5.73

Quiz 7 ANSWERS

1.

$$\int_{x_-(E)}^{x_+(E)} p_E(x')dx' = \frac{h}{2}(n+1/2)$$

$$p_E(x) = \left[2m\left(E - V(x)\right)\right]^{1/2}$$

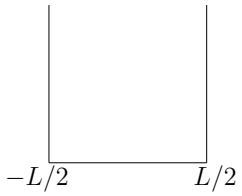
Even though the WKB quantization cannot be exact for potentials of the form

I. $V(x) = 0$ $\quad |x| \le L/2$
$V(x) = \infty$ $\quad |x| > L/2$

II. $V(x) = (2A/L)x$ $\quad |x| \le L/2$
$V(x) = \infty$ $\quad |x| > L/2$

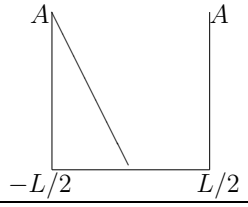
A. Evaluate the quantization integral for potentials I and II at E = A.

I

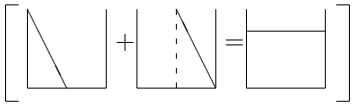


$$n = \frac{2}{h}\int_0^A [(E-V)2m]^{1/2}dx = \frac{2}{h}(2mA^{1/2}) - \frac{1}{2}$$

II



$$n = \frac{1}{2}\left[\frac{2}{h}(2mA)^{1/2} - \frac{1}{2}\right.$$



B. Which potential supports more bound energy levels at E ≤ A?

Potential #I

C. For the V(*x*) = ∞ for |*x*| > L/2 potentials in this example, does the WKB quantization integral over-estimate or under-estimate the true number of bound levels at E ≤ A? Suggest a reason in support of your answer.

There are no exponential tails in the non-classical region. WKB quantization assumes that these tails exist, resulting in a lowering of all energies, which accounts for the -1/2 in the quantization condition. There are fewer levels in ⬆⬆ than in the WKB prediction. WKB over-estimates the number of bound levels.

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