1.050 Engineering Mechanics I

Lecture 26

Beam elasticity – how to sketch the solution

Another example

Transversal shear in beams

Handout

1

1.050 – Content overview

2

1.050 – Content overview

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Drawing approach

- Start from $f_z = EI\xi_z''''$, then work your way up…
- Note sign changes:

$$\xi_z'''' \sim f_z$$
$$\xi_z''' \sim -Q_z \qquad + \rightarrow -$$
$$\xi_z'' \sim -M_y$$
$$\xi_z' \sim -\omega_y$$
$$\xi_z \sim \xi_z \qquad - \rightarrow +$$

- At each level of derivative, first plot extreme cases at ends of beam
- Then consider zeros of higher derivatives; determine points of local min/max
- ξ_z represents physical shape of the beam ("beam line")

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Review: Finding min/max of functions

$f(x)$ function of x

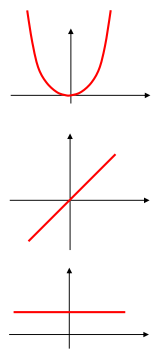
$f'(x)=0$ necessary condition for min/max

$f''(x)<0$ local **maximum**

$f''(x)>0$ local **minimum**

$f''(x)=0$ **inflection point**

Example



$f(x)=x^2$

$f'(x)=2x$

$f''(x)=2$

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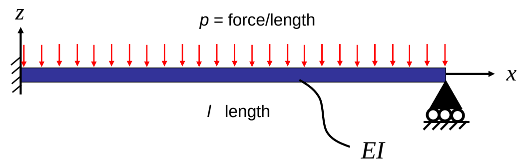
Example solved in lecture 25:



$$Q_z(x)=p\left(x-\frac{5}{8}l\right)$$

$$M_y(x)=p\left(\frac{1}{8}l^2+\frac{x^2}{2}-\frac{5}{8}lx\right)$$

$$\omega_y(x)=\frac{p}{EI}\left(\frac{1}{8}l^2x+\frac{x^3}{6}-\frac{5}{16}lx^2\right)$$

$$\xi_z(x)=-\frac{p}{EI}\left(\frac{1}{16}l^2x^2+\frac{x^4}{24}-\frac{5}{48}lx^3\right)$$

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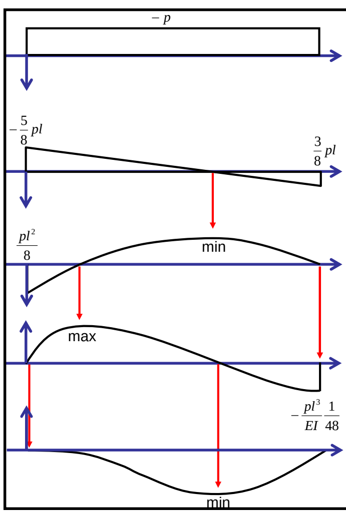

$$f_z(x)=-p\sim\xi_z''''$$

$$Q_z(x)=p\left(x-\frac{5}{8}l\right)\sim-\xi_z'''$$

$$M_y(x)=p\left(\frac{1}{8}l^2+\frac{x^2}{2}-\frac{5}{8}lx\right)\sim-\xi_z''$$

$$\omega_y(x)=\frac{p}{EI}\left(\frac{1}{8}l^2x+\frac{x^3}{6}-\frac{5}{16}lx^2\right)\sim-\xi_z'$$

$$\xi_z(x)=-\frac{p}{EI}\left(\frac{1}{16}l^2x^2+\frac{x^4}{24}-\frac{5}{48}lx^3\right)$$

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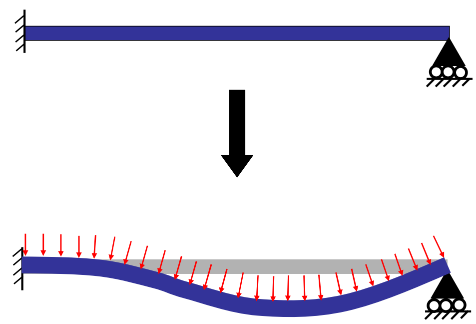

$$\xi_z(x)=-\frac{p}{EI}\left(\frac{1}{16}l^2x^2+\frac{x^4}{24}-\frac{5}{48}lx^3\right)$$

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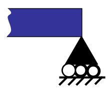
Illustration of various BCs

Free end

$\vec{F} = 0$

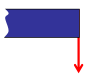
$\vec{M} = 0$



$\xi_z = 0$

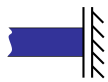
$M_y = 0$

Concentrated force



$Q_z = -P$

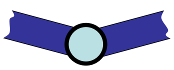
P



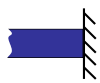
$\xi_x = 0$

$\omega_y = 0$

Hinge (bending)



$M_y = 0$



$\vec{\xi} = 0$

$\omega_y = 0$

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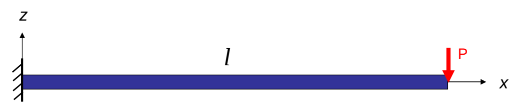
Example with point load



Step 1: BCs

$x = 0$ $\begin{cases} \xi_z(0) = 0 \\ \omega_y(0) = 0 \end{cases}$ $x = l$ $\begin{cases} Q_z(l) = -P \\ M_y(l) = 0 \end{cases}$

Step 2: Governing equation $\dfrac{d^4\xi_z}{dx^4} = 0$

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Example with point load (cont'd)

Step 3: Integrate

$$\begin{cases} \xi_z'''' = 0, \xi_z''' = C_1 = -\dfrac{Q_z}{EI} \\ \xi_z'' = C_1 x + C_2 = -\dfrac{M_y}{EI} \\ \xi_z' = C_1 \dfrac{x^2}{2} + C_2 x + C_3 = -\omega_y \\ \xi_z = C_1 \dfrac{x^3}{6} + C_2 \dfrac{x^2}{2} + C_3 x + C_4 \end{cases}$$

Step 4: Determine integration constants by applying BCs

$$\begin{cases} \xi_z(0) = 0 \rightarrow C_4 = 0 \qquad \omega_y = -\xi_z'(0) = 0 \rightarrow C_3 = 0 \\ M_y(l) = EI(\dfrac{P}{EI} l + C_2) = 0 \rightarrow C_2 = -\dfrac{Pl}{EI} \\ Q_z(l) = -C_1 EI = -P \rightarrow C_1 = \dfrac{P}{EI} \end{cases}$$

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Example with point load (cont'd)

$$\begin{cases} f_z = 0 \\ Q_z = -P \\ M_y = P(l - x) \\ \omega_y = -\left(\dfrac{P}{EI} \dfrac{x^2}{2} - \dfrac{Pl}{EI} x \right) \\ \xi_z = \dfrac{P}{EI} \dfrac{x^3}{6} - \dfrac{Pl}{EI} \dfrac{x^2}{2} \end{cases}$$

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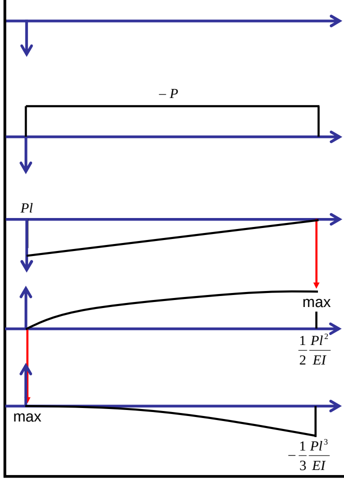

$$f_z(x) = 0 \sim \xi_z''''$$

$$Q_z(x) = -P \sim -\xi_z'''$$

$$M_y(x) = P(l - x) \sim -\xi_z''$$

$$\omega_y(x) = -\left(\frac{P}{EI}\frac{x^2}{2} - \frac{Pl}{EI}x\right) \sim -\xi_z'$$

$$\frac{1}{2}\frac{Pl^2}{EI}$$

$$\xi_z(x) = \frac{P}{EI}\frac{x^3}{6} - \frac{Pl}{EI}\frac{x^2}{2} \sim \xi_z$$

$$-\frac{1}{3}\frac{Pl^3}{EI}$$

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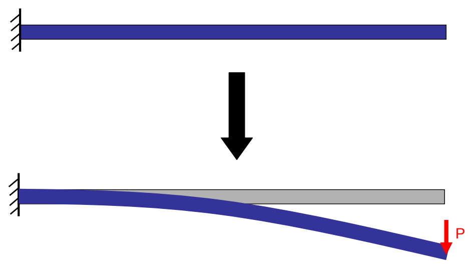

$$\xi_z(x) = \frac{P}{EI}\frac{x^3}{6} - \frac{Pl}{EI}\frac{x^2}{2}$$

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Plotting stress distribution in beam's cross-section

Given: Section quantities known as a function of position *x*

Want: Calculate stress distribution in the section

$$\sigma_{xx} = E\left(\varepsilon_{xx}^0 + \vartheta_y z\right)$$

with:
$$\begin{cases} N = ES\varepsilon_{xx}^0 \\ M_y = EI\vartheta_y \end{cases}$$

$$\sigma_{xx}(z;x) = E\left(\frac{N(x)}{ES} + \frac{M_y(x)}{EI}z\right) = \frac{N(x)}{S} + \frac{M_y(x)}{I}z$$

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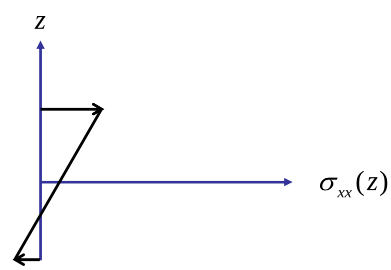
Example: Plotting stress distribution in beam's cross-section

Fixed *x*:



$$N > 0, M_y > 0 \qquad \sigma_{xx}(z) = \frac{N}{S} + \frac{M_y}{I}z$$

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