1.060 ENGINEERING MECHANICS II

Cheat-Sheet No: 1

Pressure

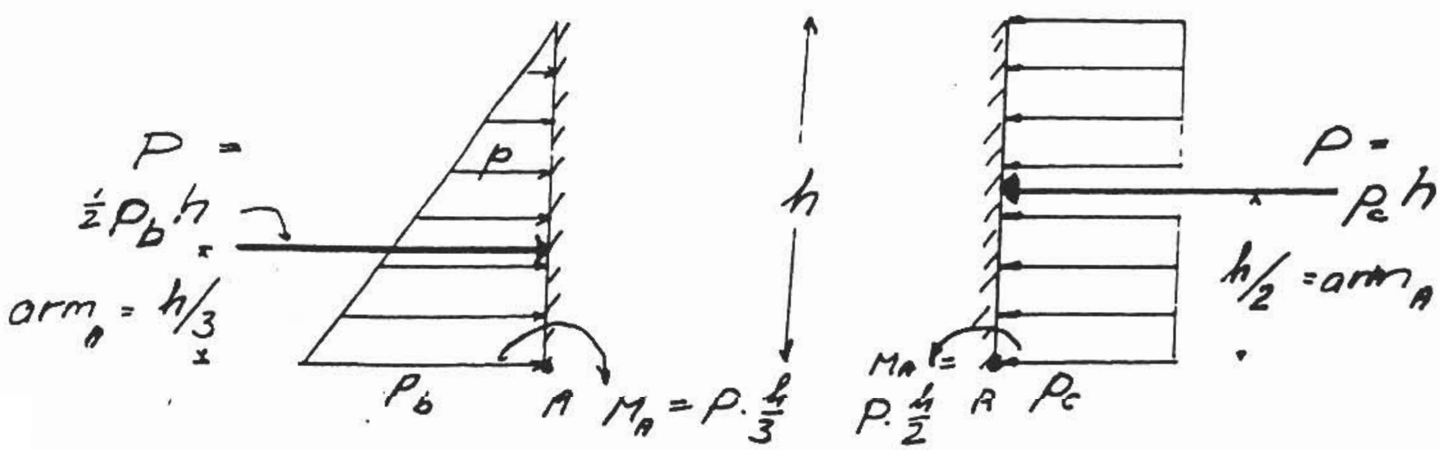
Is Isotropic – same in all directions
Is always perpendicular to surface upon which it acts

Hydrostatics

$$p + \rho g z = \text{CONSTANT} = p_o + \rho g z_o$$

Valid within fluid of constant density, at rest. Constant determined by knowledge of pressure p $(= p_o)$ at reference elevation $z = z_o$.

Hydrostatic Forces



Horizontal force on surface = Horizontal force on projection of surface onto vertical plane

Vertical force on surface = Weight of fluid "above" surface (or its translation horizontally to a location where there is fluid "above")

Moments

$$M = \text{Force} \times \text{Arm}$$

Bernoulli

Along a Streamline, s

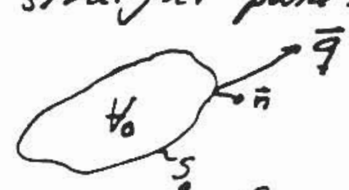
$$\frac{1}{2}\rho V_s^2 + p_s + \rho g z_s = \text{CONSTANT}$$

for steady flow ($\partial/\partial t = 0$). Constant obtained from knowledge of V_s, p_s at reference point of streamline $z = z_s$.

Perpendicular to Streamline, n

$$p_n + \rho g z_n = -\rho \int_{n_0}^{n_1} (V_s^2/R)\, dn + \text{CONSTANT}$$

for steady flow with $n \perp s$ and pointing towards center of curvature of s. R = radius of curvature of streamline. If $R \to \infty \Rightarrow$ straight streamlines $\Rightarrow$ pressure varies hydrostatically normal to straight parallel streamlines.



Conservation of Mass

$$\frac{\partial}{\partial t}\int_{\forall_0} \rho\, d\forall = -\int_{S_0} \rho(\bar{q}\cdot\vec{n})\, dS_0 = \int_{S_{in}} \rho q_\perp\, dS_{in} - \int_{S_{out}} \rho q_\perp\, dS_{out}$$

If ρ = constant over flow areas:

$$\int_A \rho q_\perp\, dA = \rho \int_A q_\perp\, dA = \rho Q = \rho \bar{V} A$$

Q = Discharge, $\bar{V}$ = average velocity = Q/A

If fluid incompressible: Volume Conservation

$$\frac{\partial \forall}{\partial t} = \sum Q_{in} - \sum Q_{out}$$

$\forall$ = volume of fluid within chosen boundaries of fixed control volume

Geometry

Area of circle : πr^2 ; Circumference = $2\pi r$

Volume of sphere : $\frac{4\pi}{3} r^3$; surface area = $4\pi r^2$

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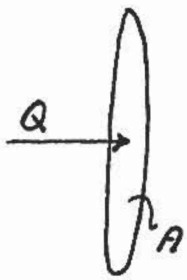
CHEAT-SHEET NO: 2

MOMENTUM

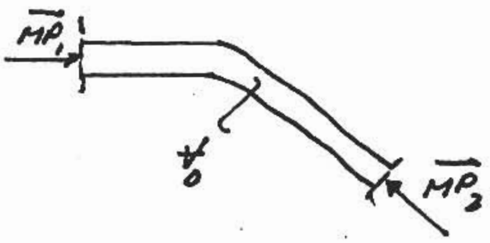
$$Q = VA \quad ; \quad V = Q/A$$



Wellbehaved flow: Streamlines straight & parallel & $\perp A$

$\overrightarrow{MP} = (\rho V^2 + p_{CG})A$, $\perp A$ towards control $\forall_0$

p_{CG} = pressure @ center of gravity of A



Equilibrium of forces (steady flow):
$\overrightarrow{MP_1} + \overrightarrow{MP_2}$ + (sum of all other forces on fluid within $\forall_0$) = 0

"Other forces": Shear forces & pressure forces on boundaries of $\forall_0$, gravity, drag forces on objects in $\forall_0$

DRAG FORCE: $F_D = \frac{1}{2}\rho C_D A_\perp V^2$

$A_\perp$ = area of body's projection on plane $\perp V$
C_D = Drag coefficient = $C_D(Re)$

BERNOULLI

$$H = \text{Total Head} = \frac{V^2}{2g} + \frac{p_{CG}}{\rho g} + z_{CG}$$

Piezometric Head = $\frac{p_{CG}}{\rho g} + z_{CG}$; Velocity Head = $\frac{V^2}{2g}$

EGL = Energy Grade Line : $z_{EGL} = H$
HGL = Hydraulic Grade Line : $z_{HGL} = H - \frac{V^2}{2g}$

Flow from 1 to 2 with wellbehaved flow @ 1 & 2

$H_1 = H_2 + \Delta H$; ΔH = headloss between 1 & 2

HEAD LOSSES

$\Delta H \approx 0$ if Short transition with Converging Flow

Pipe Friction Losses

$$\Delta H_f = f \frac{l}{D} \frac{V^2}{2g} \quad \left(D = 4 \frac{A}{P} = 4 \frac{\text{Area}}{\text{Perimeter}} = 4 \text{ Hyd. Radius} \right)$$

$$f = f\left(\frac{VD}{\nu}, \frac{\varepsilon}{D} \right) \text{ from Moody}$$

ε = pipe roughness
ν = kin. viscosity of fluid

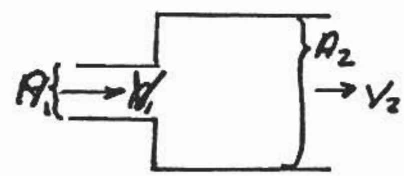
· Wall Shear Stress: $\tau_s = \frac{f}{8} \rho V^2$

Minor Losses

$$\Delta H_m = K_L \frac{V^2}{2g}$$

K_L = Minor Loss Coefficient

Expansion Loss: $\Delta H_{exp} = \dfrac{(V_1 - V_2)^2}{2g}$



Exit loss ($A_2 \gg A_1$): $K_{L,exit} = 1$

Entry loss (sharp edged orifice): $K_{L,ent} = \left(\frac{1}{C_c} - 1 \right)^2$

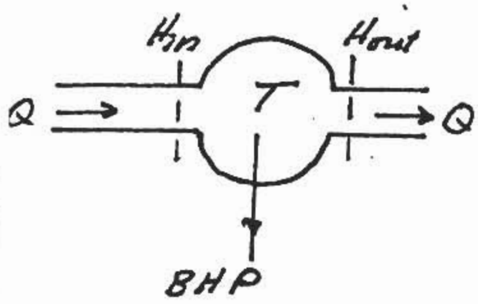
C_c = Contraction Coefficient [$C_c \approx 0.6 \rightarrow$, $C_c \approx 0.5 \rightarrow$]

ENERGY

$\dot{E}$ = rate of flow of Mech. Energy $= \rho g Q H$

$\rho g Q (H_1 - H_2)$ = rate of dissipation of Mech. Energy between ① & ② [power loss] = rate of production of internal energy

PUMPS & TURBINES



$BHP = \eta \rho g Q [H_{in} - H_{out}]$ [Turbines]

η = efficiency ≤ 1

For pump flow of energy is reversed

$\eta \, BHP = \rho g Q [H_{out} - H_{in}]$ [Pumps]

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Cheat Sheet for Test No: 3

UNIFORM FLOW:

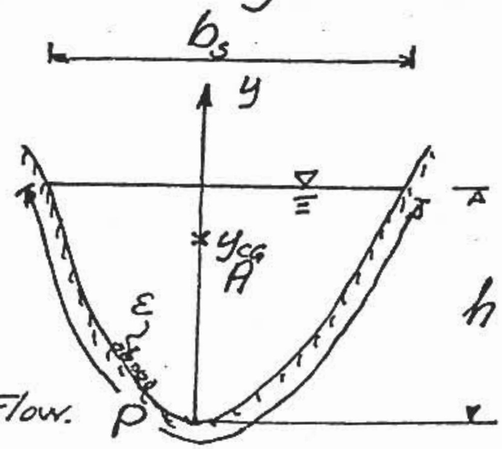
$$V = \sqrt{\frac{8g}{f}}\, R_h^{1/2} S_o^{1/2} = C\, R_h^{1/2} S_o^{1/2} = \frac{1}{n} R_h^{2/3} S_o^{1/2} = \frac{Q}{A}$$

Darcy-Weisbach, Chezy, Manning

S_o = channel slope

R_h = hydraulic Radius $= \frac{A}{P}$

$$Fr^2 = \frac{Q^2 b_s}{g A^3} = \frac{V^2}{g(A/b_s)} = \frac{V^2}{g h_m}$$



S_f = slope of EGL = S_o ⇒ Uniform Flow.

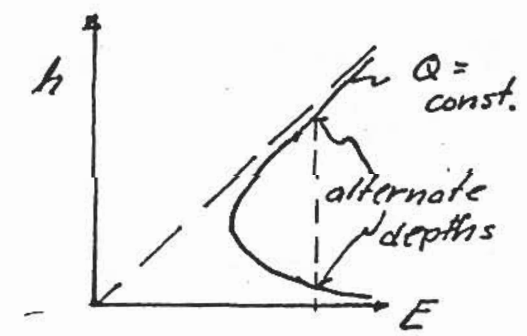
n = Manning's $n = 0.038\, \varepsilon^{1/6}$ (SI-units)

ENERGY PRINCIPLE

$$H = \frac{V^2}{2g} + h + z_o$$

E = Specific Energy $= H - z_o$

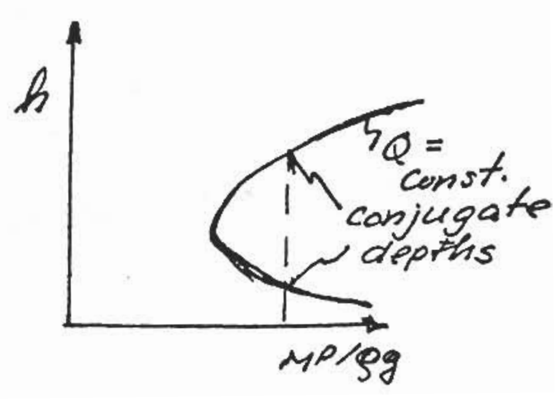
$$E = \frac{Q^2}{2gA^2} + h$$



MOMENTUM PRINCIPLE

$$MP/(\rho g) = \frac{Q^2}{gA} + (h - y_{CG})A$$

y_{CG} : y-value of centroid of A



(OVER)

HYDRAULIC JUMP (UNASSISTED) IN RECTANGULAR CHANNEL

$Fr_1 > 1$; $Fr_2 < 1$

$$\frac{h_2}{h_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right); \quad \frac{h_1}{h_2} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_2^2}\right)$$

$$\Delta H_{jump} = H_1 - H_2 = E_1 - E_2 = \left(\frac{V_1^2}{2g} + h_1\right) - \left(\frac{V_2^2}{2g} + h_2\right) = \frac{(h_2 - h_1)^3}{4h_1 h_2}$$

GRADUALLY VARIED FLOW

$$\frac{dh}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

$S_o = S_f$ for uniform flow $\Rightarrow$ Normal Depth

S_f replaces S_o in formulas for V

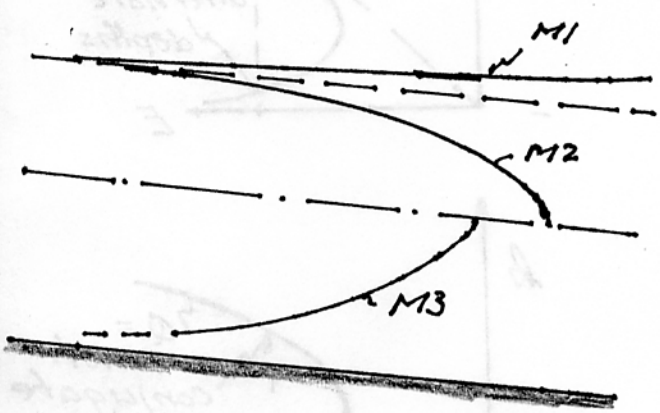
$Fr^2 = 1 \Rightarrow$ Critical Depth

$$S_f = \frac{f}{8g}\frac{Q^2}{A^3/P} = \frac{1}{C^2}\frac{Q^2}{A^3/P} = \frac{n^2 Q^2}{A^{10/3}/P^{4/3}} = \frac{\tau_s}{\rho g\, A/P}$$

Darcy-Weisbach Chezy Manning

GRADUALLY VARIED FLOW PROFILES

Mild Slope



Steep Slope