

1.050 Engineering Mechanics I

Lecture 26

Beam elasticity – how to sketch the solution
 Another example
 Transversal shear in beams

Handout

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1.050 – Content overview

I. Dimensional analysis

1. On monsters, mice and mushrooms
2. Similarity relations: Important engineering tools

Lectures 1-3
Sept.

II. Stresses and strength

3. Stresses and equilibrium
4. Strength models (how to design structures, foundations.. against mechanical failure)

Lectures 4-15
Sept./Oct.

III. Deformation and strain

5. How strain gages work?
6. How to measure deformation in a 3D structure/material?

Lectures 16-19
Oct.

IV. Elasticity

7. Elasticity model – link stresses and deformation
8. Variational methods in elasticity

Lectures 20-31
Oct./Nov.

V. How things fail – and how to avoid it

9. Elastic instabilities
10. Plasticity (permanent deformation)
11. Fracture mechanics

Lectures 32-37
Dec.

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1.050 – Content overview

I. Dimensional analysis

II. Stresses and strength

III. Deformation and strain

IV. Elasticity

- Lecture 20: Introduction to elasticity (thermodynamics)
- Lecture 21: Generalization to 3D continuum elasticity
- Lecture 22: Special case: isotropic elasticity
- Lecture 23: Applications and examples
- Lecture 24: Beam elasticity
- Lecture 25: Applications and examples (beam elasticity)
- Lecture 26: ... cont'd and closure
- ...

V. How things fail – and how to avoid it

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Drawing approach

- Start from $f_z = EI\xi_z''''$, then work your way up...

- Note sign changes:

$$\begin{aligned} \xi_z'''' &\sim f_z && \text{+} \rightarrow \text{-} \\ \xi_z''' &\sim -Q_z && \\ \xi_z'' &\sim -M_y && \\ \xi_z' &\sim -\omega_y && \text{-} \rightarrow \text{+} \\ \xi_z &\sim \xi_z && \end{aligned}$$

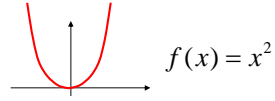
- At each level of derivative, first plot extreme cases at ends of beam
- Then consider zeros of higher derivatives; determine points of local min/max
- ξ_z represents physical shape of the beam ("beam line")

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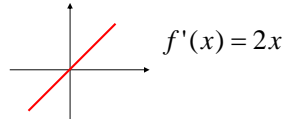
Review: Finding min/max of functions

$f(x)$ function of x

Example



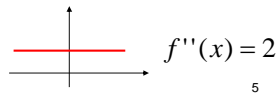
$f'(x) = 0$ necessary condition for min/max



$f''(x) < 0$ local **maximum**

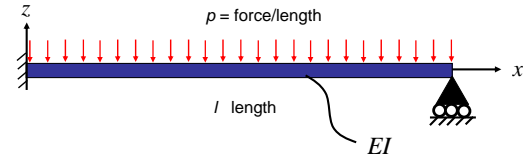
$f''(x) > 0$ local **minimum**

$f''(x) = 0$ **inflection point**



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Example solved in lecture 25:



$$Q_z(x) = p \left(x - \frac{5}{8}l \right)$$

$$\omega_y(x) = \frac{p}{EI} \left(\frac{1}{8}l^2x + \frac{x^3}{6} - \frac{5}{16}lx^2 \right)$$

$$M_y(x) = p \left(\frac{1}{8}l^2 + \frac{x^2}{2} - \frac{5}{8}lx \right)$$

$$\xi_z(x) = -\frac{p}{EI} \left(\frac{1}{16}l^2x^2 + \frac{x^4}{24} - \frac{5}{48}lx^3 \right)$$

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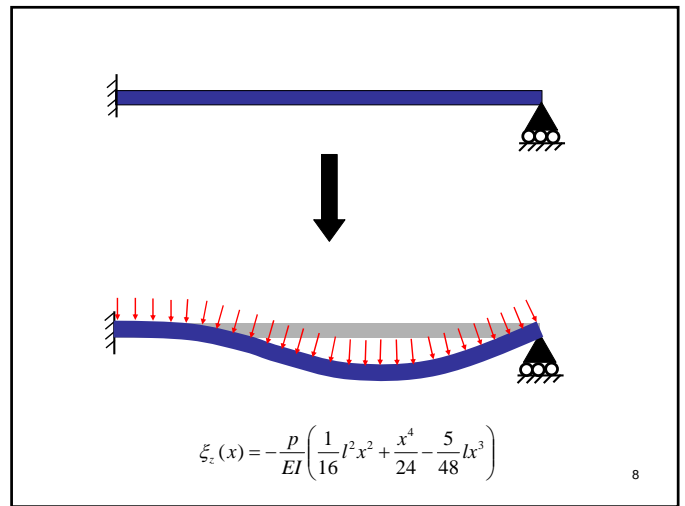
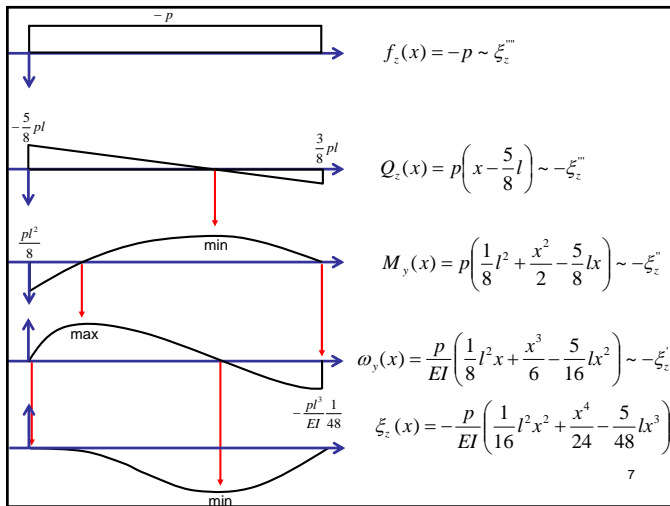


Illustration of various BCs

Free end

$$\begin{aligned} \bar{F} &= 0 \\ \bar{M} &= 0 \end{aligned}$$




$$\begin{aligned} \xi_z &= 0 \\ M_y &= 0 \end{aligned}$$

Concentrated force



$$Q_z = -P$$




$$\begin{aligned} \xi_x &= 0 \\ \omega_y &= 0 \end{aligned}$$

Hinge (bending)

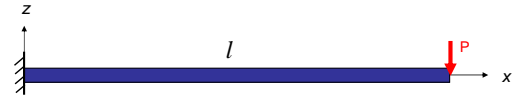


$$M_y = 0$$



$$\begin{aligned} \bar{\xi} &= 0 \\ \omega_y &= 0 \end{aligned}$$

Example with point load



Step 1: BCs

$$x=0 \begin{cases} \xi_z(0) = 0 \\ \omega_y(0) = 0 \end{cases} \quad x=l \begin{cases} Q_z(l) = -P \\ M_y(l) = 0 \end{cases}$$

Step 2: Governing equation

$$\frac{d^4 \xi_z}{dx^4} = 0$$

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Example with point load (cont'd)

Step 3: Integrate

$$\begin{cases} \xi_z'''' = 0, \xi_z''' = C_1 = -\frac{Q_z}{EI} \\ \xi_z'' = C_1 x + C_2 = -\frac{M_y}{EI} \\ \xi_z' = C_1 \frac{x^2}{2} + C_2 x + C_3 = -\omega_y \\ \xi_z = C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4 \end{cases}$$

Step 4: Determine integration constants by applying BCs

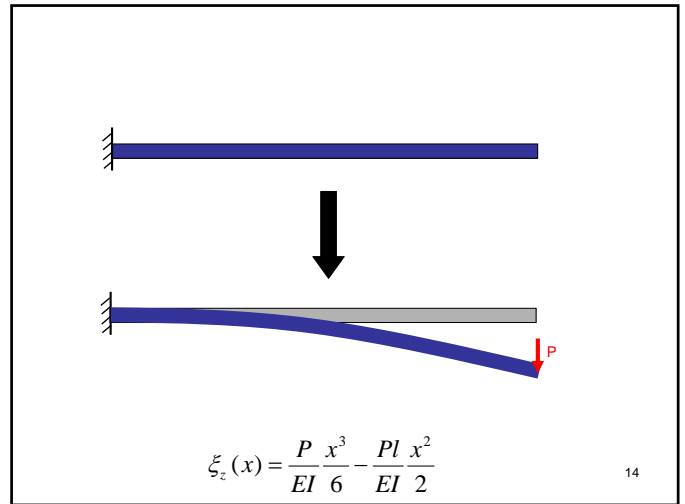
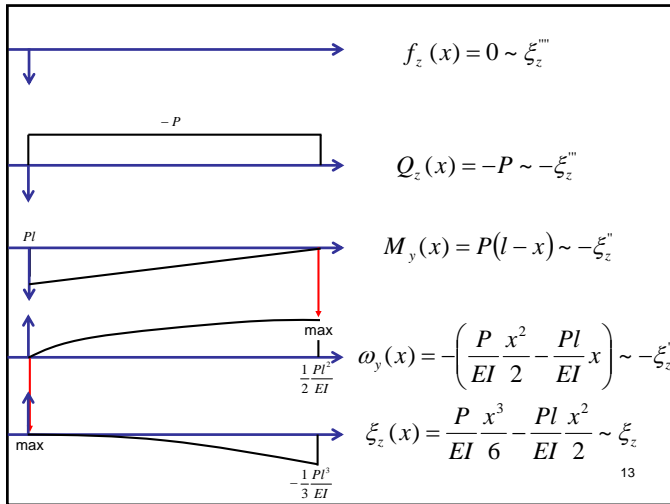
$$\begin{cases} \xi_z(0) = 0 \rightarrow C_4 = 0 & \omega_y = -\xi_z'(0) = 0 \rightarrow C_3 = 0 \\ M_y(l) = EI \left(\frac{P}{EI} l + C_2 \right) = 0 \rightarrow C_2 = -\frac{Pl}{EI} \\ Q_z(l) = -C_1 EI = -P \rightarrow C_1 = \frac{P}{EI} \end{cases}$$

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Example with point load (cont'd)

$$\begin{cases} f_z = 0 \\ Q_z = -P \\ M_y = P(l-x) \\ \omega_y = -\left(\frac{P}{EI} \frac{x^2}{2} - \frac{Pl}{EI} x \right) \\ \xi_z = \frac{P}{EI} \frac{x^3}{6} - \frac{Pl}{EI} \frac{x^2}{2} \end{cases}$$

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Plotting stress distribution in beam's cross-section

Given: Section quantities known as a function of position x

Want: Calculate stress distribution in the section

$$\sigma_{xx} = E(\varepsilon_{xx}^0 + \vartheta_y z)$$

with:
$$\begin{cases} N = ES\varepsilon_{xx}^0 \\ M_y = EI\vartheta_y \end{cases}$$

$$\sigma_{xx}(z; x) = E\left(\frac{N(x)}{ES} + \frac{M_y(x)}{EI} z\right) = \frac{N(x)}{S} + \frac{M_y(x)}{I} z$$

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Example: Plotting stress distribution in beam's cross-section

Fixed x :

$N > 0, M_y > 0 \quad \sigma_{xx}(z) = \frac{N}{S} + \frac{M_y}{I} z$

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