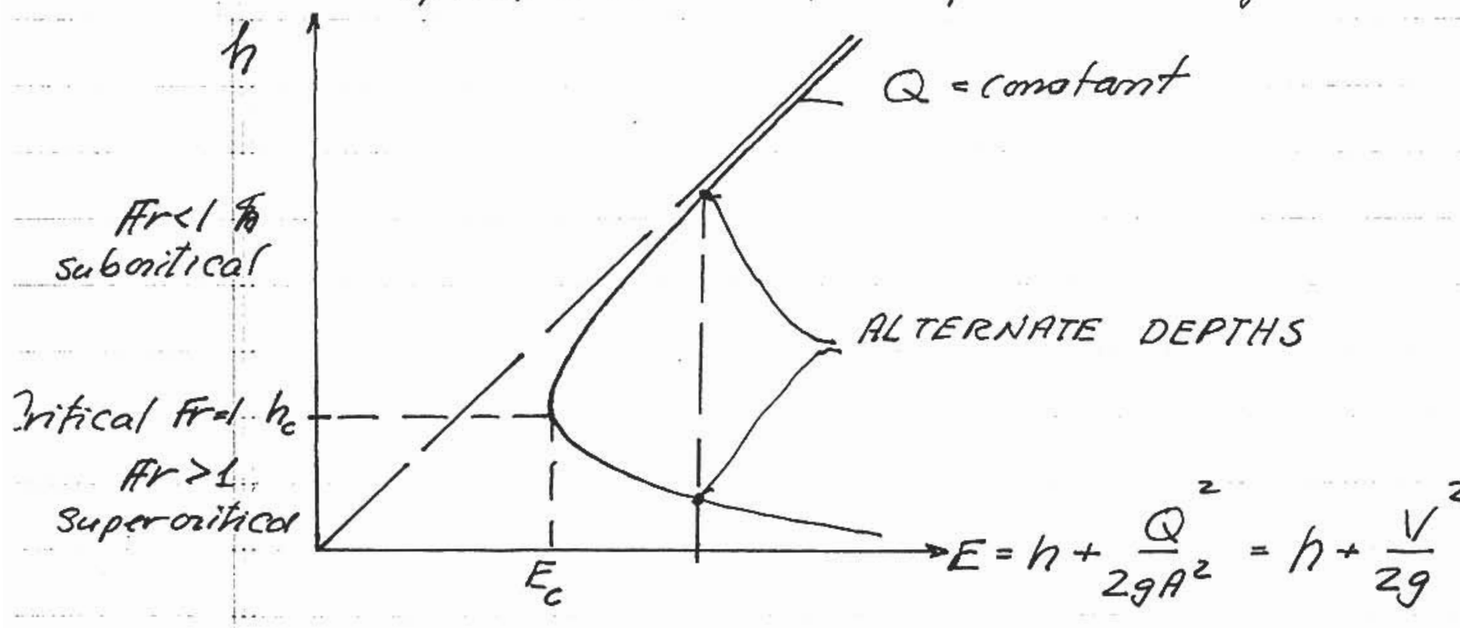
LECTURE # 26

1.060 ENGINEERING MECHANICS II

Specific Head (E) - Depth (h) Diagram



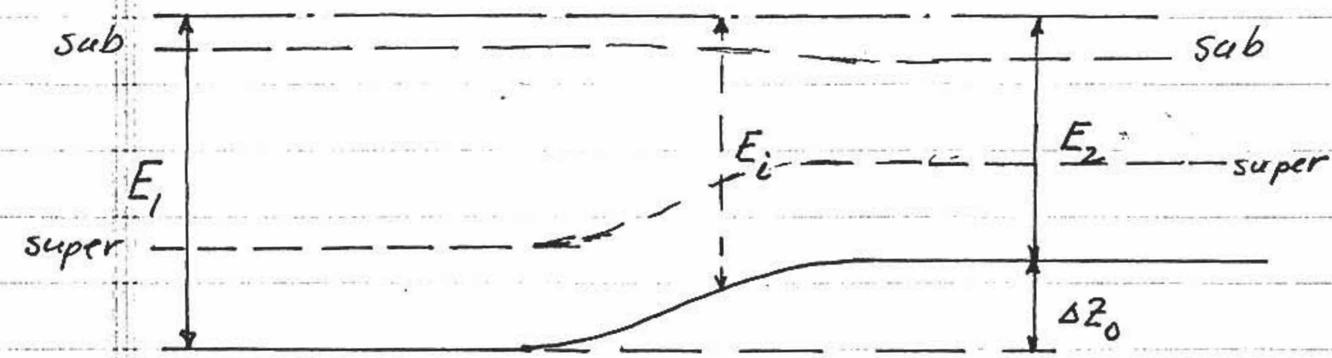
For given Q and given channel geometry a specified value of Specific Head, E, can result in:

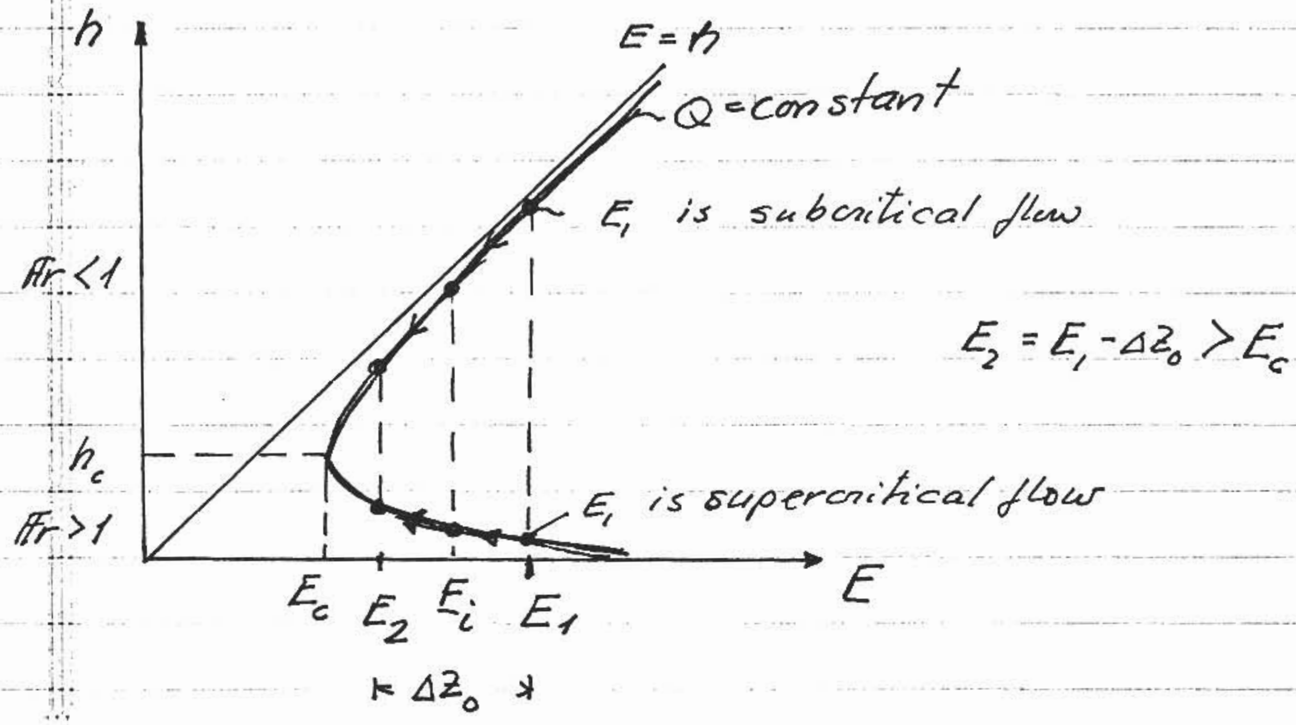
$E > E_c \Rightarrow$ Two solutions for h. These are known as ALTERNATE DEPTHS. One corresponds to sub the other to a supercritical flow, i.e. one $h > h_c$ and the other $h < h_c$.

$E = E_c \Rightarrow$ One solution only - corresponds to critical flow, i.e. $h = h_c$ = critical depth

$E < E_c \Rightarrow$ NO SOLUTION, i.e. the flow, as specified in terms of Q and E, is physically impossible!

Now, we are properly prepared to return to our TRANSITION problem, which "motivated" us to look at E vs h





If E_1 is

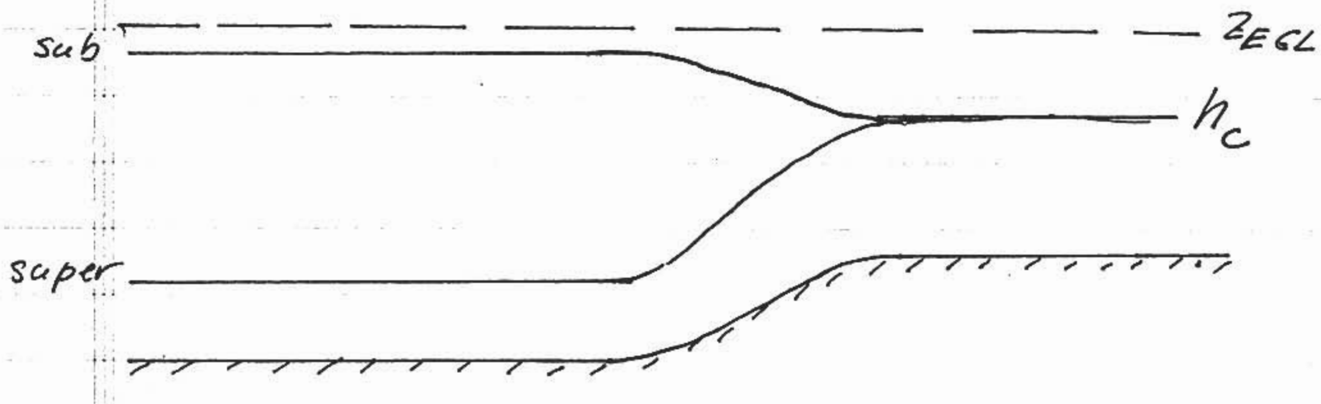
<u>E_1 is Subcritical</u>: Solution for <u>E_2</u> is <u>subcritical</u>.

Free surface slopes downward in flow direction, to produce a pressure gradient that accelerates the flow (Q = const. & h decreases ⇒ V increases)

<u>E_1 is Supercritical</u>: Solution for <u>E_2</u> is <u>supercritical</u>

Flow is "streaking" uphill against gravity and is slowed down – V is decreasing. To maintain constant Q = VA, h must increase.

If $E_2 = E_c$ after transition, flow can just pass the "hump"



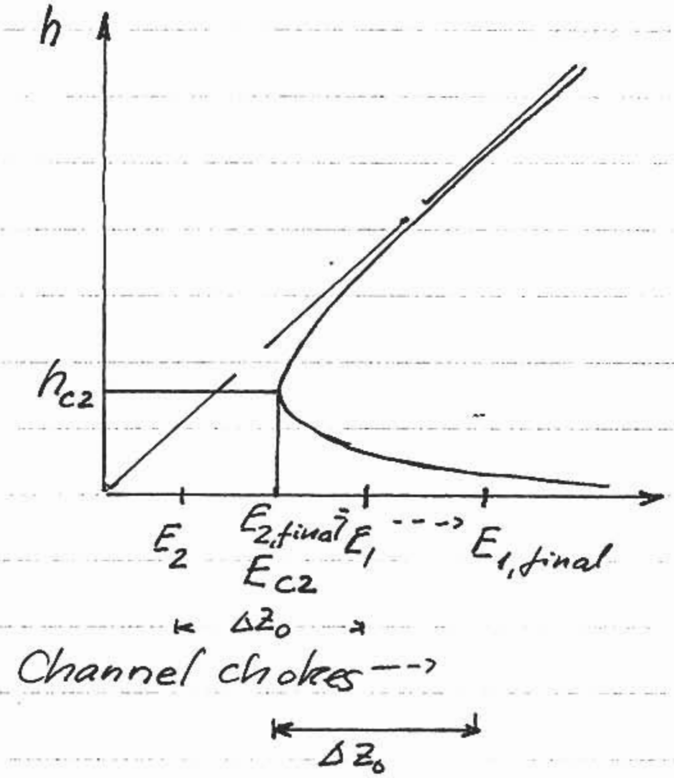
Notice: For supercritical flow velocity is actually decreasing in direction of flow, i.e. transition may be short, but flow is actually <u>not</u> converging but diverging. Possibility of headloss due to expansion is present - we assumed $\Delta H = 0$!

If $E_2 < E_c$ after transition, there is no physically possible solution for the given value of E_2, which in turn is related to E_1 through $E_2 = E_1 - \Delta z_0$. Only way to get flow over the "hump" is to increase E_1 relative to its specified value! The channel "chokes", backs up the water upstream until E_1 is just large enough to make flow proceed, i.e.

$$E_{1\,final} = E_{c2} + \Delta z_0$$

so that

$$E_2 = E_1 - \Delta z_0 = E_{c2}$$

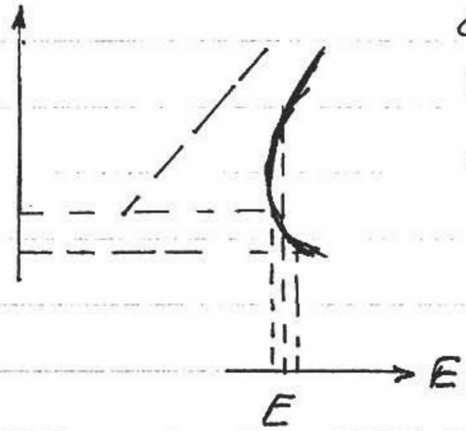


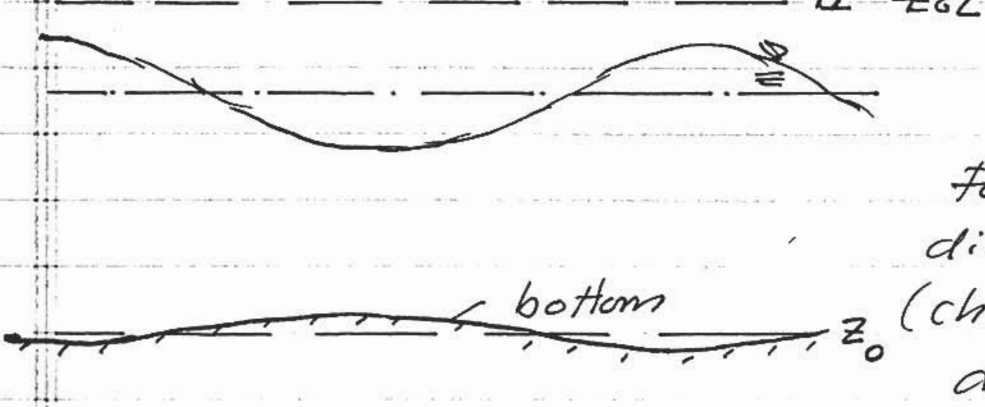
$E_2 < E_c$ initially. Flow can not pass, channel chokes and backs up water upstream until $E_{1,final}$ is just enough to produce $E_{2,final} = E_{1,final} - \Delta z_0 = E_c$. Flow can now proceed as critical flow.

CRITICAL FLOW PECULIARITY

If a flow is nearly uniform, and close to critical, i.e. $E \sim E_c$, then a small change in E, e.g. $\pm \delta E$, will result in a large change in h, $\pm \delta h$, since

$$\frac{\partial E}{\partial h} = \frac{\delta E}{\delta h} \approx 0 \text{ if } E \sim E_c$$





~ Critical Flow is very sensitive to small disturbances (changes) in depth!

ENERGY/BERNOULLI PRINCIPLE IN OPEN CHANNELS

Short Transition of Non-Separating Flows

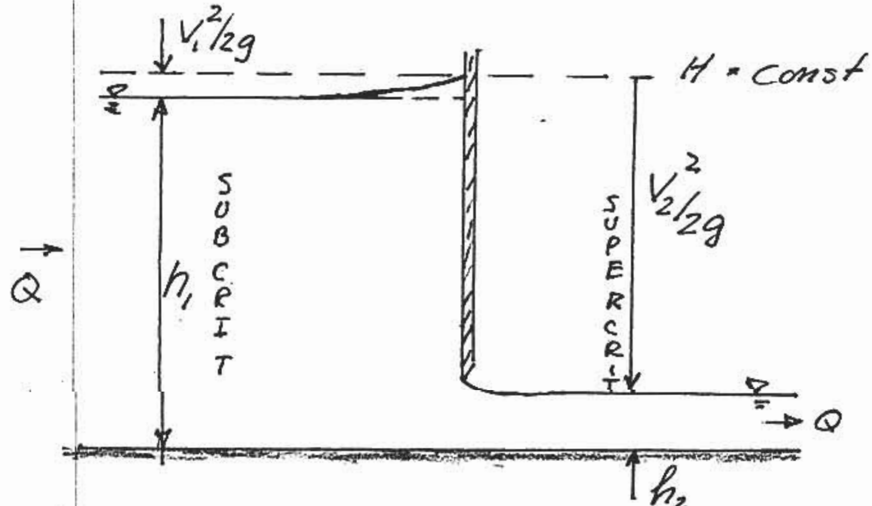
"Short" means that head loss due to friction may be neglected.

"Short" also means that any change in elevating due to the general slope of the channel may be neglected over the short transition.

"Non-Separating" means converging (for sure) or so gently expanding that flow separation from boundaries is not expected.

Summary: Locally channel is treated as horizontal and $\Delta H \approx 0$ across transition.

Flow under a Gate

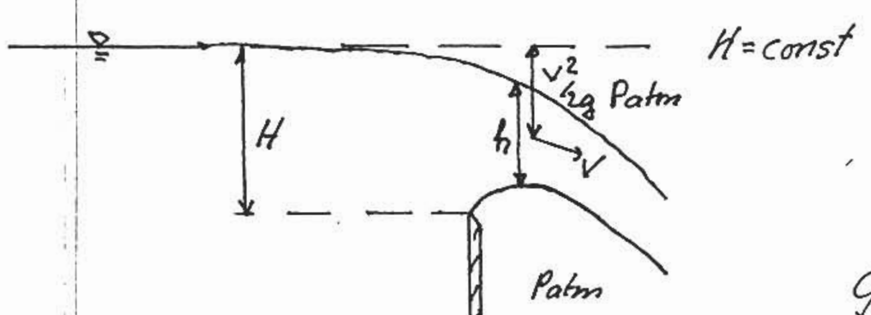


$$V_2 \approx \sqrt{2g(h_1 - h_2)}$$

$$q = h_2 V_2$$

$$h_2 = C_c h_g \approx 0.6 h_g$$

Flow over a Gate (weir)

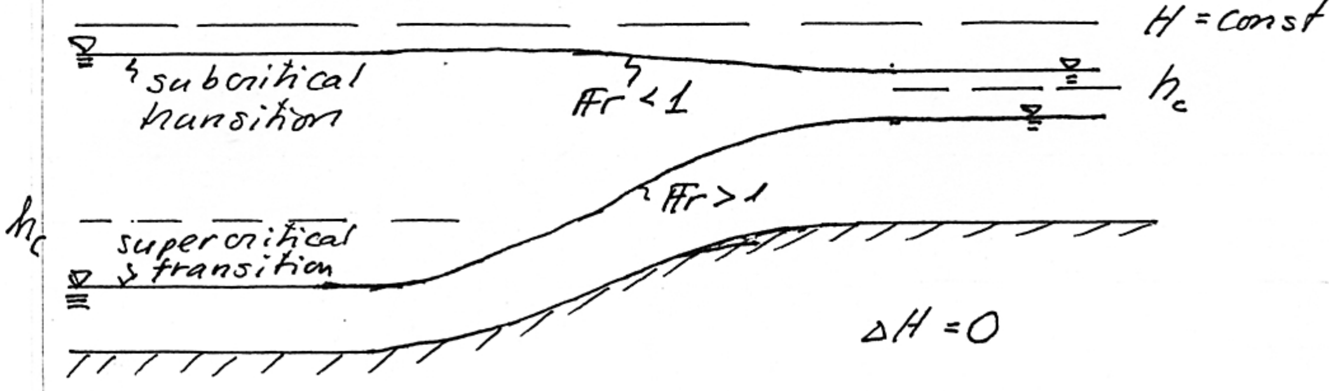


$$V \propto \sqrt{gH}$$

$$h \propto H$$

$$q = hV \propto \sqrt{g}\, H^{3/2}$$

Short Transitions



Excellent for subcritical flow transitions from one channel cross-section (e.g. man-made rectangular concrete channel) at ① through a transition to a natural (e.g. a trapezoidal channel) at ②. That is, transition may involve a change in bottom elevation as well as a change in channel cross-section's geometry. So long as changes are gradual and do not result in severe decreases in velocity in direction of flow (negligible expansion losses) the basic assumption: $\Delta H \approx 0$ for transition is reasonably good.

For supercritical flow transitions: WATCH OUT! The smooth variations in h may potentially be completely unrealistic, and transitions exhibiting hydraulic jump, waves and non-uniform flows downstream may result. Consult advanced texts on Open Channel Flow for supercritical transition.