1.022 Introduction to Network Models

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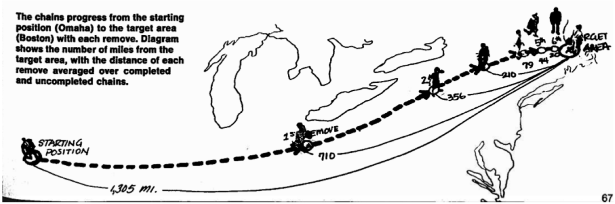
Lecture 11

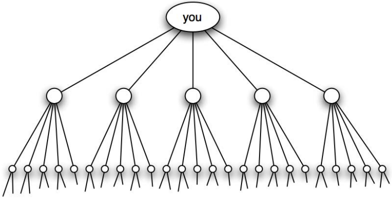
Small-world phenomenon

- Stanley Milgram's experiment $\Rightarrow$ six degrees of separation

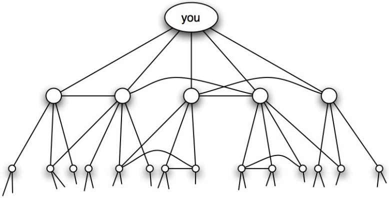


- Get letter from 'starter' to target by forwarding to acquaintances
  - $\Rightarrow$ Letters arrive with a median of six steps
- Two rather surprising facts
  - $\Rightarrow$ 1) Short paths between two nodes exist in abundance
  - $\Rightarrow$ 2) People without global knowledge can find these paths

- Pure exponential growth produces a small world



- Triadic closure reduces the growth rate

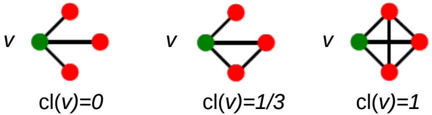


- Can a model exhibit both many closed triads and very short paths?

- Q: What fraction of v's neighbors are themselves connected?
- The clustering coefficient $\text{cl}(v)$ of $v \in V$ is

$$\text{cl}(v) = \frac{2|E_v|}{d_v(d_v - 1)} \in [0, 1]$$

$\Rightarrow$ $|E_v|$ is the number of edges among v's neighbors



- An indication of the extent to which edges 'cluster'
- The global (average) clustering coefficient is

$$\text{cl}(G) = \frac{1}{|V|} \sum_{v \in V} \text{cl}(v)$$

Do we really need another model for this?

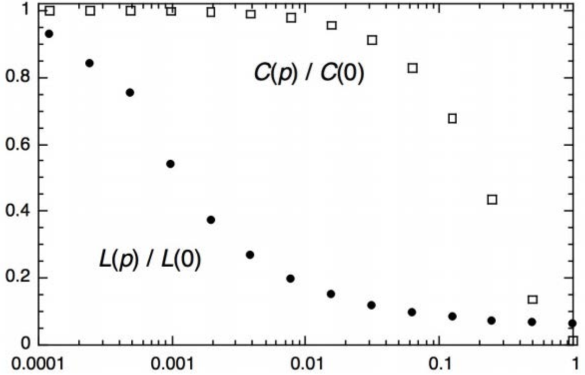
Network	size	av. shortest path	Shortest path in fitted random graph	Clustering (averaged over vertices)	Clustering in random graph
Film actors	225,226	3.65	2.99	0.79	0.00027
MEDLINE co-authorship	1,520,251	4.6	4.91	0.56	1.8×10^{-4}
E.Coli substrate graph	282	2.9	3.04	0.32	0.026
C.Elegans	282	2.65	2.25	0.28	0.05

Watts-Strogatz small-world model

- Reconciling short paths and high clustering coefficients
- Desired number of nodes n, average degree K and probability p
  - $\Rightarrow$ 1) Construct a circle of n nodes
  - $\Rightarrow$ 2) Connect each node to its K closest neighbors
  - $\Rightarrow$ 3) With probability p rewire each edge uniformly

- Small p: regular lattice. Large p: close to ER graph
  - $\Rightarrow$ There is a sweet spot in between

- Plot clustering coefficient and average shortest path
  - $\Rightarrow$ As a function of the rewiring probability p
- What happens for $p \approx 0.01$?



Watts, Duncan J., and Steven H. Strogatz. "Collective dynamics of 'small-world' networks." *Nature* 393 (1998): 441–42.

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