cont.)

- **Build Markov chain**
- **Decisions, u:**
  - ➤ u(j) = 1 if book flight j, = 0 otherwise
- **Cost(s(j))**

  = AAT(f) − PAT(p) if s(j)∈ $\Im$

  = 0 otherwise

- **Can restrict the decision space to chose only one itinerary**

Optimal routing algorithm (Cont.)

- **State space size: O(2^F)**

- **10 flights in recovery list means at most 1024 states**

- **Can include bumping cost in decisions: Assume that you have estimated the value of one hour of delay for PNR p. You can reward a passenger to free up his/her seat. What is the best (itinerary, reward) decisions to minimize airline returns (passenger delay cost – "bumping" rewards)**

Passenger routing algorithm performance

- **PMIX provides the optimal passenger routings; We found that PDC is close to optimality (PMIX) to route the passengers**

- **When passengers are disrupted at the hub (flight cancellation or missed connection), PDC provides the optimal recovery most of the time because only one route typically goes from the hub to destination airport (hub and spoke topology); Only when passengers are disrupted at the origin spoke (first flight canceled), does PDC might provide sub-optimal solution**

origin   destination

Questions?
Discussion items?

Flight delays and flight cancellations
Passenger bookings for each scheduled itinerary

Disrupted?

Yes

No

Build the list of disrupted passengers, L

Assign all non-disrupted passengers to their planned itineraries

Sort L according to service policy

Remove seats from remaining inventory

Record passenger delay

Is L = ∅ ?

Yes

END

No

Take next disrupted passenger in L

Passenger Delay Statistics

Find best recovery itinerary and assign passenger

Remove seat from remaining inventory

Record passenger delay