1.00 Lecture 30

Matrices

Reading for next time: Numerical Recipes, pp. 37-42 (online)
http://www.nrbook.com/a/bookcpdf.php

Matrices

- Matrix is 2-D array of m rows by n columns

$$\begin{vmatrix} a_{00} & a_{01} & a_{02} & a_{03}\ldots & a_{0n-1} \\ a_{10} & a_{11} & a_{12} & a_{13}\ldots & a_{1n-1} \\ a_{20} & a_{21} & a_{22} & a_{23}\ldots & a_{2n-1} \\ \ldots & \ldots & \ldots & \ldots\ \ldots & \ldots \\ a_{m-1,0} & a_{m-1,1} & a_{m-1,2} & a_{m-1,3}\ldots & a_{m-1,n-1} \end{vmatrix}$$

  - In math notation, we use index 1, … m and 1, … n.
  - In Java, we usually use index 0, … m-1 and 0, …n-1
- They often represent a set of linear equations:

$$a_{00}x_0 + a_{01}x_1 + a_{02}x_2 + \ldots + a_{0,n-1}x_{n-1} = b_0$$
$$a_{10}x_0 + a_{11}x_1 + a_{12}x_2 + \ldots + a_{1,n-1}x_{n-1} = b_1$$
$$\ldots$$
$$a_{m-1,0}x_0 + a_{m-1,1}x_1 + a_{m-1,2}x_2 + \ldots + a_{m-1,n-1}x_{n-1} = b_{m-1}$$

  - n unknowns x are related by m equations
  - Coefficients a are known, as are right hand side b

Matrices, p.2

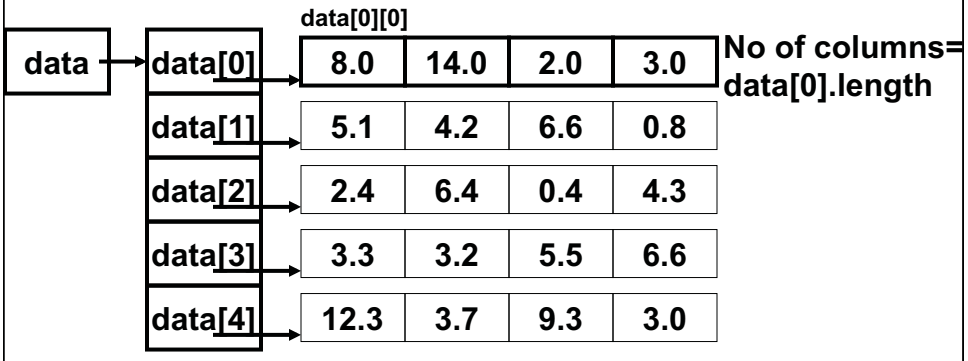
- In this lecture we cover basic matrix representation and manipulation
  - Used most often to prepare matrices for use in solving linear systems, which we cover in next lecture
- Java has 2-D arrays, declared as, for example

```
double[][] squareMatrix= new double[5][5];
```

  - But there are no built-in methods for them
- So, it's helpful to create a Matrix class:
  - Create methods to add, subtract, multiply, form identity matrix, etc.
  - Used for matrices with a few hundred rows or so
- Sparse matrices are handled differently:
  - Almost all large matrices (millions of rows or columns) are extremely sparse (99%+ of entries are zeros)
  - Store (i, j, value) in a list or 1-D array or other data structure

2-D Arrays

```
double[][] data= new double[5][4];
```



No. of rows=
data.length

A 2-D array is:
a reference to a 1-D array of references to 1-D arrays of data.
This is how we'll store the matrix data in class Matrix

Matrix class, p.1

```
public class Matrix {
    private double[][] data;      // Reference to array

    public Matrix(int m, int n) {
        data = new double[m][n];
    }

    public void setIdentity() {
        int nrows = data.length;
        int ncols = data[0].length;
        for (int i = 0; i < nrows; i++)
            for (int j = 0; j < ncols; j++)
                if (i == j)
                    data[i][j]= 1.0;
                else
                    data[i][j]= 0.0;
    }  // Should check that matrix is square
```

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Matrix class, p.2

```
public int getNumRows() {  return data.length; }

public int getNumCols() {  return data[0].length; }

public double getElement(int i, int j) { return data[i][j]; }

public void setElement(int i, int j, double val) {
    data[i][j] = val; }

public void incrElement(int i, int j, double incr) {
    data[i][j] += incr; }

public Matrix add(Matrix b) {
    Matrix result = null;
    int nrows = data.length;
    int ncols = data[0].length;
    if (nrows== b.data.length && ncols== b.data[0].length) {
        result = new Matrix(nrows, ncols);
        for (int i = 0; i < nrows; i++)
          for (int j = 0; j < ncols; j++)
            result.data[i][j]= data[i][j]+ b.data[i][j]; }
    return result;
}  // Objects of same class see each others' private data
```

$$\begin{vmatrix} 1 & 3 & 5 \\ 0 & 2 & 6 \\ 0 & 5 & 1 \end{vmatrix} + \begin{vmatrix} 3 & 2 & 0 \\ 0 & 4 & 3 \\ 2 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 4 & 5 & 5 \\ 0 & 6 & 9 \\ 2 & 8 & 2 \end{vmatrix}$$

Matrix class, p.3

$$\begin{vmatrix} 1 & 3 \\ 0 & 2 \\ 0 & 5 \end{vmatrix} \cdot \begin{vmatrix} 1 & 3 & 5 \\ 0 & 2 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 9 & 23 \\ 0 & 4 & 12 \\ 0 & 10 & 30 \end{vmatrix}$$

```
public Matrix mult(Matrix b) {
   Matrix result = null;
   int nrows = data.length;
   int ncols = data[0].length;
   if (ncols == b.data.length) {
       result = new Matrix(nrows, b.data[0].length);
       for (int i= 0; i < nrows; i++)
           for (int j=0; j < result.data[0].length; j++) {
               double t = 0.0;
               for (int k= 0; k < ncols; k++) {
                   t += data[i][k] * b.data[k][j];   }
               result.data[i][j]= t;  }   }
       return result;
   }
   public void print() {
       for (int i= 0; i < data.length; i++) {
           for (int j= 0; j < data[0].length; j++)
               System.out.print(data[i][j] + " ");
           System.out.println();  }
       System.out.println();
       }
   }
```

MatrixTest

```
public class MatrixTest {
   public static void main(String argv[]) {
       Matrix mat1 = new Matrix(3,3);
       Matrix mat2 = new Matrix(3,3);
       mat1.setIdentity();
       mat2.setIdentity();
       Matrix res;

       res = mat1.add(mat2);

       System.out.println("mat1:");
       mat1.print();
       System.out.println("mat2:");
       mat2.print();
       System.out.println("mat1 + mat2:" );
       res.print();

       // Similar code for multiplication

       // Add your exercise code here
   }
}
```

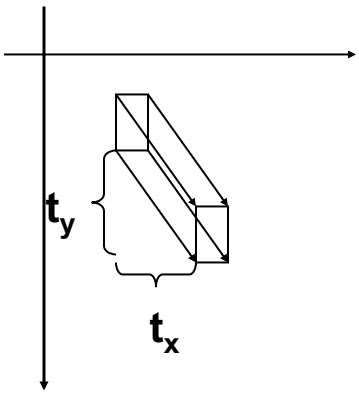
Exercise 1

- Download Matrix and MatrixTest from Web site
- Write an instance method multScalar() to multiply a matrix by a scalar (double) in class Matrix
- Invoke your method from MatrixTest main()
- Hints for writing multScalar()
  - Use add() as a rough guide
  - Find the number of rows and columns in the matrix
  - Create a new Matrix object to return as the result
  - Loop through all entries (nested `for` loops) to multiply by the scalar
  - Return the result
- Modify MatrixTest main() method:
  - Add a line to use the multScalar() method
  - Add another line to print the resulting matrix, using its print() method

Exercise 2 Introduction

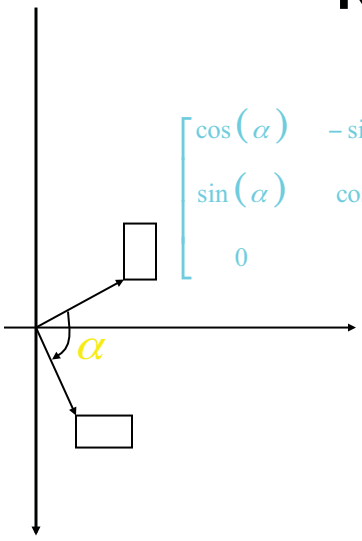
- Implement graphics transforms from last Swing lecture
- Instead of using Java's rotate() and scale() methods, you'll create matrices to represent rotation and scaling, multiply them, and apply them to a shape.
- With some perseverance, your matrix manipulations will yield the same result as Java's methods

Translation



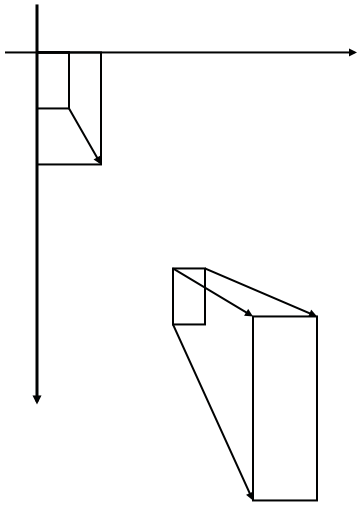
$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Rotation



$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos(\alpha) - y\sin(\alpha) \\ x\sin(\alpha) + y\cos(\alpha) \\ 1 \end{bmatrix}$$

Scaling



$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x * x \\ s_y * y \\ 1 \end{bmatrix}$$

Composing Transformations

- **Suppose we want to scale point (x, y) by 2 and then rotate by 90 degrees.**

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \right)$$

rotate **scale**

Composing Transformations, 2

Because matrix multiplication is associative, we can rewrite this as

$$\left(\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Exercise 2

- **Download TransformTest, TransformPanel**
  - **TransformPanel rotates (by 18 º) and scales (by 2) a rectangle using Java affine transforms**
  - **Run it to see the result. (Don't use this code for exercise.)**
- **Download TransformTest1, TransformPanel1**
  - **These are skeletons for doing the rotations and scaling through matrix multiplication yourself. You will:**
  - **Create two matrices (rotate, scale) with your Matrix class**
  - **Scale by 2 in the x and y directions and rotate by 18º (π/10)**
    - **Look at the scaling and rotation matrices in previous slides**
  - **Multiply the 2 matrices, save them in Matrix result. Order matters in general**
    - **Try it both ways here—it's simple enough to give same result**
  - **Pass the values as arguments to AffineTransform() as shown in TransformPanel1 code on the next slides**
  - **See if your AffineTransform produces the same result**

Exercise 2: TransformTest

```
import java.awt.*;
import javax.swing.*;

public class TransformTest {
    public static void main(String args[]) {
        JFrame frame = new JFrame("Rectangle transform");
        frame.setDefaultCloseOperation(JFrame.EXIT_ON_CLOSE);
        frame.setSize(500,500);
        Container contentPane= frame.getContentPane();
        TransformPanel panel = new TransformPanel();
        contentPane.add(panel, BorderLayout.CENTER);
        frame.setVisible(true);
    }
}
```

Exercise 2: TransformPanel

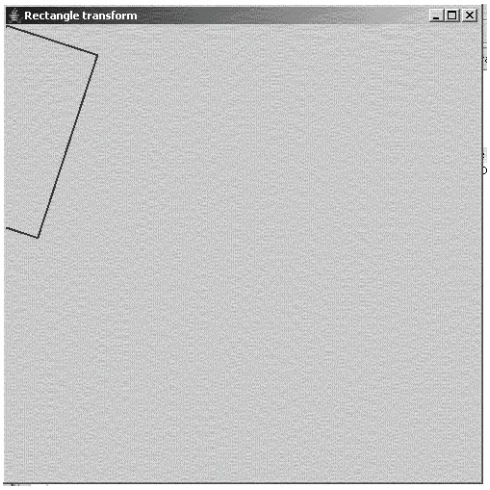
```
import javax.swing.*;
import java.awt.*;
import java.awt.geom.*;     // For 2D classes

public class TransformPanel extends JPanel {
    public void paintComponent(Graphics g) {
       super.paintComponent(g);
       Graphics2D g2= (Graphics2D) g;
       Rectangle2D rect= new Rectangle2D.Double(0, 0, 50, 100);

       g2.setPaint(Color.BLUE);
       AffineTransform baseXf = new AffineTransform();
       // Scale by 2 in x, y directions, then rotate by 18 degrees
       baseXf.rotate(Math.PI/10.0);
       baseXf.scale(2.0, 2.0);
       g2.transform(baseXf);
       g2.draw(rect);
    }
}
```

Replace these lines

Exercise 2: TransformPanel, p.2

```
public class TransformPanel1 extends JPanel {
  public void paintComponent(Graphics g) {
      // Same initial lines: superclass, cast g2, new rectangle
      g2.setPaint(Color.MAGENTA);
      Matrix s = new Matrix(3, 3);
      // Set its elements to scale rectangle by 2
      // Your code here
      s.print();
      Matrix r = new Matrix(3, 3);                // Rotate
      double a = Math.PI / 10;                    // 18 degree angle
      // Set elements to rotate 18 degrees; use Math.sin() and cos()
      // Your code here
      r.print();
      // Multiply r and s to get Matrix result
      // Your code here
      result.print();
      double m00 = result.getElement(0, 0);
      double m01 = result.getElement(0, 1);
      // Etc.  Coefficients inserted in COLUMN order. Done for you.
      AffineTransform baseXf =
             new AffineTransform(m00, m10, m01, m11, m02, m12);
      g2.transform(baseXf);         // Only 6 elements vary in xform
      g2.draw(rect);  }  }
```

Exercise 2 Desired Result



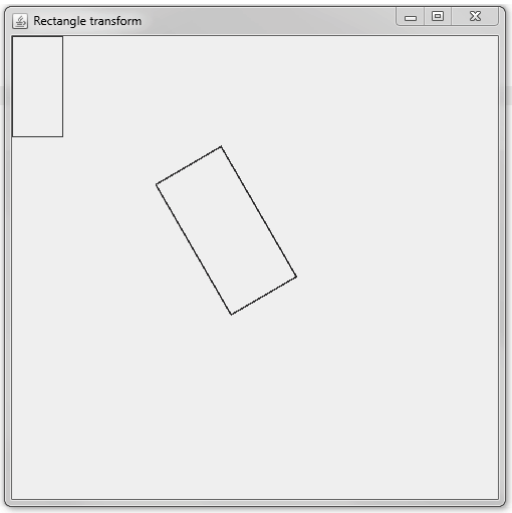
Exercise 3

- Modify TransfomPanel
  - Almost the same as exercise 4, lecture 21
  - Initially, rectangle is 50 by 100, at origin
  - Apply the following transforms:
    - Translate rectangle 50 pixels east, 200 pixels south
    - Scale by factor of 1.5, but leave upper left corner of rectangle in same position
    - Rotate by 30 degrees counterclockwise around the origin
      - Not around the upper left corner, as in lecture 21, which would require translating to origin and back again
      - Counterclockwise, not clockwise, to stay on the panel
  - Draw the original rectangle in red
  - Draw the transformed rectangle in blue
  - Remember to apply transforms by premultiplying by each one in order

Exercise 3 cont.

- Mechanics:
  - Copy your exercise 2 solution into class TransformPanel2
  - Copy TransformTest into TransformTest2
    - Have its main() create a TransformPanel2 object
  - Do not use AffineTransform methods rotate(), scale() or translate()
  - Create r, s and t matrices and multiply them appropriately to create a result matrix holding the combined transform
  - Use the first 6 coefficients of the result matrix (m00, m10, etc.) in the AffineTransform constructor, as in exercise 2

Exercise 3 output



Exercise 3 previous code

```
// Same imports as before: swing, awt, awt.geom

public class TransformPanel extends JPanel {
    public void paintComponent(Graphics g) {
       super.paintComponent(g);
       Graphics2D g2= (Graphics2D) g;
       Rectangle2D rect= new Rectangle2D.Double(0, 0, 50, 100);
       g2.setPaint(Color.RED);
       g2.draw(rect);
       g2.setPaint(Color.BLUE);
       AffineTransform baseXf = new AffineTransform();
       baseXf.rotate(-Math.PI/6.0);   // 3. Rotate 30° at origin
       baseXf.translate(50,200);      // 2. Move 50, 200 pixels
       baseXf.scale(1.5, 1.5);        // 1. Do scaling at origin
       g2.transform(baseXf);
       g2.draw(rect);
    }
}      // Rotation (step 3) different than earlier exercise:
       // Rotate counterclockwise, not clockwise, around origin
```

1.00 / 1.001 / 1.002 Introduction to Computers and Engineering Problem Solving
Spring 2012