# 1.206J/16.77J/ESD.215J Airline Schedule Planning

## Problem Set #1

### Barnhart C.

### Due date: Monday, March 10, 2003

## Problem #1: Express package operation

Assume a shipping company has 2 factories (A and B) and 3 distribution centers (C, D and E) that are connected with directed arcs (e.g., aircraft routes). Each arc, $(i,j)$, can carry a maximum number of units of flow $u(i,j)$. Every unit of commodity has to pay $c(i,j)$ to flow on arc $(i,j)$, independently of its type. A commodity k is defined as a shipment type from an origin factory $O(k)$ to a destination distribution center $D(k)$. We denote $n(k)$ to be the number of units of commodity k from $O(k)$ to $D(k)$. The objective is to minimize the total shipping cost for the company subject to conservation of flows, arc capacity limits and flow non-negativity.
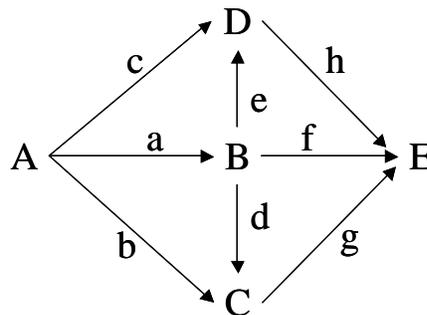


**Fig. 1: Network representation**

| k | From O(k) | To D(k) | n(k) |
|----|-----------|---------|------|
| k1 | A | C | 6 |
| k2 | A | D | 6 |
| k3 | A | E | 15 |
| k4 | B | C | 5 |
| k5 | B | D | 5 |
| k6 | B | E | 10 |

**Tab. 1: Commodity table**

| Arc i | From | To | c ($000/unit) | u |
|-------|------|-----|---------------|-----|
| a | A | B | 0.5 | 10 |
| b | A | C | 2 | 10 |
| c | A | D | 2 | 10 |
| d | B | C | 0.5 | 15 |
| e | B | D | 1 | 14 |
| f | B | E | 2 | 6 |
| g | C | E | 1 | 10 |
| h | D | E | 1 | 9 |

**Table 2: Arcs table**

Questions:

1. Prove that if a network is acyclic, there always exists a topological ordering of the nodes in the network.

2. Write the node-arc multi-commodity flow problem formulation and define all notations (e.g., parameters, decision variables, etc.).

3. Solve the problem using OPL Studio. Give the optimal objective value as well as the values of the optimal decision variables and the optimal dual variables. Check that the optimality conditions are satisfied. Use OPL structure to write only the non-zero coefficients (as explained during the tutorial) and comment on the benefit of using it for large scale problems.

4. Assume that the company can invest $1,000/unit to increase the arc capacity $u_{(A,B)}$ by one unit (i.e., from 10 to 11). Without resolving the problem, can you tell if this investment would benefit the company? Explain your reasoning and check if your decision is correct by solving a new LP.

5. Assume that the accounting department asks you to quickly access (i.e., _without resolving the problem_) whether it might benefit the company to use arc f given a reduction in the cost of commodity k3 by $600/unit. Explain your reasoning. Verify your assessment by changing the cost coefficient and resolving the problem.

6. Write the path formulation of the multi-commodity flow problem. Show that the node-arc and the path formulations are equivalent (hint: show similarity of feasible space and objective function).

7.  Assume now that each shipment has to go through only one single path.  Solve the problem using **<u>only</u>** the LP solver.  Describe and explain every step of your algorithm.

8.  Write the sub-network formulation of the problem.  Assuming again continuous variables, solve the formulation using OPL Studio.  Discuss and compare the advantages and disadvantages of the node-arc, path and sub-network formulations.


## Problem #2: Passenger Mix Model

In this problem, you are asked to formulate and implement the Passenger Mix Model described in class.  This problem serves as an exercise on row and column generation.


<u>PART A: Solving the initial Restricted Master Problem for the PMM Model</u>

1.  Compute unconstrained demand for each flight, based on the unconstrained demand by itinerary (see itinerary worksheet for inputs).

2.  Formulate the initial Restricted Master Problem (RMP) using the Keypath formulation in which demand constraints are relaxed, using the template provided in the Excel file. (Assume that spill is recaptured only on the null itinerary in this initial RMP).

    +  Fill in the objective function coefficients.

    +  Fill in the constraint matrix with appropriate coefficients (The size of this matrix is the number of flights multiplied by the number of itineraries).

    +  Compute the Right-Hand-Side (RHS) values.

3.  Solve the initial RMP using OPL Studio and print out the solution.


<u>PART B: Introducing Recapture and Implementing Row and Column Generation</u>

1.  Compute the associated objective function coefficients for all possible columns omitted from the RMP in PART A using recapture rate information given in the Recapture Rate worksheet. (Assume the recapture rate from itinerary i to the null itinerary is 1.0, and for all other pairs of itineraries not included in the Recapture Rate worksheet, the recapture rate is zero).

2.  Define dual variables for the formulation in PART A and write the expression (i.e., equation) for computing reduced cost of a column.

3.   Given the current solution, check for violated constraints and possible negative reduced cost columns. (Show your work.) If there are violated constraints or columns with negative reduced cost, continue to Step 4. If not, skip to Step 7.

4.   Add all negative reduced cost columns and/or violating rows by expanding the constraint matrix from PART A to reflect the additions. Go to Step 5.

5.   Solve the modified model using OPL Studio as outlined in PART A. Print the Solution. Print File and mark clearly the columns and rows newly added. Go to Step 6.

6.   Repeat Step 3

7.   In order to check if your final solution is indeed optimal, build the entire model with all possible rows and constraints and solve it using OPL Studio. Print out the solution file. Explain why you found the same solution.

8.   Why can't you use a shortest path algorithm to find the column to enter the Master Problem?

9.   Now, consider the slightly modified problem. Suppose that fares are flight based (i.e., it cost $c(f)$ for a passenger to fly on flight f) and there is no recapture. The fare associated therefore, with an itinerary corresponds to the sum of the fares associated with the flight leg traversed. Describe but do not compute the steps of an efficient algorithm to solve the problem. In a real size airline planning, how would you consider, in the column generation problem, the constraint that itineraries cannot have more than 3 flight legs? Discuss complexity issues.

_____