LECTURE # 25

1.060 ENGINEERING MECHANICS II

THE ENERGY (BERNOULLI) PRINCIPLE

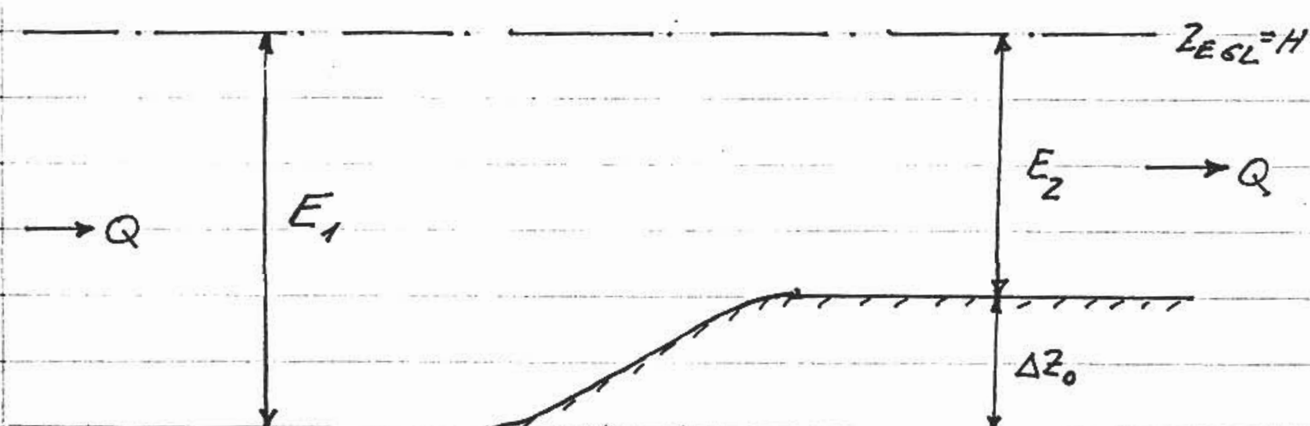
In Open Channel Flow it turns out to be convenient to refer to the EGL elevation above the channel bottom (located at $z = z_0$ above datum). This elevation difference

$$H_S = H - z_0 = z_{EGL} - z_0 = h\cos\beta + \frac{Q^2}{2gA^2} \approx h + \frac{Q^2}{2gA^2}$$

is referred to as the SPECIFIC HEAD, $E = H_S$

SHORT TRANSITION OF CONVERGING FLOW

From "old" Bernoulli principle : $H_1 = H_2$ $(\Delta H = 0)$

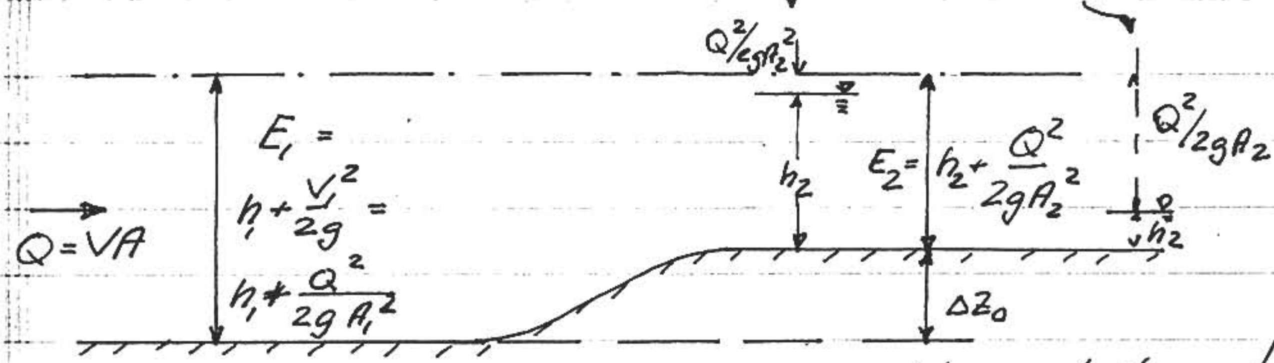


$$\underbrace{E_1 = h_1 + \frac{Q^2}{2gA_1^2}}_{\text{KNOWN}} \qquad \underbrace{E_2 = E_1 - \Delta z_0}_{\text{KNOWN}}$$

$$E_2 = h_2 + \frac{Q^2}{2gA_2^2} = E_1 - \Delta z_0 \,;\; A_2 = A_2(h_2)$$

NOT A UNIQUE SOLUTION!

$E_2 \simeq h_2$ if A_2 large; $E_2 \simeq Q^2/2gA_2^2$ if h_2 & A_2 small!



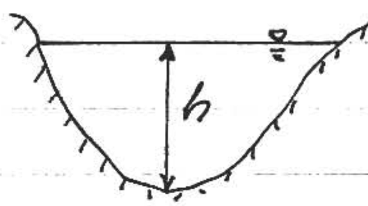
Two possible solutions!

MOTHER NATURE DOES NOT LIKE THIS!

To determine which one is the true solution we need to introduce the:

SPECIFIC HEAD – DEPTH DIAGRAM

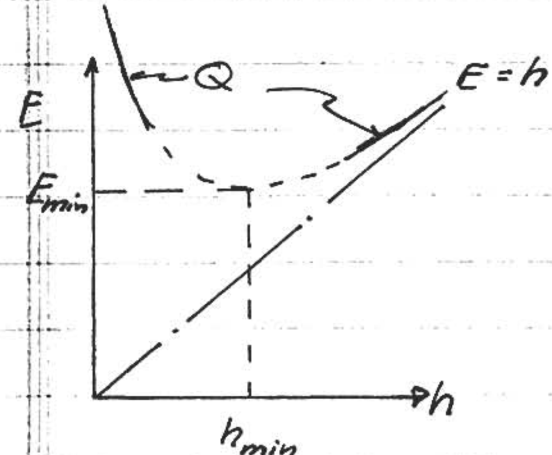
Q = Discharge is specified



$A = A(h)$ is specified

$$E = h + \frac{Q^2}{2gA^2} \quad \text{varies with } h!$$

$$h \to \infty \quad A \to \infty \quad \Rightarrow \quad E \simeq h \quad \to \infty$$
$$h \to 0 \quad A \to 0 \quad \Rightarrow \quad E \simeq Q^2/(2gA^2) \to \infty$$

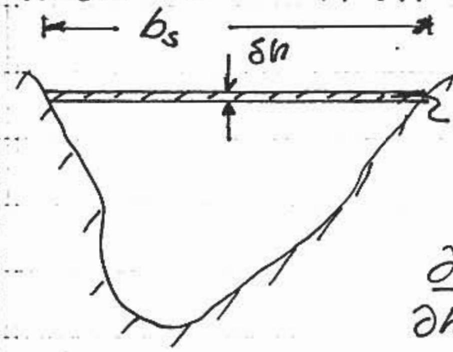


E must have a minimum value, given by

$$\frac{\partial E}{\partial h} = \frac{\partial}{\partial h}\left(h + \frac{Q^2}{2g} A(h)^{-2}\right) = 0$$

$$\frac{\partial E}{\partial h} = 1 + \frac{Q^2}{2g}\left(-2A^{-3}\frac{\partial A}{\partial h}\right) = 1 - \frac{Q^2}{gA^3}\frac{\partial A}{\partial h} = 0$$

determines value of h for which $E = E_{min}$, but what is $\partial A/\partial h$?



$$\delta A = b_s\, \delta h$$

$$\frac{\partial A}{\partial h} = \frac{\delta A}{\delta h} = b_s$$

$\frac{\partial A}{\partial h} = b_s$ = surface width of channel.

So,

$$E = E_{min} \text{ for } \frac{Q^2 b_s}{gA^3} = 1$$

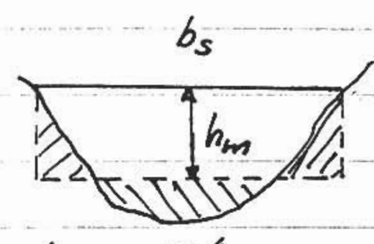
Flow condition corresponding to E_{min} is referred to as <u>CRITICAL FLOW</u>, and it is defined as

$$\frac{Q^2 b_s}{gA^3} = 1, \quad \text{i.e. with } b_s = b_s(h) \;\&\; A = A(h)$$

$\frac{Q^2 b_s}{gA^3} = 1$ defines <u>$h = h_c$ = CRITICAL DEPTH</u>

But $Q = VA$ so

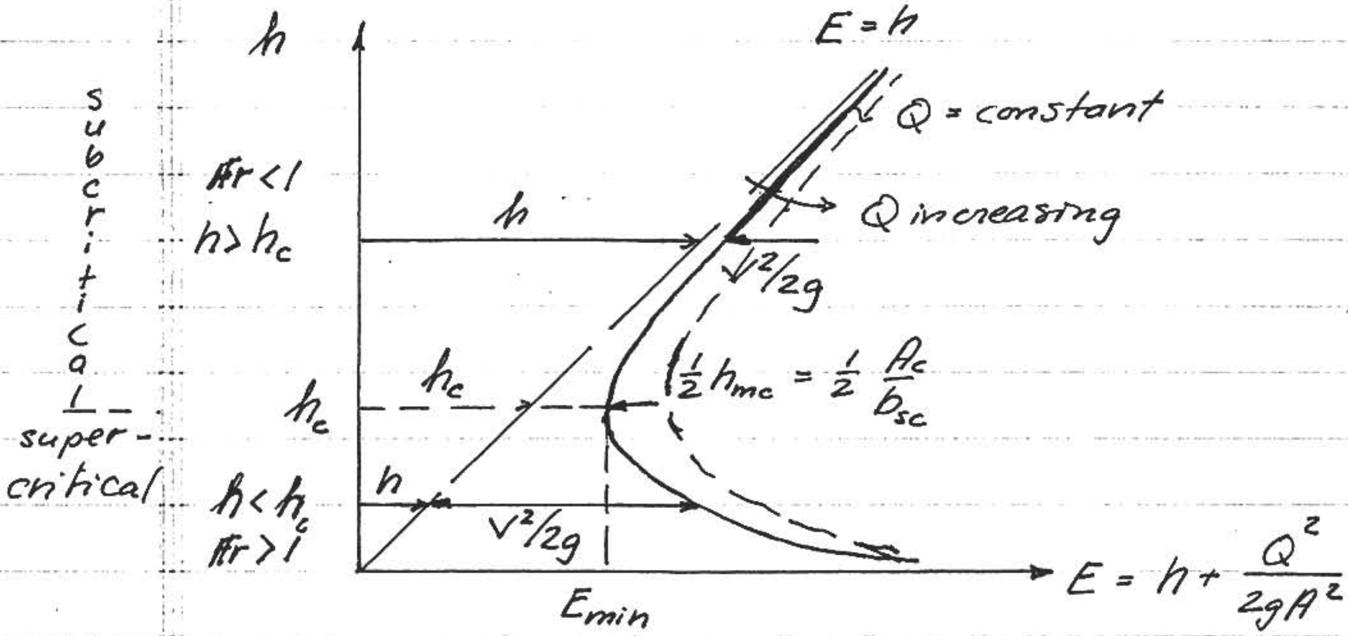
$$\frac{Q^2 b_s}{gA^3} = \frac{V^2}{g(A/b_s)} = \frac{V^2}{g h_m}$$



$h_m = A/b_s$ = mean depth

$$\frac{Q^2 b_s}{gA^3} = \frac{V^2}{g h_m} = Fr^2 \Rightarrow \underline{Fr} = \frac{V}{\sqrt{g h_m}} = \underline{\text{FROUDE NUMBER}}$$

SPECIFIC HEAD – DEPTH DIAGRAM



Two "types" of flow:

$$h > h_c \Rightarrow Fr^2 = \frac{Q^2}{g(A^3/b_s)} < 1 \therefore \underline{\text{Subcritical Flow}}$$

where "most of E" is contributed by h with only a small velocity head

$$h < h_c \Rightarrow Fr^2 = \frac{Q^2}{g(A^3/b_s)} > 1 : \underline{\text{Supercritical Flow}}$$

where "most of E" is contributed by the velocity head and h is minor.

$$h = h_c \Rightarrow Fr^2 = \frac{Q^2}{g\, A_c^3/b_{sc}} = 1 : \underline{\text{Critical Flow}}$$

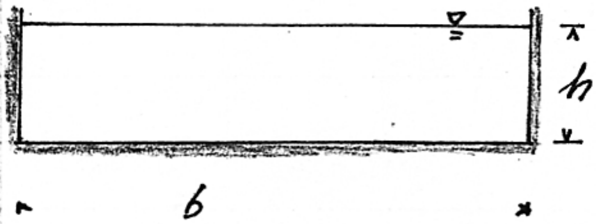
But, since flow is critical $\frac{Q^2}{2gA_c^2} = \frac{V_c^2}{2g} = \frac{1}{2}\frac{A_c}{b_{sc}} = \frac{1}{2}h_{mc}$

Thus, for critical flow

$$E = E_c = h_c + \frac{V_c^2}{2g} = h_c + \frac{1}{2}\frac{A_c}{b_{sc}} = h_c + \frac{1}{2}h_{mc}$$

for critical flow.

For a rectangular channel, we have



$b_s = b$ = constant;

$$h_m = \frac{A}{b_s} = \frac{bh}{b} = h$$

So, for a rectangular channel

$$E_c = h_c + \frac{Q^2}{2gA_c^2} = h_c + \frac{1}{2} h_{mc} = h_c + \frac{1}{2} h_c = \frac{3}{2} h_c$$

Thus, if a channel is rectangular and E = specific head, is known, the depth corresponding to critical flow is

$$h_c = \text{critical depth} = \frac{2}{3} E$$

Since $Fr = V_c / \sqrt{g h_{mc}} = V_c / \sqrt{g h_c} = 1$, the discharge per unit width is given by

$$(Q/b) = q = V_c h_c = \sqrt{g}\, h_c^{3/2}$$

Also, for a rectangular channel, we have

$$\frac{Q^2}{2gA^2} = \frac{(Q/b)^2}{2gh^2} = \text{velocity head} = \frac{1}{2} h_{mc} = \frac{1}{2} h_c$$

if flow is critical. Thus,

$$h_c = \left(\frac{(Q/b)^2}{g} \right)^{1/3}$$