1.010 Uncertainty in Engineering
Fall 2008

Brief Notes #1
Events and Their Probability

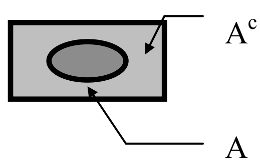
• Definitions

Experiment: a set of conditions under which some variable is observed
Outcome of an experiment: the result of the observation (a sample point)
Sample Space, S: collection of all possible outcomes (sample points) of an experiment
Event: a collection of sample points

• Operations with events

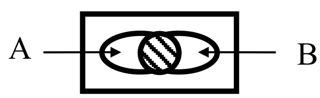
1. **Complementation**

A^c



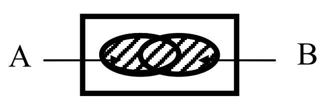
2. **Intersection**

$A \cap B$



3. **Union**

$A \cup B$



• Properties of events

1. Mutual Exclusiveness - intersection of events is the null set ($A_i \cap A_j = \varnothing$, for all $i \neq j$)
2. Collective Exhaustiveness (C.E.) - union of events is sample space ($A_1 \cup A_2 \cup ... \cup A_n = S$)
3. If the events $\{A_1, A_2, ... , A_n\}$ are both mutually exclusive and collectively exhaustive, they form a partition of the sample space, S.

• Probability of events

• Relative frequency f_E and limit of relative frequency F_E of an event E

$$f_E = \frac{n_E}{n}$$

$$F_E = \lim_{n \to \infty} f_E = \lim_{n \to \infty} \frac{n_E}{n}$$

• Properties of relative frequency (the same is true for the limit of relative frequency

1. $0 \le f_E \le 1$
2. $f_S = 1$
3. $f_{(A \cup B)} = f_A + f_B$ if A and B are mutually exclusive

- Properties/axioms of probability

1. $0 \le P(A) \le 1$
2. $P(S) = 1$
3. $P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive

- Two consequences of the axioms of probability theory

1. $P(A^c) = 1 - P(A)$
2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, for any two events A and B,
   $\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

- **Conditional Probability**

Definition:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Therefore, $P(A \cap B)$ can also be obtained as $P(A \cap B) = P(B)P(A|B) = P(A)\, P(B|A)$

- **Total Probability Theorem**

Let $\{B_1, B_2, ..., B_n\}$ be a set of mutually exclusive and collectively exhaustive events and let A be any other event. Then the marginal probability of A can be obtained as:

$$P(A) = \sum_i P(A \cap B_i) = \sum_i P(B_i) P(A \mid B_i)$$

- **Independent events**

A and B are independent if:
$P(A|B) = P(A)$, or equivalently if
$P(B|A) = P(B)$, or if
$P(A \cap B) = P(A)\, P(B)$

- **Bayes' Theorem**

$$P(A \mid B) = P(A)\frac{P(B \mid A)}{P(B)}$$

Using Total Probability Theorem, P(B) can be expressed in terms of P(A), $P(A^c) = 1 - P(A)$, and the conditional probabilities P(B|A) and $P(B|A^C)$:

$$P(B) = P(A)P(B \mid A) + P(A^C)P(B \mid A^C)$$

So Bayes' Theorem can be rewritten as:

$$P(A \mid B) = P(A)\frac{P(B \mid A)}{P(A)P(B \mid A) + P(A^C)P(B \mid A^C)}$$