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Quiz 35 $\boxed{\text{ANSWERS}}$

For p^3 configuration:

The $M_L = 0, M_S = 1/2$ block contains $\|1\alpha 0\alpha -1\beta\|, \|1\alpha 0\beta -1\alpha\|$, and $\|1\beta 0\alpha -1\alpha\|$. For $M_L = 0$, $\mathbf{L}^2$ may be replaced by $\mathbf{L}_+\mathbf{L}_-$.

$$\mathbf{L}^2\|1\alpha 0\alpha -1\beta\| = \hbar^2\left[2\|1\alpha 0\alpha -1\beta\| - 2\|1\alpha 0\beta -1\alpha\|\right]$$
$$\mathbf{L}^2\|1\alpha 0\beta -1\alpha\| = \hbar^2\left[4\|1\alpha 0\beta -1\alpha\| - 2\|1\beta 0\alpha -1\alpha\| - 2\|1\alpha 0\alpha -1\beta\|\right]$$
$$\mathbf{L}^2\|1\beta 0\alpha -1\alpha\| = \hbar^2\left[2\|1\beta 0\alpha -1\alpha\| - 2\|1\alpha 0\beta -1\alpha\|\right]$$

A. Set up the $\mathbf{L}^2$ matrix for the $M_L = 0$, $M_S = 1/2$ block.

Row Label $\mathbf{L}^2$ matrix

$$\begin{array}{c} \|1\alpha 0\alpha -1\beta\| \\ \|1\alpha 0\beta -1\alpha\| \\ \|1\beta 0\alpha -1\alpha\| \end{array} \quad \hbar^2\begin{pmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{pmatrix}$$

B. Find the normalized eigenvector of $\mathbf{L}^2$ that corresponds to $\left|{}^2D\ M_L = 0, M_S = 1/2\right\rangle$

$$\left(\mathbf{L}^2\right)\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \hbar^2 6\begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad 1 = \left[|a|^2 + |b|^2 + |c|^2\right]^{1/2}$$

$$\hbar^2\underbrace{\begin{pmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{pmatrix}}_{\mathbf{L}^2}\overbrace{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}^{\text{eigenvector}} = \overbrace{\hbar^2 6}^{\text{eigenvalue}}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$L = 2$ $\mathbf{L}^2 = \hbar^2 2\cdot 3 = \hbar^2 6$

$2a - 2b = 6a$ $-2b = 4a$ $b = -2a$

$-2a + 4b - 2c = 6b$ not needed

$-2b + 2c = 6c$ $-2b = 4c$ $b = -2c$

a = c

$[a^2 + b^2 + c^2]^{1/2} = 1$

$[a^2 + 4a^2 + a^2]^{1/2} = 1$ $[6a^2]^{1/2} = 1$ $a = 6^{-1/2}$

Thus: $a = 6^{-1/2}$, $b = -2\cdot 6^{-1/2}$, $c = 6^{-1/2}$

Verify:

$$\hbar^2\begin{pmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{pmatrix} 6^{-1/2}\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \hbar^2 6^{-1/2}\begin{pmatrix} 6 \\ -12 \\ 6 \end{pmatrix}$$

$$= 6\hbar^2 6^{-1/2}\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$