Name

5.73

Quiz 6 ANSWERS

Harmonic Oscillator:

$V(x)=kx^2/2$

$E = T + V$

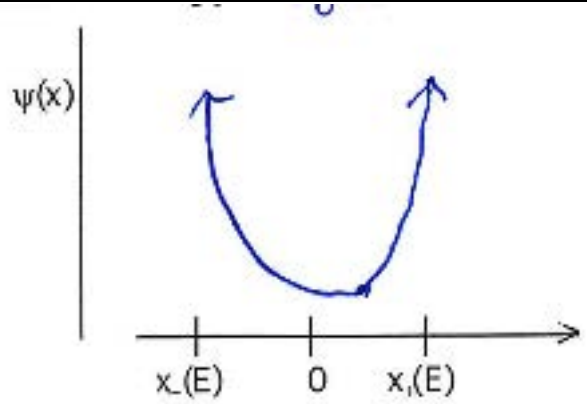
$T = p^2/2m$

$p_E(x) = \left[2m\left(E - V(x)\right)\right]^{1/2}$ classical mechanical momentum

$x_{\pm}(E) = \pm\left[2E/k\right]^{1/2}$ turning points

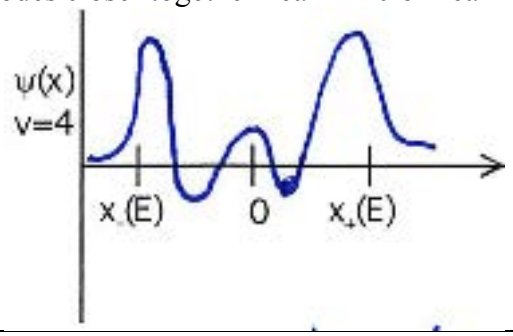
$\omega=\left[k/m\right]^{1/2}$

A. Draw a cartoon of the classical mechanical wavefunction, $\psi(x)$, where $P(x) = |\psi(x)|^2$. Recall that the classical probability $P(x) \propto 1/v(x)$. Pay special attention to $\psi(x)$ at the two turning points, $x_{\pm}(E)$, and at $x = 0$.



(*continued other side*)

B. Convert your classical mechanical cartoon from part A to a qualitatively correct quantum mechanical cartoon for the v = 4 eigenstate. Use deBroglie's equation, $\lambda(x) = h/p(x)$, generalized to allow λ and p to be functions of x. How many nodes? Are the nodes closer together near x = 0 or near $x = x_{\pm}(E)$?



v = 4: 4 nodes, nodes closest together near *x* = 0.

C. Make *extremely crude approximations* to estimate the fraction of time an oscillator at energy $E_n = (n+1/2)\hbar\omega$ can be found between the two center-most nodes. The period of a harmonic oscillator is $\tau = \frac{2\pi}{\omega}$, the node spacing is $\lambda = \frac{h}{p_E}$, and the velocity is $p_E(0)/m$.

$$\lambda(0) = \frac{h}{p(0)}$$

$$p(0) = \left[\frac{9}{2}\hbar\omega 2m\right]^{1/2}$$

$$T = \frac{1}{v} = \frac{2\pi}{\omega}$$

$$\delta t = \lambda(0)/v(0) = \frac{h/p(0)}{p(0)/m} = hm/p_0^2$$

Fraction of time

$$\frac{\delta t}{T/2} = \frac{hm/p_0^2}{\pi/\omega} = \frac{h\omega m}{\pi p_0^2}$$

$$= \frac{h\omega m}{\pi\frac{9}{2}\hbar\omega 2m}$$

$$= \frac{2}{9}$$

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