LECTURE # 24

1.060 ENGINEERING MECHANICS II

UNIFORM FLOW COMPUTATIONS

Geometry of Prismatic Channel, i.e.
$A(h)$, $P(h)$, $R_h = A/P = R_h(h)$, "known."
Channel Slope S_o "known"
Channel Roughness ε (or Manning's 'n') "known"
from Table 10.1 or "n" $= 0.038\,\varepsilon^{1/6}$ (SI)

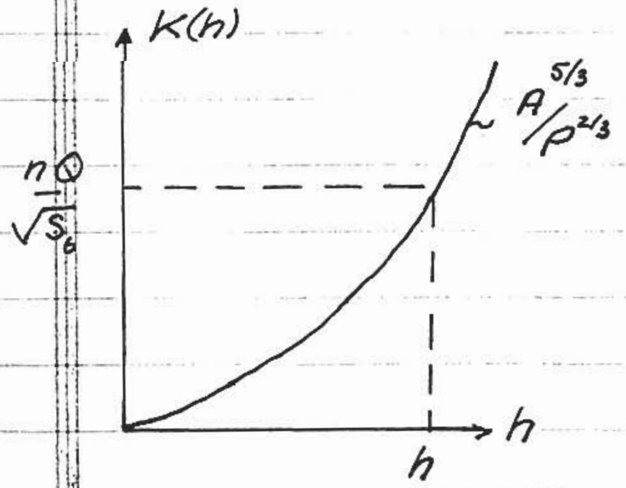
Type 1 Problem: h specified $\Rightarrow$ Find Q

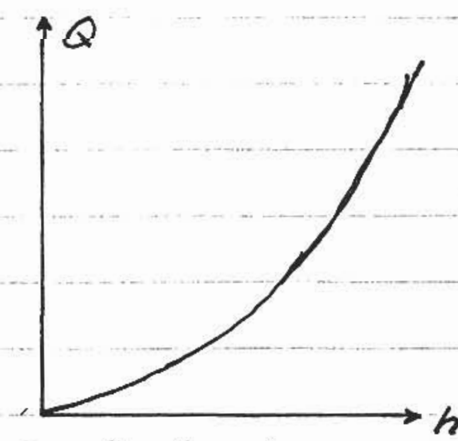
$$Q = V \cdot A = \frac{1}{n} R_h^{2/3}(h)\, A(h) \sqrt{S_o} = \frac{1}{n} \frac{[A(h)]^{5/3}}{[P(h)]^{2/3}} S_o^{1/2}$$

Not a hard problem.

Type 2 Problem: Q specified $\Rightarrow$ Find stage, i.e. h

$$Q = VA = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} \sqrt{S_o} \Rightarrow \frac{A^{5/3}}{P^{2/3}} = K(h) = \frac{nQ}{\sqrt{S_o}}$$

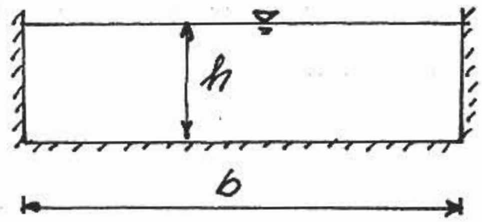




Depth-Discharge Relationship

Simple Geometry (Iterative Solution)

Rectangular Channel



$A = bh$, $P = b + 2h$

$R_h = A/P = bh/(b+2h) = h/(1+2h/b)$

Very Wide Rec. Channel $h \ll b$

$R_h \approx h$

$$Q = AV = \frac{1}{n} R_h^{2/3} A \sqrt{S_o} = \frac{1}{n} \frac{h^{2/3}}{(1+2h/b)^{2/3}} hb \sqrt{S_o}$$

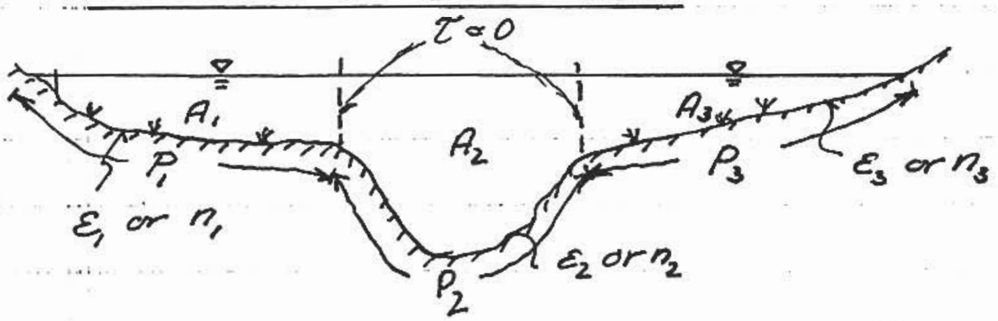
$$\frac{Qn}{\sqrt{S_o}\, b} = \frac{h^{5/3}}{(1+2h/b)^{2/3}} \Rightarrow h = \underbrace{\left(\frac{Q\,n}{b\sqrt{S_o}}\right)^{3/5}}_{K} \left(1+2\frac{h}{b}\right)^{2/5}$$

Solve by iteration:

$$h^{(n+1)} = K\left(1+2\frac{h^{(n)}}{b}\right)^{2/5}$$

starting with $h^{(0)} = 0$.

COMPOSITE CHANNELS

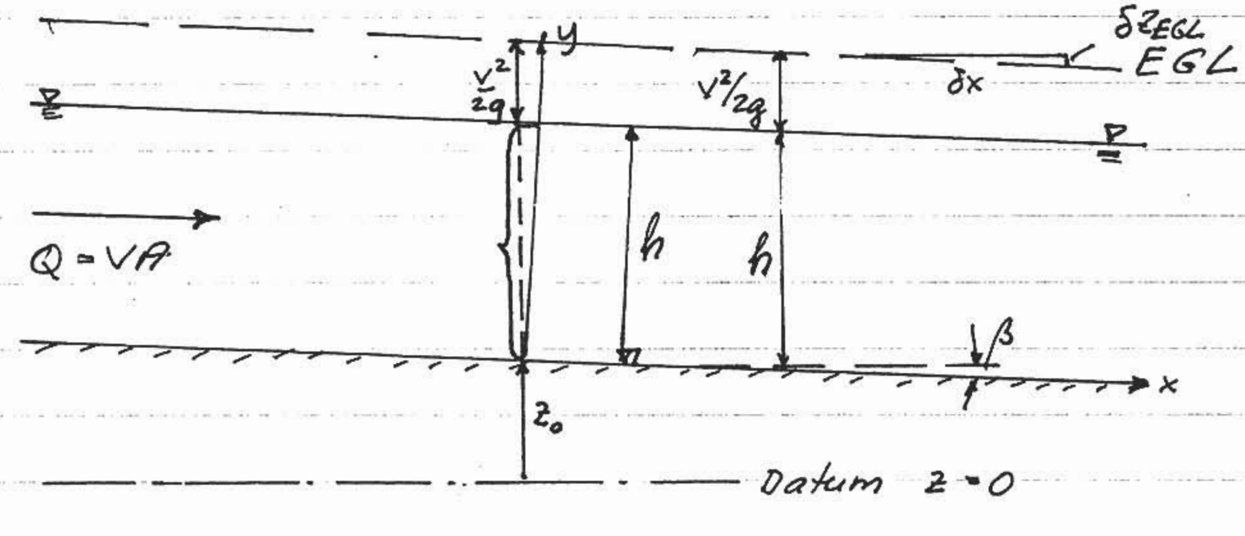


No shear stress along fluid-air and fluid-fluid interfaces.

$$Q = \sum Q_n = \sum \frac{1}{n_n} A_n \left(\frac{A_n}{P_n}\right)^{2/3} \sqrt{S_o}$$

ENERGY AND HYDRAULIC GRADE LINES

Uniform, Steady Flow



z_{EGL} = vertical distance of Energy Grade Line (Total Head Line) above datum = $H = z + \frac{p}{\rho g} + V^2/2g$,

BERNOULLI

Since flow is steady, uniform and wellbehaved $p + \rho g z$ = constant $\perp$ streamlines. Within fluid we have: $z = z_o + y\cos\beta$, so

$p + \rho g z = p + \rho g(z_o + y\cos\beta) = const = \rho g(z_o + h\cos\beta)$

since $p = 0$ at free surface. So, within fluid

$$\frac{p}{\rho g} + z = \frac{p + \rho g z}{\rho g} = z_o + h\cos\beta = z_{HGL}$$

So,

$$z_{EGL} = H = z_o + h\cos\beta + \frac{V^2}{2g} = z_o + h\cos\beta + \frac{Q^2}{2gA^2}$$

Since, in most cases $\cos\beta \simeq 1.000..$,

$$z_{EGL} = H = h + \frac{Q^2}{2gA^2} + z_o$$

EGL is parallel to bottom

In a uniform steady flow

HGL = hydraulic grade line

has

$$z_{HGL} = z_0 + h\cos\beta$$

i.e. it is parallel to the bottom and a vertical distance $h\cos\beta$ above it. Since, in most cases $\cos\beta \simeq 1$, the hydraulic grade line is "identical" to the free surface.

EGL = energy grade line

has

$$H = z_{EGL} = z_0 + h\cos\beta + \frac{V^2}{2g} = z_0 + h\cos\beta + \frac{Q^2}{2gA^2}$$

and is located a distance of $V^2/2g = Q^2/2gA^2 =$ the velocity head above the HGL, i.e. it too is parallel to the bottom.

$$\text{Slope of bottom} = -\frac{\partial z_0}{\partial x} = \sin\beta = S_0 =$$
$$\text{Slope of EGL} = \underline{-\partial H/\partial x = S_f = S_0}$$

is valid for uniform, steady flow.

From the generalized pipe flow expression

$$-\frac{\partial H}{\partial x} = \frac{\tau_s P}{\rho g A} = \frac{\tau_s}{\rho g R_h} = S_f$$

we have

$$S_f = \frac{f}{8g}\frac{P}{A^3} Q^2 \qquad \text{(Darcy-Weisbach)}$$

for Darcy-Weisbach friction, $\tau_s = \frac{1}{8}\rho f V^2$.
For Chezy's Equation, $C = \sqrt{8g/f}$, we

$$S_f = \frac{1}{C^2}\frac{P}{A^3} Q^2 \qquad \text{(Chezy's Equation)}$$

where we refer to this equation as Chezy's Equation only when C = Chezy Coefficient is treated as a constant

Finally, we have for Manning's Equation

$$S_f = n^2 \frac{P^{4/3}}{A^{10/3}} Q^2 \qquad \text{(Manning's Equation)}$$

i.e. the bottom shear stress is, in terms of Manning's "n" given by

$$\tau_s = \rho g n^2 \left(\frac{P}{A}\right)^{1/3} V^2 = \rho g n^2 R_h^{-1/3} V^2$$

For uniform, steady flow $S_f = S_0$ and the above formulae for S_f reduce to the equations we use to determine uniform, steady flow characteristics.