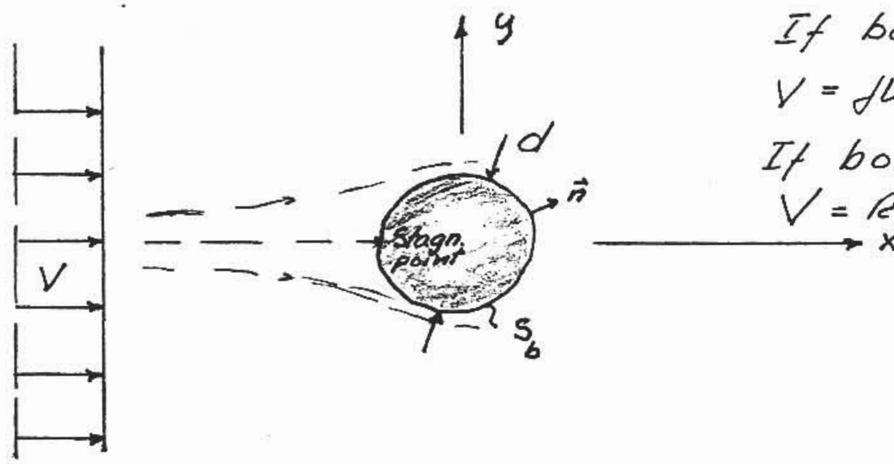
LECTURE #20

1.060 ENGINEERING MECHANICS II

FORCES ON SUBMERGED BODIES IN A FLOWING FLUID



If body is fixed
V = fluid velocity.
If body is moving
V = Relative velocity = $V_f - V_B$

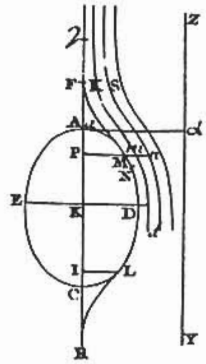
Integration of pressure and shear forces over the surface of the body gives

F_x = force in line with approach flow =
F_D = drag force = $\int_{S_b} (\tau_{s,x} - p_s \vec{n}_x)\, dS$

F_y = force $\perp$ to approach flow =
F_L = lift force = $\int_{S_b} (\tau_{s,y} - p_s \vec{n}_y)\, dS$

For an ideal fluid (incompressible & $\nu = 0$) stream line pattern is symmetrical, i.e.
$p_s(-x,y) = p_s(x,y) = p_s(x,-y) = p(-x,-y)$, and there is no net force on the body. This is D'Alembert's Paradox (1752):
No drag force on a body in an ideal fluid flow!

Jean le Rond d'Alembert



Stagnation zone after d'Alembert.

1752

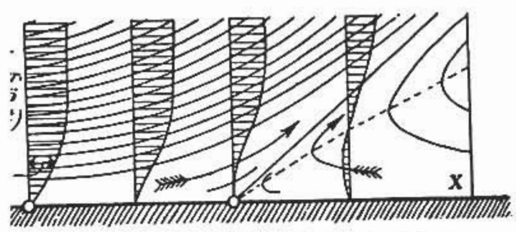
Thus, I do not see, I admit, how one can satisfactorily explain by theory the resistance of fluids. On the contrary, it seems to me that the theory, in all rigor, gives in many cases zero resistance; a singular paradox which I leave to future Geometers for elucidation.

The "Geometer" who provided the "elucidation" was Ludwig Prandtl and the "future" was 1904

Ludwig Prandtl

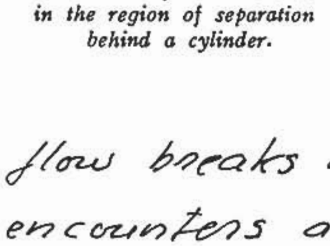
I have set myself the task of investigating systematically the motion of a fluid of which the internal resistance can be assumed very small. In fact, the resistance is supposed to be so small that it can be neglected wherever great velocity differences or cumulative effects of the resistance do not exist. This plan has proved to be very fruitful, for one arrives thereby at mathematical formulations which not only permit problems to be solved but also give promise of providing very satisfactory agreement with observation.

... the investigation of a particular flow phenomenon is thus divided into two interdependent parts: there is on the one hand the *free fluid*, which can be treated as inviscid according to the vorticity principles of Helmholtz, and on the other hand the transition layers at the fixed boundaries, the movement of which is controlled by the free fluid, yet which in turn give the free movement its characteristic stamp by the emission of vortex sheets.



Prandtl's concept of the velocity distribution near a point of separation.

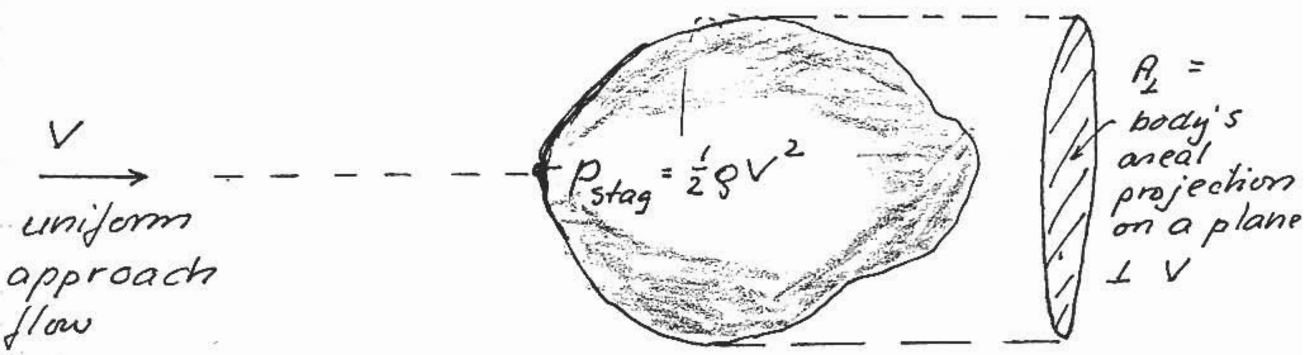
1904



Formation of vortex sheets in the region of separation behind a cylinder.

The apparent symmetry of the flow breaks down when the real (viscous) fluid encounters an adverse pressure gradient on the lee-side of the body and separation occurs.

Definition of a Drag Coefficient



$$F_D = \underbrace{\left(\tfrac{1}{2}\rho V^2\right)}_{\text{dynamic pressure at stagnation point scales pressure}} \underbrace{\left(A_\perp\right)}_{\text{Scales area of body upon which dynamic pressure acts}} \cdot \underbrace{C_D}_{\text{DRAG COEFFICIENT}}$$

$$\underline{F_D = \tfrac{1}{2}\rho\, C_D\, A_\perp V^2}$$

(Note similarity with wall friction in pipes
$F_\tau = \frac{1}{8}\rho f \underbrace{(P \cdot \ell)}_{"A_\tau} V^2$)

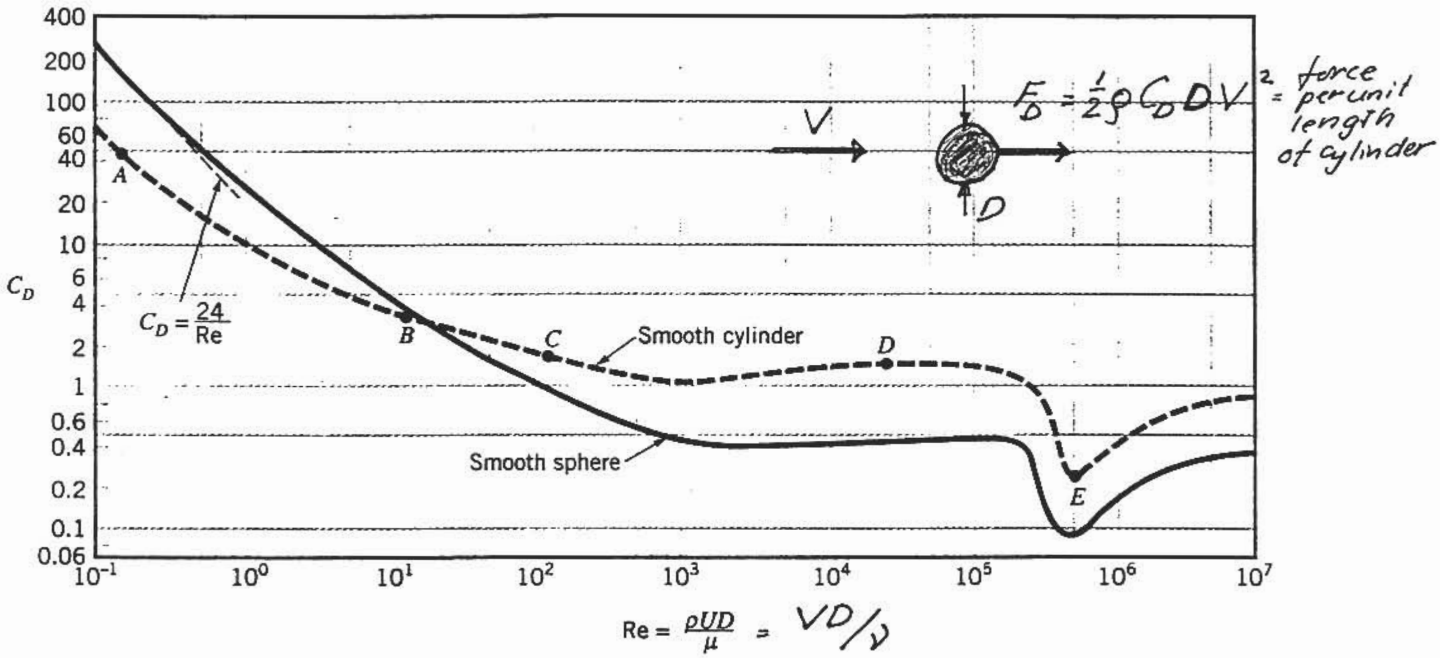
From dimensional analysis (see Recitation #1) we have:

C_D = Drag coefficient $= C_D\left(Re = \frac{VL}{\nu},\ \varepsilon/L,\ \text{shape of Body}\right)$

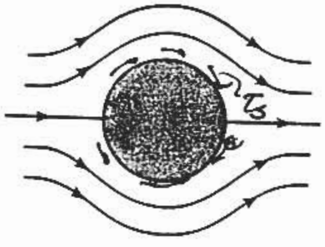
where L = characteristic scale of $A_\perp$

$$\left.\begin{array}{l} L \sim \sqrt{A_\perp} \text{ for 3-D} \\ \\ L \sim \text{"short length" for 2-D} \end{array}\right\} \quad L = D \text{ for } \left\{\begin{array}{l} \text{sphere} \\ \\ \text{circ. cylinder} \end{array}\right.$$

Nature of Drag Force on Circular Cylinder



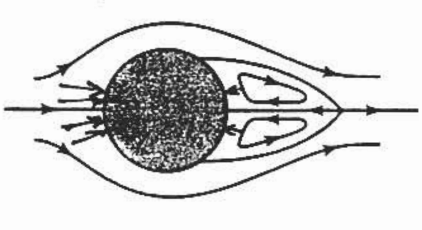
$Re <\sim 1$ (A)



No separartion

Highly viscous (laminar flow) referred to as "creeping flow". No separation and although pressure is involved in producing a drag force, one may think of this as "shear stress drag" for simplicity.

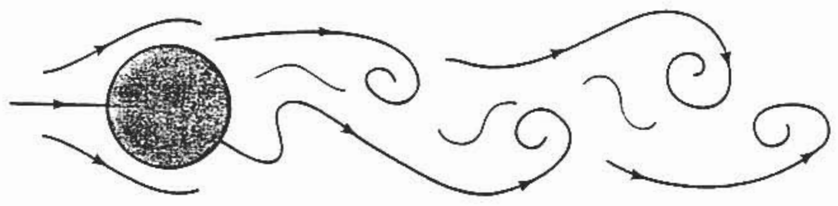
$Re \approx O(10)$ (B)



Steady separation bubble

Flow separation takes place on lee side of cylinder and forms a "separation bubble" with two counter rotating vortices. Pressure in "bubble" is, less than pressure on upstream counterpart. This pressure difference provides most of the drag force.

Re = O(100) (c)



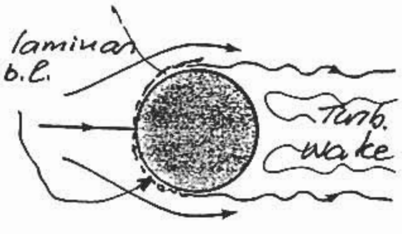
Oscillating Karman vortex street wake

The two counter-rotating vortices in the separation bubble (Re ~ O(10)) start to compete for space with each other. One grows more than the other, gets too large for its own good, and detaches from the cylinder and is advected downstream. With the big guy gone, the little guy on the other side sees his chance and takes over. But he too gets too big for his own good and detaches. Result: the formation of von Karman's Vortex Street with vortices of alternating rotation forming a "wake" that can persist for a large distance downstream of body.

The drag force is pressure dominated and due to the detachment of vortices from alternating sides of the cylinder the drag force fluctuates in magnitude (dynamic forcing). Also, since the vortex shedding upsets the equilibrium ⊥ flow direction, there will be an oscillating transverse force, i.e. a lift force.

Note: You may have "heard" this effect if you have been near a taut wire in high winds: It is referred to then as "strumming".

~ $1{,}000 < Re < O(10^5)$ (D)



Laminar boundary layer, wide turbulent wake

As Re increases from case B ($O(10)$) to $O(10^3)$ the point of separation on the lee side moves up/down, i.e. the area of low pressure on the lee side increases, but when $Re \approx 10^3$ separation has reached its limit (top/bottom of cylinder) and the entire rear side of the cylinder is in the wake. The flow within the wake is turbulent, and drag force is pressure dominated.

Since area affected by low pressure in the wake stays constant for $Re > 10^3$, and pressures both front and back, are scaled by ρV^2, it stands to reason that $C_D \approx$ CONSTANT for this range of Re.

Note: $C_D \approx$ constant, i.e. independent of Reynolds number, is analogous to Darcy-Weisbach's friction factor "f" for fully rough turbulent flow - but the reasons are not the same.

Up to $Re \approx O(10^5)$ the viscous flow near the upstream surface of the cylinder (the boundary layer) is laminar if surface is smooth.

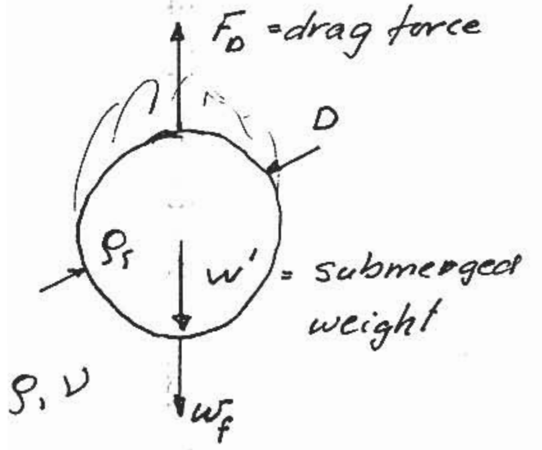
$Re \geq Re_{crit} \approx O(10^5)$ (E)

Turbulent boundary layer, narrow turbulent wake

When Re reaches a value of order 10^5 the boundary layer on the upstream surface turns turbulent. A turbulent flow is far more efficient in transporting momentum across streamlines than a laminar flow. Thus, when the flow encounters the adverse pressure gradient (after passing from front to rear of cylinder) high velocity fluid from further away from the surface is "injected" into the near surface flow and thereby delaying separation. Result is that when $Re = Re_{crit} = O(10^5)$ the low pressure wake region rather than the entire rear area becomes significantly reduced, i.e. the area over which the pressure difference between front and back act, "suddenly" decreases and results in a decrease in drag force reflected by the sudden drop in drag coefficient. As Re continues to increase the wake area increases and so does C_D.

Note: Onset of turbulence in upstream boundary layer is affected by surface roughness. Thus, the sudden drop in drag force and C_D is a function of ε/D. Same effect is seen for spheres and this explains why golf balls have a "pitted" surface (less drag $\Rightarrow$ longer drive!).

Fall (or Settling) Velocity of a Spherical Particle



When steady state has been reached force equilibrium gives

W' = submerged weight = $(\rho_s - \rho) g \frac{\pi}{6} D^3$ = drag force = F_D = $\frac{1}{2} \rho C_D \frac{\pi}{4} D^2 w_f^2$

w_f = fall velocity of sphere ($\rho_s > \rho$ is assumed)

$$w_f = \sqrt{(\rho_s/\rho - 1) g D} \sqrt{\frac{4}{3 C_D}}$$

but $C_D = C_D \left(Re_D = \frac{w_f D}{\nu}\right)$ i.e. a function of w_f. So, iteration is required!
Guess $C_D = C_D^{(1)}$ (~ 0.4 appears reasonable $10^3 < Re_D < 10^5$)
get $w_f^{(1)} \Rightarrow$ Then $Re_D^{(1)} = w_f^{(1)} D/\nu$ to get $C_D^{(2)} = C_D^{(2)}(Re_D^{(1)})$
and improved $w_f = w_f^{(2)}$ etc. etc.

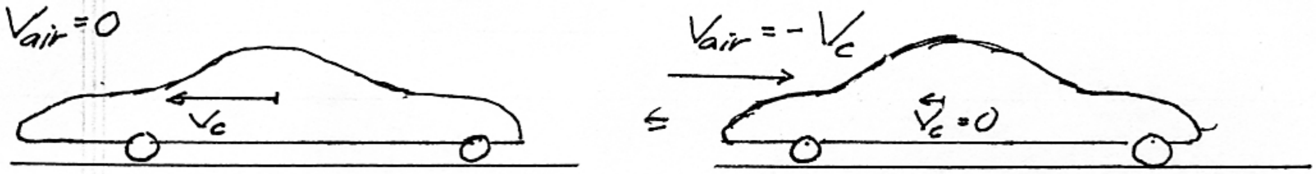
However, if $Re_D < 1$, we have $C_D = \frac{24}{Re_D} = \frac{24}{\frac{w_f D}{\nu}}$
and we can obtain an explicit solution:

STOKES LAW

$$w_f = \frac{(\rho_s/\rho - 1) g D^2}{18 \nu} \qquad \left(Re_D = \frac{w_f D}{\nu} \leq 1\right)$$

Fall velocity is used to evaluate the time for suspended solids to settle to the bottom in waste water treatment, e.g. fluid w. suspended solids into settling tank - if long enough residence time in settling tank - fluid only out!

Implications of Drag Resistance



Relative velocity between air and car is what counts.

V_c = speed of car = 90 mph [no cops around] ≈ 40 m/s

C_D = 0.29 [Acura Legend – I wanted to be in a Legend in my own time!]

$A_\perp$ = frontal area = $1.3 \times 1.5 = 2\ m^2$

$\rho = \rho_a$ = air density = $1.2\ kg/m^3$

F_D = Drag Force = $\frac{1}{2} \rho_a C_D A_\perp V_c^2 = 560\ N$

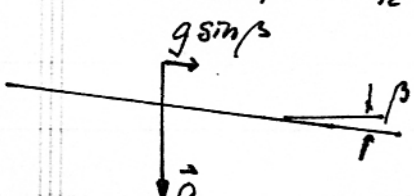
$F_D V_c$ = Rate of Work done to overcome air resistance =

$560 \cdot 40 = 22.3 \cdot 10^3$ (Nm/s = Watts) ≅ 30 HP!

So, who needs 200 HP or more?

F_D = force due to gravity on a sloping road of slope $S_R = \sin\beta = (\text{Mass} \cdot \text{Gravity})^{-1} F_D$

Mass ≈ 10^3 kg



$$\sin\beta = \frac{F_D}{1{,}000 \cdot 9.8} = 0.057 = 1/17.5$$

We need 200 HP to get up steep hills and to accelerate when going uphill.

0-60 mph in 6 sec ⇒

$$E_{kin} = \frac{1}{2} \text{mass} \cdot V^2 \Rightarrow \frac{DE_{kin}}{Dt} = \underline{\text{mass} \cdot V \frac{dV}{dt} = \text{Power req.}} > 175\ HP!$$