INTRODUCTION TO CHEMOSTAT THEORY

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The chemostat, independently developed by Monod (1950) and Novick and Szilard (1950), is a continuous culture device used to describe the growth characteristics of nutrient-limited microorganisms under steady state conditions.

Consider a well-mixed vessel containing a culture of microorganisms. Let v be the volume of the culture and let N be the cell density. The flow rate, f, is the rate at which fresh medium is pumped into the culture and the rate at which the contents are removed, maintaining a constant volume, v, in the vessel. f/v is the dilution rate of the culture, D. All nutrients in the influent medium are in excess except one, the limiting nutrient, which enters at a concentration S_i. The concentration of the limiting nutrient in the vessel is S, and the cells have a specific growth rate r.

The rate of change of the cell density in the vessel can be written as

$$\frac{dN}{dt} = rN - DN \tag{1}$$

which is the growth rate of the cells minus the rate at which they are being washed out of the culture vessel. In a steady state, equation (1) is equal to zero, thus

$$rN = DN \tag{2}$$

and

$$r = D \tag{3}$$

That is, in steady state, the growth rate of the cells will be equal to the dilution rate of the culture. r and D have the unit time $^{-1}$, which is the specific growth rate of the cells (you can think of it as cells per cell per day.)

The rate of change of the concentration of the limiting nutrient in the vessel can be written

$$dS/dt = DS_i - DS - rNQ \tag{4}$$

where Q is the cell quota, or the amount of limiting nutrient per cell. In steady state, equation (4) equals zero, and

$$D(S_i - S) = rNQ \tag{5}$$

which, since D = r, reduces to

$$N = (S_i - S)/Q \tag{6}$$

Thus, cell density in the culture is a function of the cell quota and the concentration of the limiting nutrient in the culture vessel.

Monod (1950) showed that the growth of bacteria limited by a single substrate can be described by the equation

$$r = r_{max}\, S/ (K_S + S) \qquad (7)$$

where r_{max} is the maximum possible growth rate of a species and K_S is the substrate concentration at $r_{max}/2$. Since D = r, equation (7) can be rewritten as

$$S = K_S\, D/ (r_{max} - D) \qquad (8)$$

Combining equations (6) and (8) yields

$$N = [S_i - K_S\, D/r_{max} - D] / Q \qquad (9)$$

Thus, in a system where Q is a constant, if K_S, r_{max}, and Q are known for a given species, steady state values of N and S can be predicted for any dilution rate using equations (8) and (9).

$r = hr^{-1}$	specific growth rate of cells
N = cells ml^{-1}	cell density
S_i, S = g ml^{-1}	influent, effluent dissolved concentration of limiting nutrient
Q = g $cell^{-1}$	cell quota - amount of limiting nutrient in the cell
Y = 1/Q cells g^{-1}	yield coefficient
D = day^{-1} = f/v	dilution rate
f = ml day^{-1}	flow rate
V = ml	volume of culture

Novick, A, and L. Szilard, 1950. Experiments with the chemostat in spontaneous mutations of bacteria. *Proc. Nat. Acad. Sci*. 36:340-345.

Monod, 1950. La technique de culture continue theorie et applications. *Annls. Inst. Pasteur*, Paris,79:390-401.

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