

5.73

Quiz 28 ANSWERS

1.

Bohr radius	$r_n = a_0 n^2$
Rydberg Equation	$E_n = -\frac{\mathfrak{R}}{n^2}$
Amplitude of wavefunction inside core: $\psi_n(r) \propto n^{-3/2}$ (innermost lobe)	
$E_{n+\delta/2} - E_{n-\delta/2} = \frac{2\delta\mathfrak{R}}{n^3}$	

- A. How does the transition probability (proportional to $|\langle 1s|r|np \rangle|^2$) for the $1s \rightarrow np$ Rydberg series scale with n ? [HINT: $1s$ is *entirely* inside the core; only the innermost lobe of np is inside the core.]

$$\langle 1s|r|np \rangle \propto n^{-3/2}$$

$$|\langle 1s|r|np \rangle|^2 \propto n^{-3}$$

- B. The transition probability density (proportional to $|\langle 1s|r|np \rangle|^2 / [E_{n+1/2} - E_{n-1/2}]$) is supposed to be constant as $n \rightarrow \infty$ and beyond into the ionization continuum. Show that this is true. [HINT: Use the n -scaling of the numerator from Part A; use the n -scaling of the denominator from the box above.]

$$E_{n+1/2} - E_{n-1/2} = \frac{2\mathfrak{R}}{n^3} \text{ because } \delta = 1$$

$$\frac{|\langle 1s|r|np \rangle|^2}{E_{n+1/2} - E_{n-1/2}} \propto \frac{n^{-3}}{\frac{2\mathfrak{R}}{n^3}} = \frac{1}{2\mathfrak{R}} \text{ independent of } n$$

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