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Quiz 25

1.

$$\mathbf{L}\cdot\mathbf{S}=\tfrac{1}{2}\left[\mathbf{J}^2-\mathbf{L}^2-\mathbf{S}^2\right]$$
$$\mathbf{L}\cdot\mathbf{S}=\mathbf{L}_z\mathbf{S}_z+\tfrac{1}{2}\left[\mathbf{L}_+\mathbf{S}_-+\mathbf{L}_-\mathbf{S}_+\right]$$
$$\mathbf{H}^{\mathrm{SO}}=\zeta\mathbf{L}\bullet\mathbf{S}$$
$$\mathbf{H}^{\mathrm{Zeeman}}=-\gamma\beta_z\left(\mathbf{L}_z+2\mathbf{S}_z\right)$$

A. For a $^2F_{5/2}$ state:

(i) What is **S**?

(ii) What is **L**?

(iii) What is **J**?

(iv) Evaluate $\left\langle {}^2F_{5/2},M_J\left|\mathbf{H}^{\mathrm{SO}}\right|{}^2F_{5/2},M_J\right\rangle$.

(v) Evaluate $\left\langle {}^2F_{7/2},M_J=7/2\left|\mathbf{H}^{\mathrm{Zeeman}}\right|{}^2F_{7/2},M_J=7/2\right\rangle$.

B. Apply $\mathbf{J}^- = \mathbf{L}^- + \mathbf{S}^-$ to both sides of $\left|{}^2F_{7/2}, M_J = 7/2\right\rangle = \left|{}^2F, M_L = 3, M_S = 1/2\right\rangle$ where the two basis states are the "extreme" states, respectively in the coupled and uncoupled basis sets:

$$\mathbf{J}^-\left|{}^2F_{7/2}, M_J = 7/2\right\rangle = \left(\mathbf{L}^- + \mathbf{S}^-\right)\left|{}^2F, M_L = 3, M_S = 1/2\right\rangle.$$

Find the normalized combination of $\left|{}^2F, M_L = 2, M_S = 1/2\right\rangle$ and $\left|{}^2F, M_L = 3, M_S = -1/2\right\rangle$ that corresponds to $\left|{}^2F_{7/2}, M_J = 5/2\right\rangle$.

C. Verify that $|{}^2F_{5/2}, M_J{=}5/2\rangle = -7^{-1/2}|{}^2F, M_L = 2, M_S = 1/2\rangle + (6/7)^{1/2}|{}^2F, M_L = 3, M_S = -1/2\rangle$ is orthogonal to $\left|{}^2F_{7/2}, M_J = 5/2\right\rangle$ which you obtained in part B.

D. Evaluate $\left\langle {}^2F_{5/2}, M_J = 5/2\right|\mathbf{H}^{\text{Zeeman}}\left|{}^2F_{5/2}, M_J = 5/2\right\rangle$.

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